How I Abandoned the Textbook: A Primary Source Driven Modeling Course

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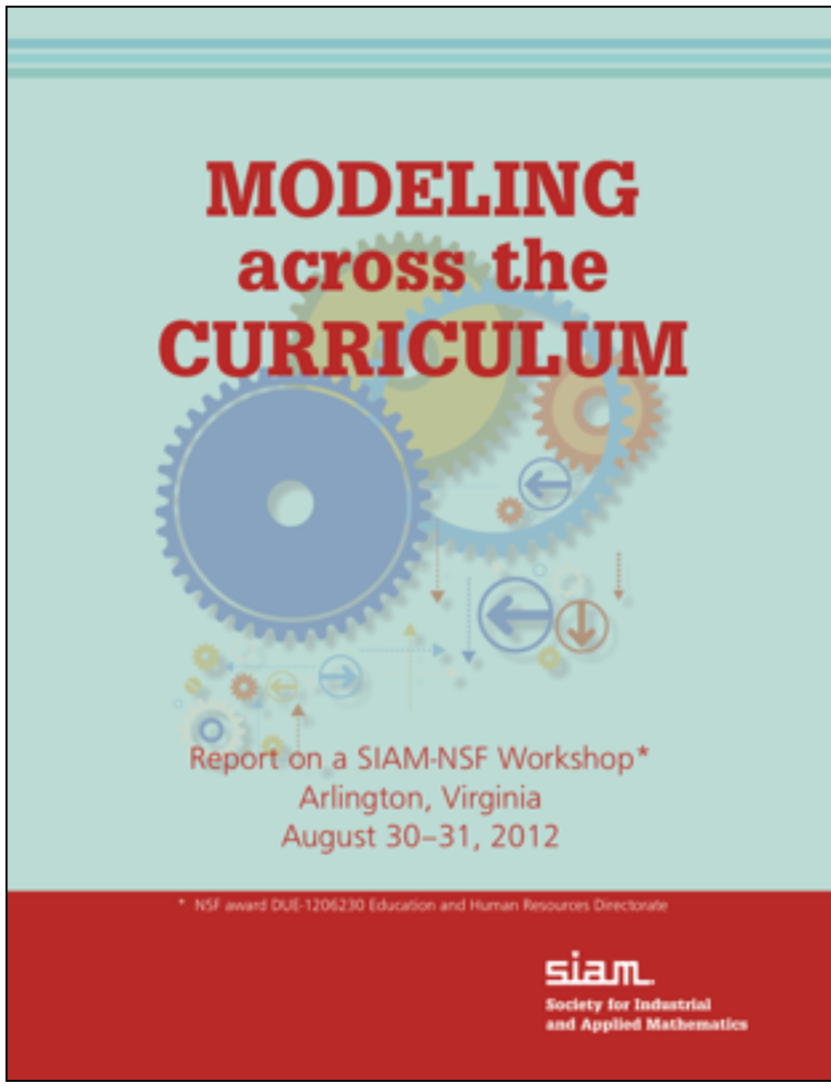
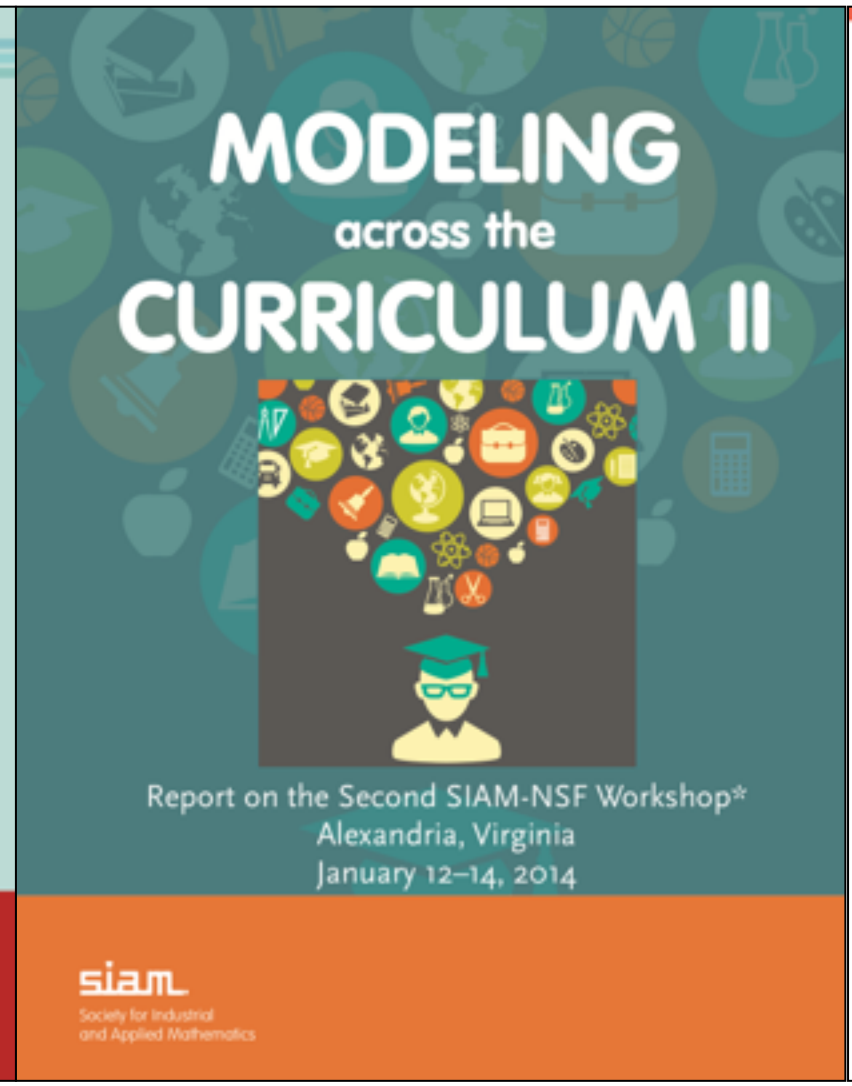
Why are we talking about modeling?





