Populus model: Lotka-Volterra competition

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1 + \alpha N_2}{K_1} \right)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2 + \beta N_1}{K_2} \right)$$

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N_1	population density of species 1
N_2	population density of species 2
$\frac{dN_1}{dt}$	the slope of N_1 versus t (time)
r_1	per-capita growth rate of species 1 without competition
K_1	carrying capacity of species 1 (smaller K means more **intra**specific competition)
α	the effect of species 2 on the per-capita growth of species 1
	(larger α means more **inter**specific competition)
β	the effect of species 1 on the per-capita growth of species 2

If there is no species 2 ($N_2 = 0$), then equation 1 just becomes logistic (density dependent) population growth.

The Populus defaults are set up to allow coexistence of the two species (a stable equilibrium):
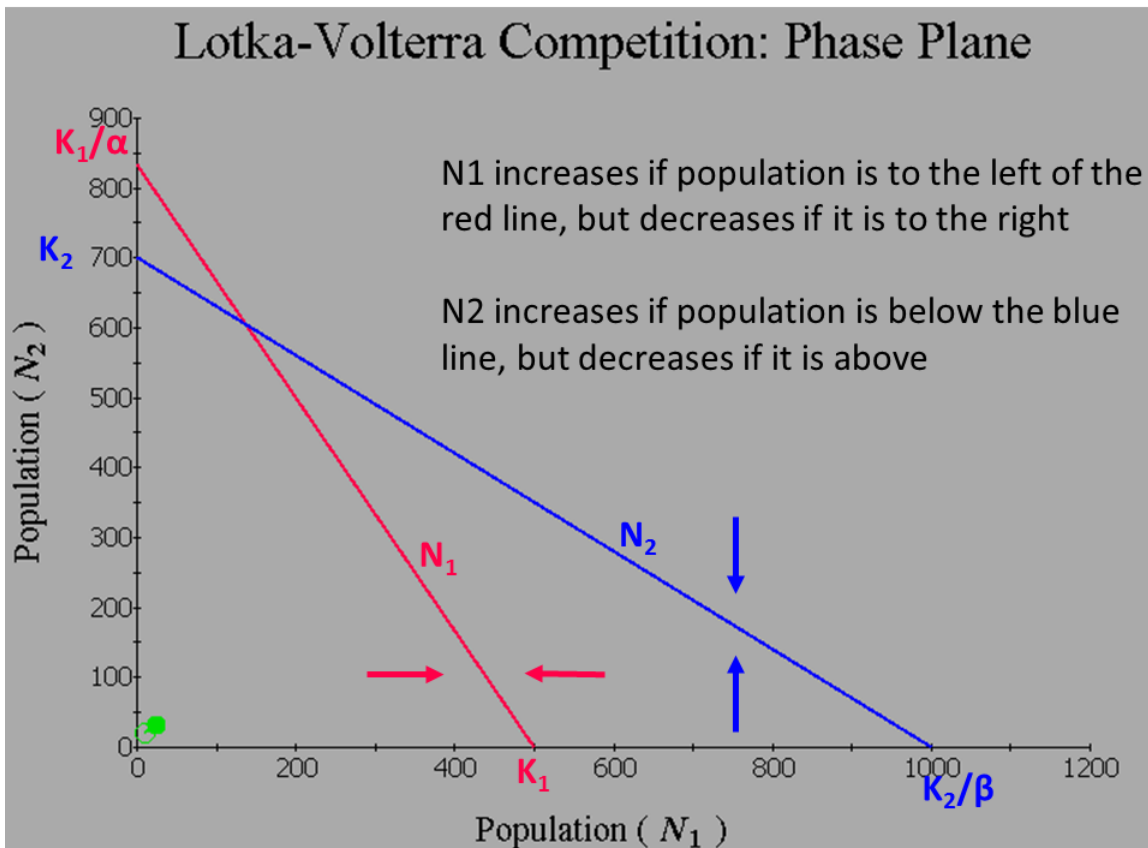

Note that you can only cross the blue line horizontally (because when you hit it vertically, the blue population N_2 stops changing). Likewise, the red line can only be crossed vertically (N_1 is not changing as you move vertically but N_2 is).