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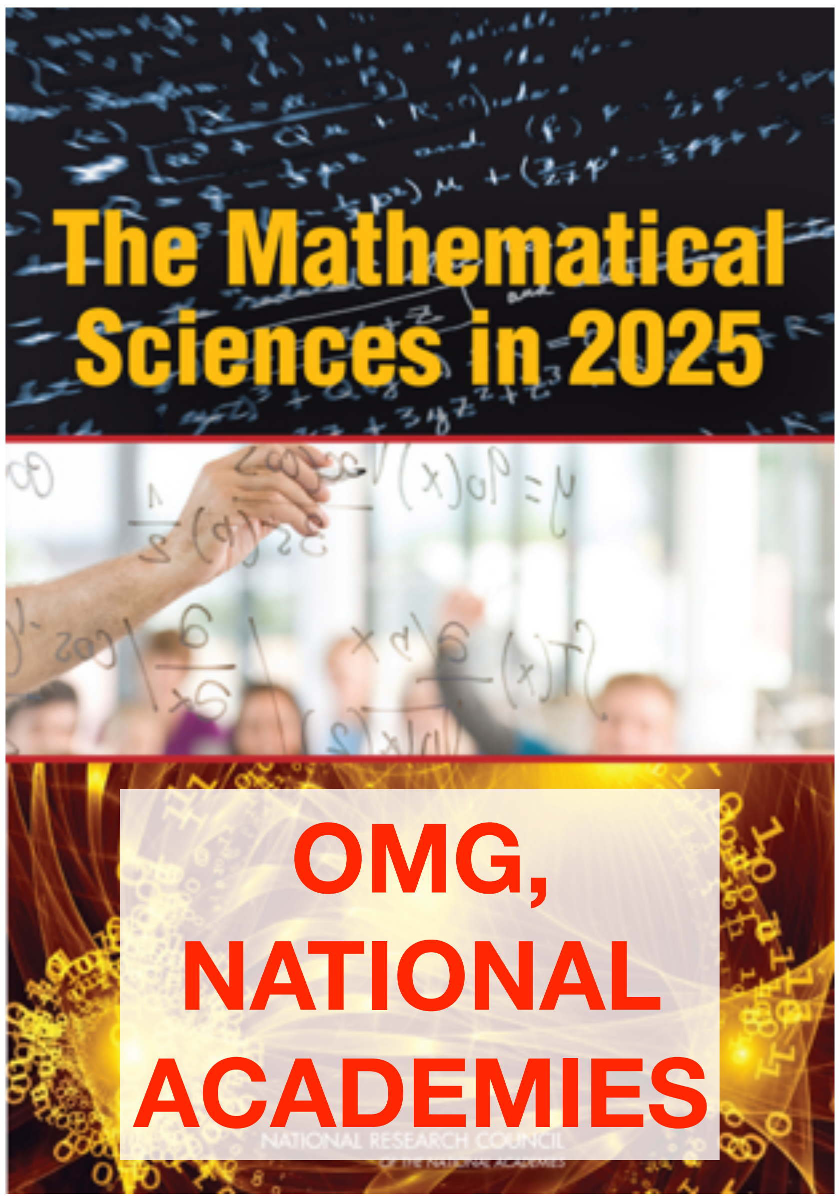

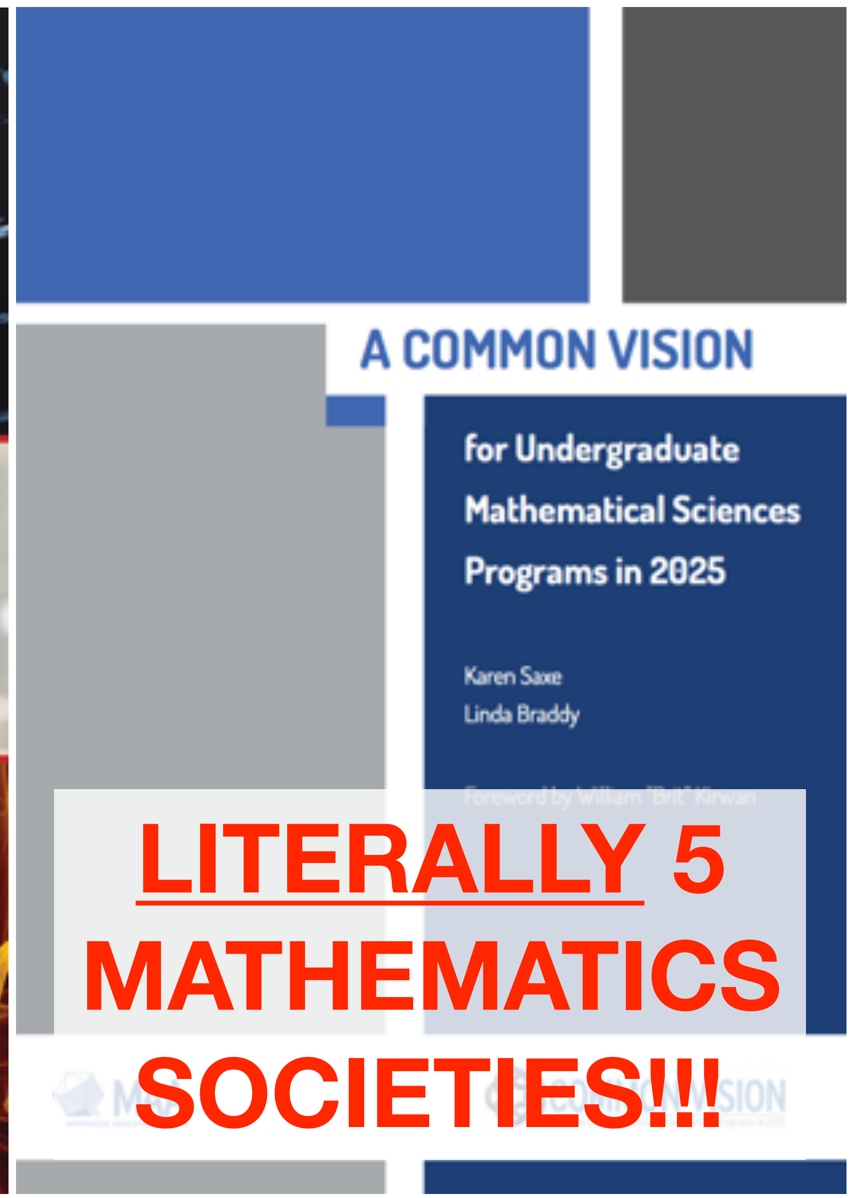

"Many areas of science, engineering, and industry are now concerned with building and evaluating mathematical models, exploring them computationally, and analyzing enormous amounts of observed and computed data."

"Virtual experimentation is replacing many aspects of real-world implementation, and the demand for modelers is rapidly increasing."

What is modeling, anyway?

data, first principles, …

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

mathematics, computation…

Course goals and description. Mathematical modeling means (1) translating a real-life problem into a mathematical object, and (2) studying that object using mathematical techniques, and (3) interpreting the results in order to learn something about the real-life problem. Mathematical modeling is used in biology, economics, chemistry, geology, sociology, political science, art, and countless other fields.