I set the time to 1 in that example so it is easier to see the isoclines. If you add the trajectory (in green), and run to a stable state, it looks like this:
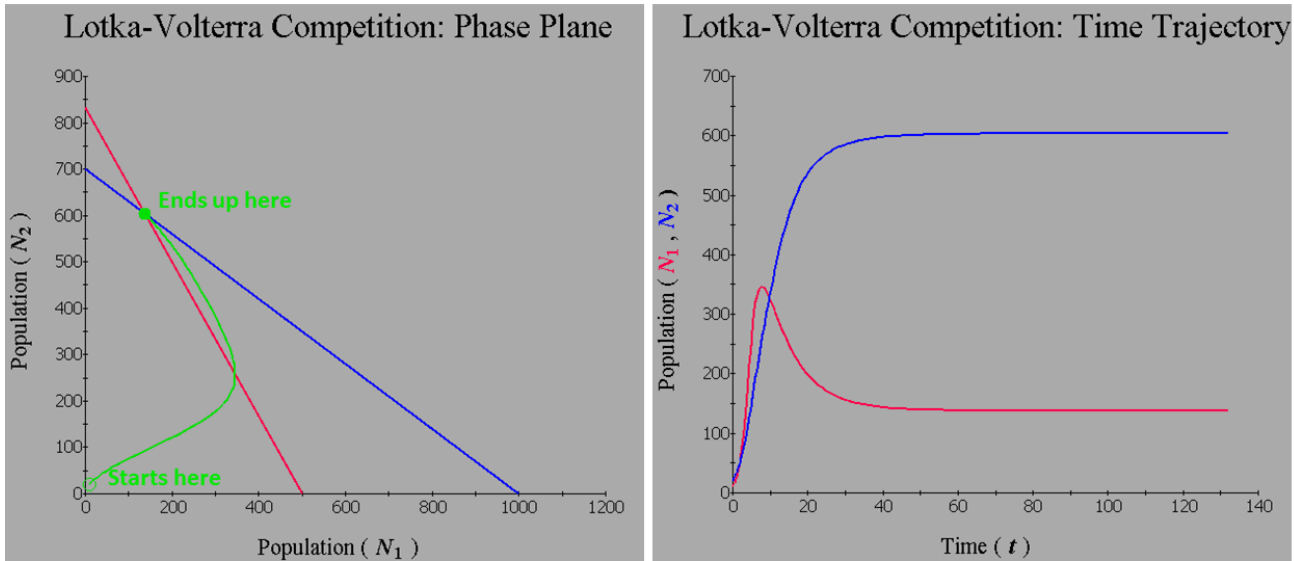
If the isoclines cross in the opposite direction, the outcome of competition depends on the starting population sizes. Changing the values in Populus so that $K_1 = 300$ and $\beta = 3$ will give you that case (all other parameters stay the same).
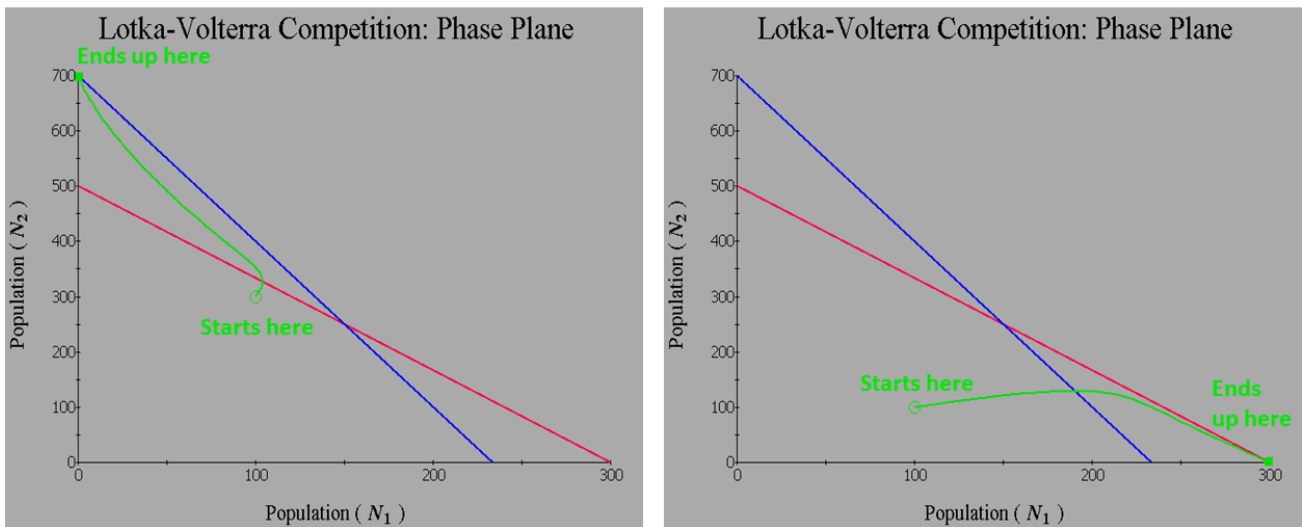
To get coexistence, we need a species to have a greater effect on the other species than it has on itself. In other words, we need:

$K_1 < K_1/\alpha$ and/or $K_2 < K_2/\beta$

We also need them to cross (so that there is an equilibrium where they intersect, and not just where one or the other is zero). For this to occur we need either

$K_1 < K_2/\beta$ and $K_2 < K_1/\alpha$ OR $K_1 > K_2/\beta$ and $K_2 > K_1/\alpha$