What is modeling, anyway?

data, first principles, …

difficult, messy, non-algorithmic, underemphasized

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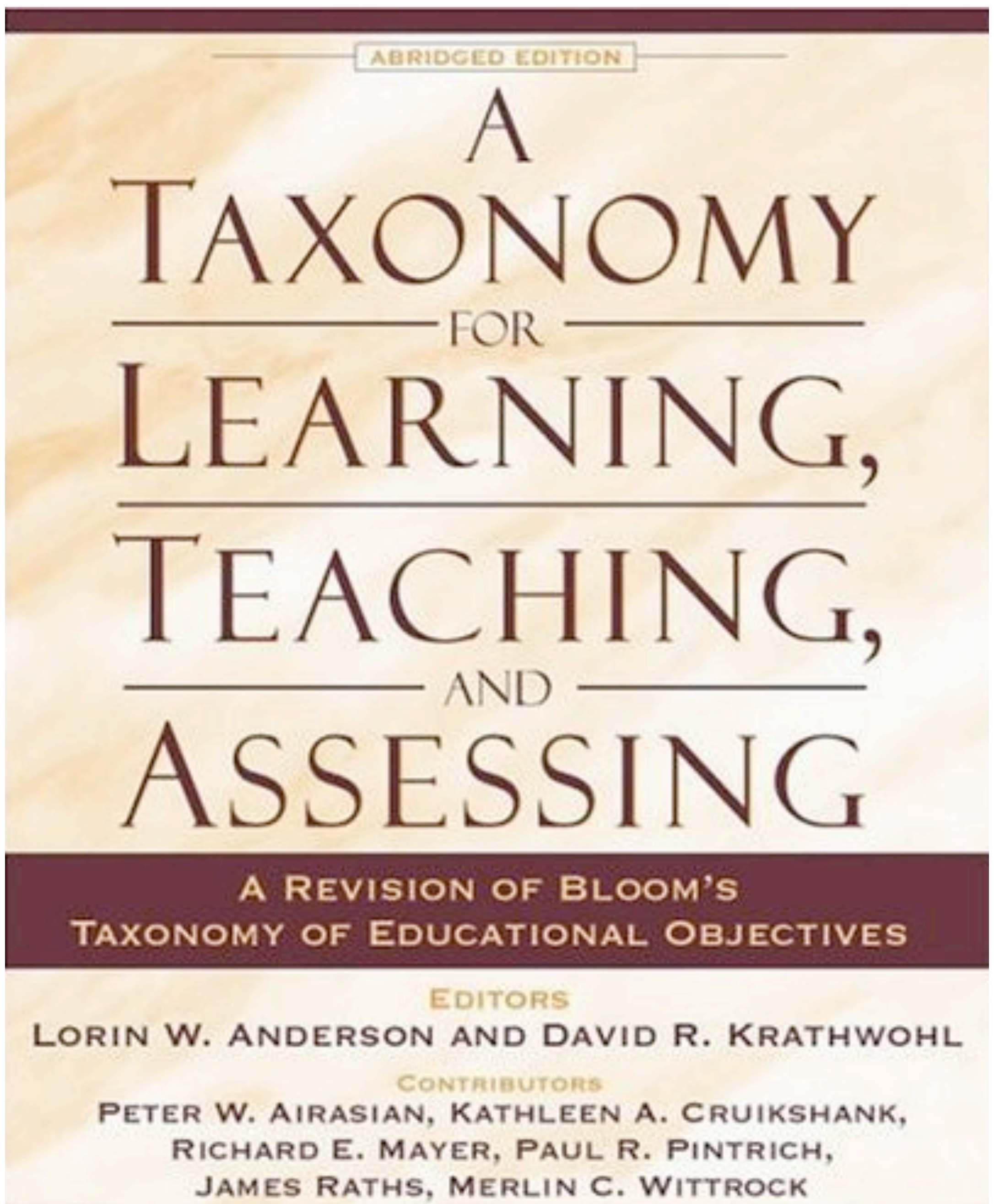
Curriculum: Think about your learning goals.



Students will be able to verb noun.

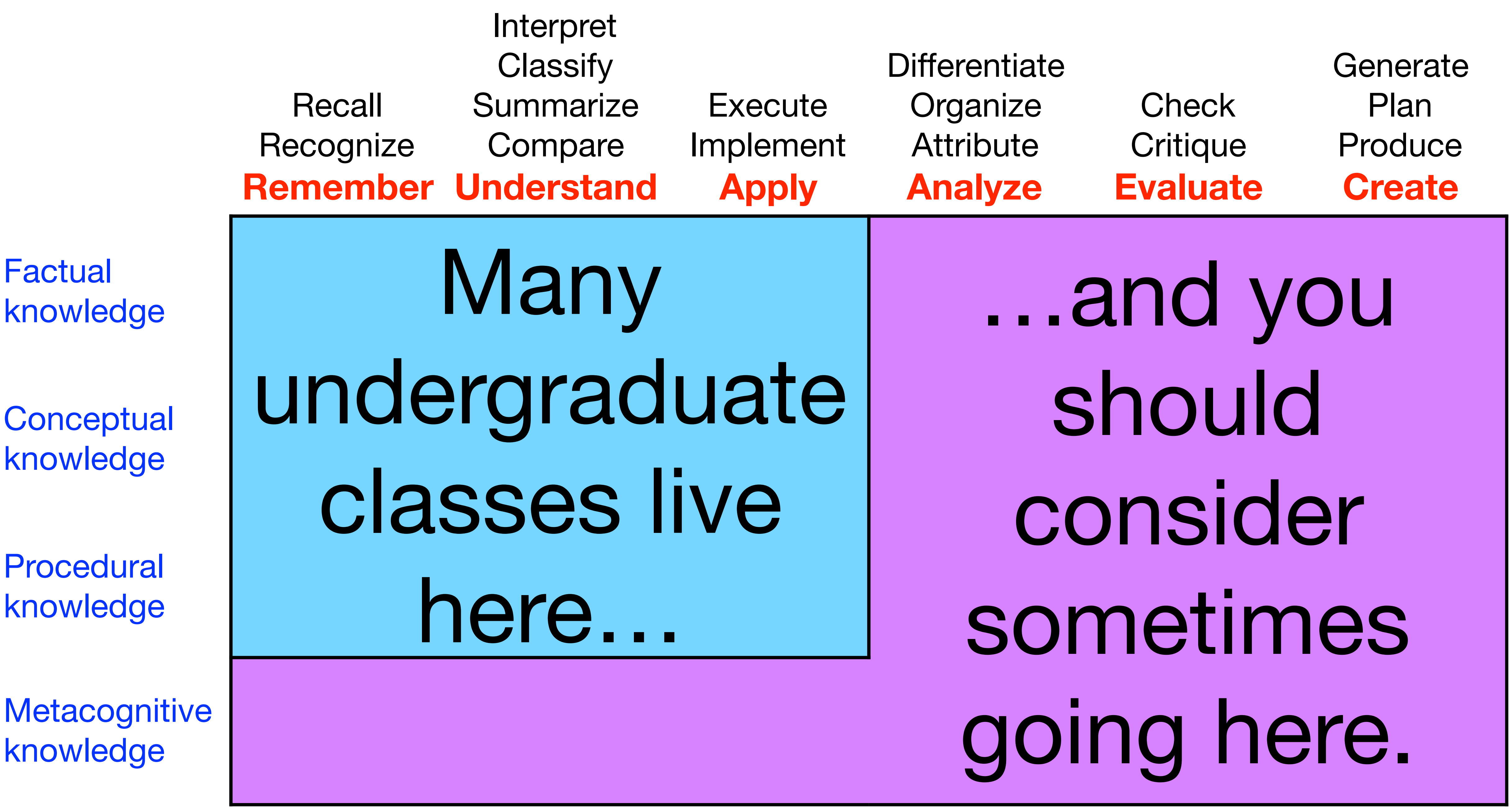
cognitive process → verb

knowledge type → noun

Curriculum: Think about your learning goals.

	Recall Recognize **Remember**	Interpret Classify Summarize Compare **Understand**	Execute Implement **Apply**	Differentiate Organize Attribute **Analyze**	Check Critique **Evaluate**	Generate Plan Produce **Create**
Factual knowledge						
Conceptual knowledge						
Procedural knowledge						
Metacognitive knowledge						

Curriculum: Think about your learning goals.



Pedagogy: My philosophy, broadly.

Common conception of college learning

Reality of successful college learning

Institutional/Course Context

- Macalester College, St. Paul, MN
- 2000 Students
- Department of Mathematics, Statistics, and Computer Science
- 3 majors, 4 minors
- > 200 declared majors, > 80 graduates per/year, teach 1/8 of college enrollments
- Math 432: Mathematical Modeling
- **Capstone course, 15 - 30 students**
- **Requires programming & (differential equations | computational linear algebra)**

Course goals (note absence of specific math content)

By the end of this course, you should be able to:

- **Articulate** a philosophy of mathematical modeling
- **Recognize** problems susceptible to mathematical modeling
- **Recognize** and **implement** different types of mathematical models
- **Be comfortable** with open-ended scientific work
- **Read** and **synthesize** applied mathematical literature
- **Communicate** applied mathematics clearly, precisely, and appropriately

Course structure (syllabus is here)

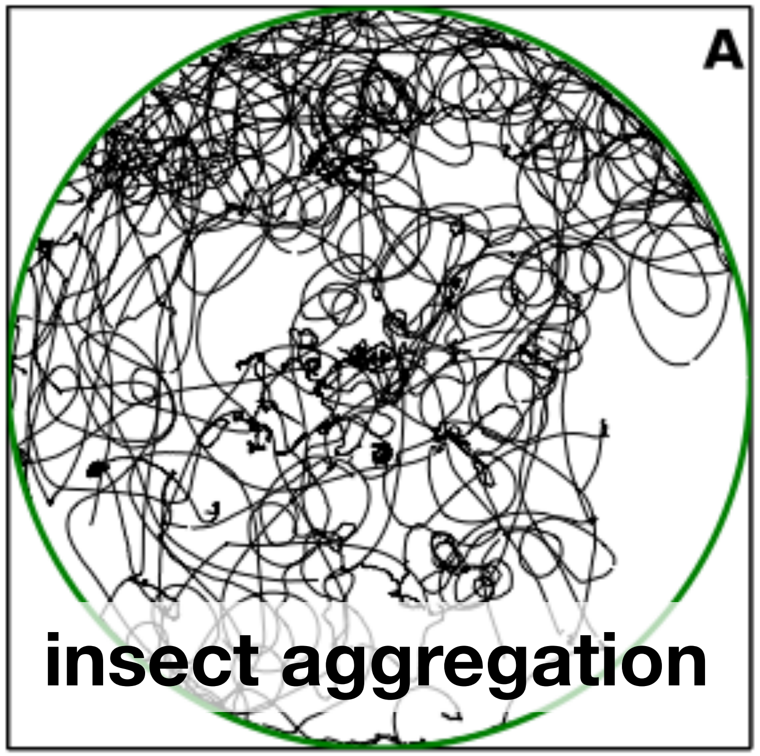
What	How	Groups	Presentation	When
Look at examples of modeling	Student-run journal club	Randomly assigned	Live	8 hrs First 1/2 of term
Model with guidance	Structured mini-modeling exercises	Randomly assigned	Recorded	9 hrs First 1/2 of term
Attack open-ended problems	Research	Student selected	Live	21 hrs Second 1/2 of term

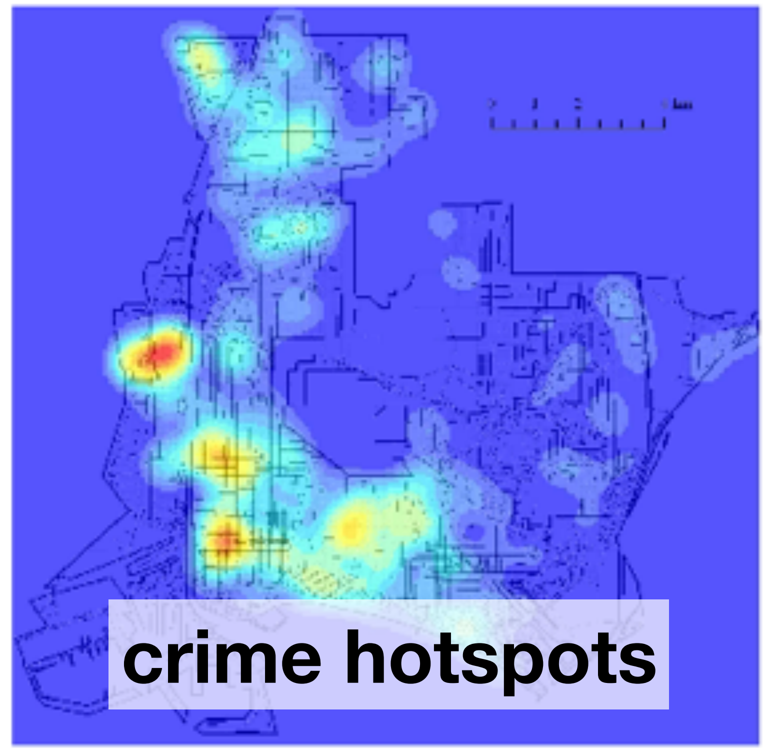
Journal Club (examples are here)

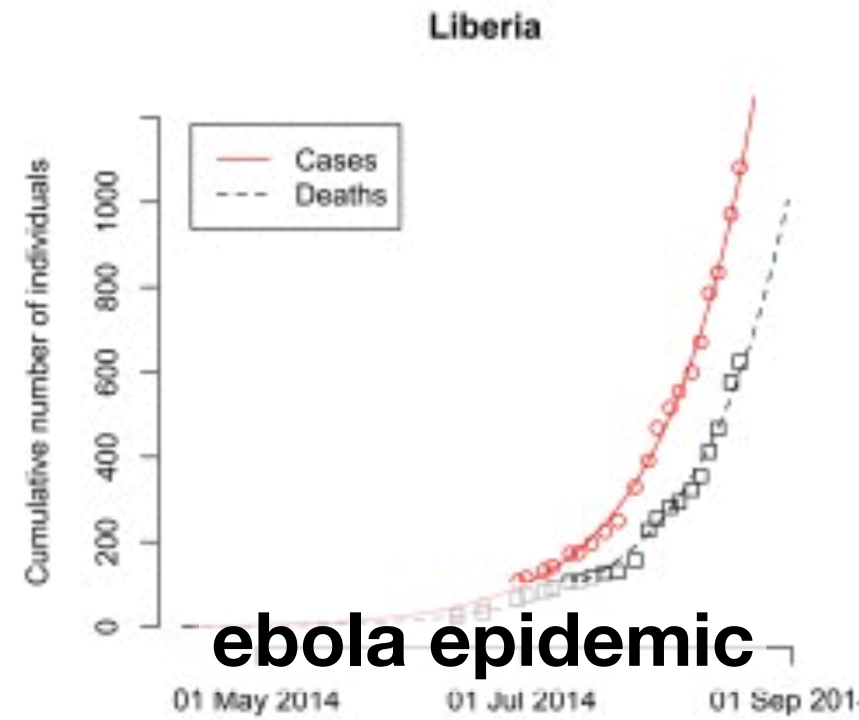
When	What
Before class	Everyone reads, reflects, writes. Facilitators plan class.
During class	Facilitators run discussion and learning activities.
After class	Instructor gives feedback to facilitators. Facilitators write advice for successors.

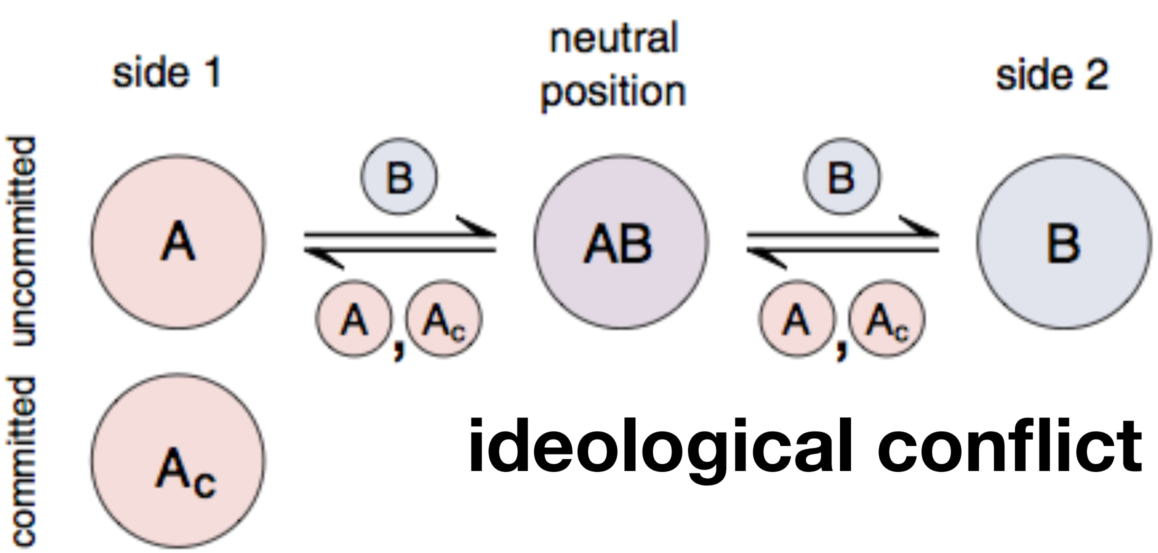
1. What system is being modeled? Explain it as thoroughly as possible in straightforward terms.
2. What is the motivation, and/or what can be gained by studying this system?
3. What previous work is this study built on (references and main points/results)?
4. Of what general type is the model? Classify it. What mathematical objects are used?
5. What assumptions are made in constructing the model? What's included/excluded? Is the model as minimal as it might be? Too minimal?
6. What mathematical techniques are used to study the model? Don't just state -- explain the most important mathematical steps.
7. What are the main results? What is learned about the system?
8. What questions remain? Has follow-up work been done, and if so, what is it (references and main points)?
9. What reflections do you have on this paper?
10. What questions do you have about this paper?

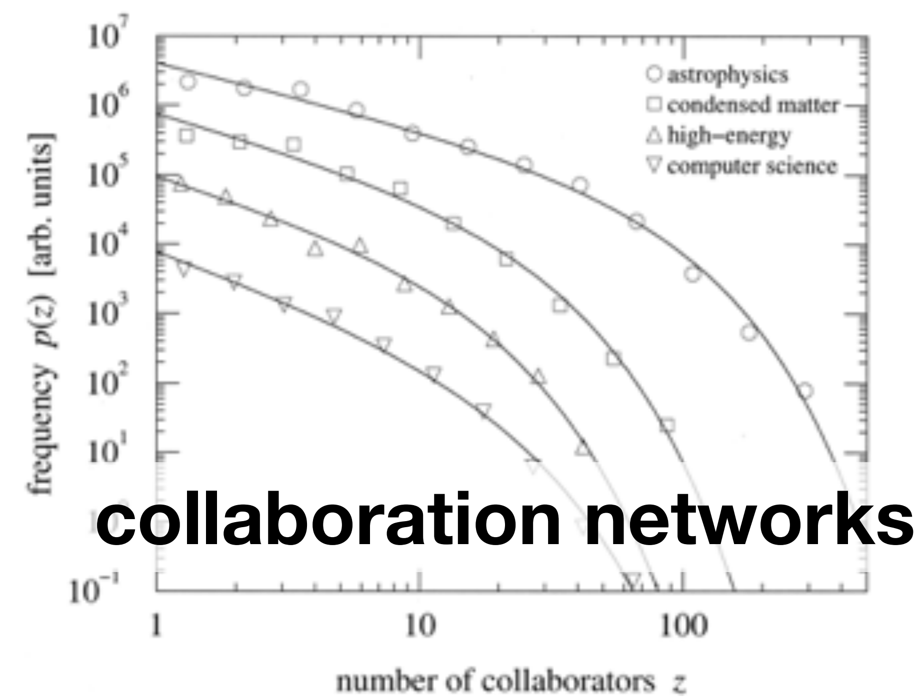
Journal Club (examples are here)







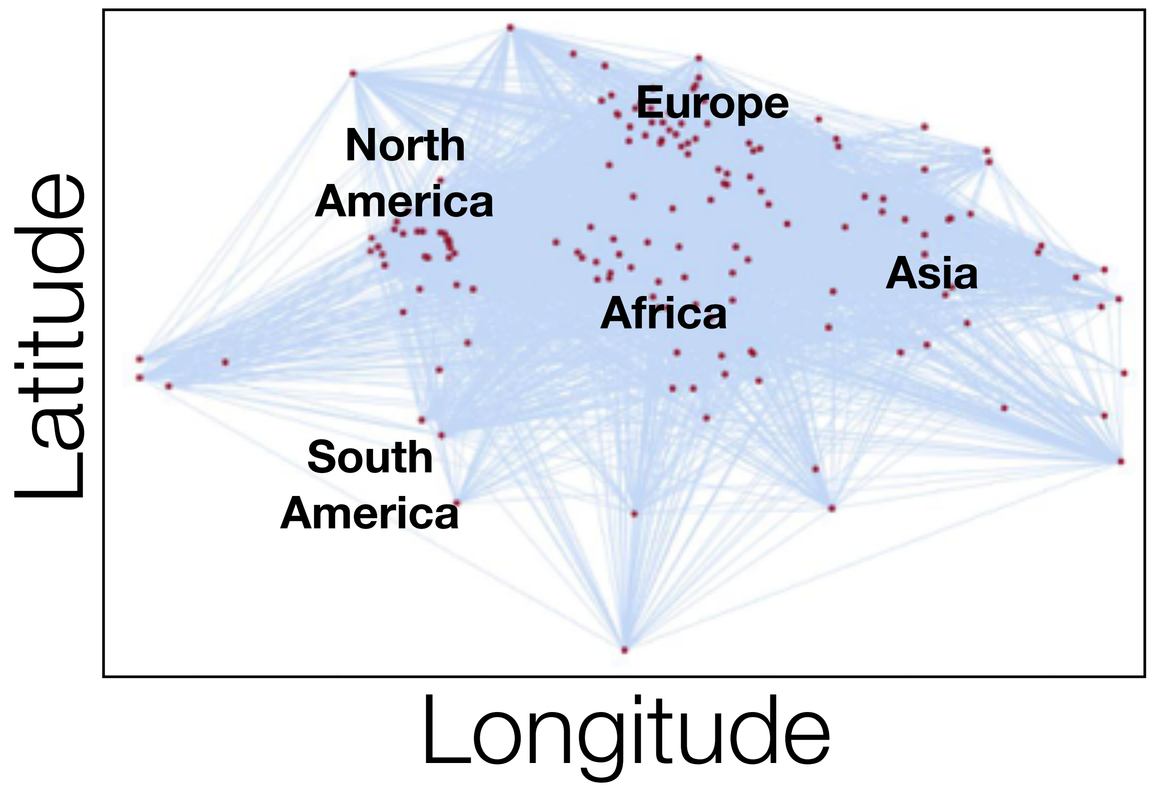




Mini-Modeling (examples are here)

When	What
Before class	Students read exercise.
During class	Students work on exercise.
After class	Finish exercise; produce presentation; receive written feedback

Network of
Free Trade Agreements



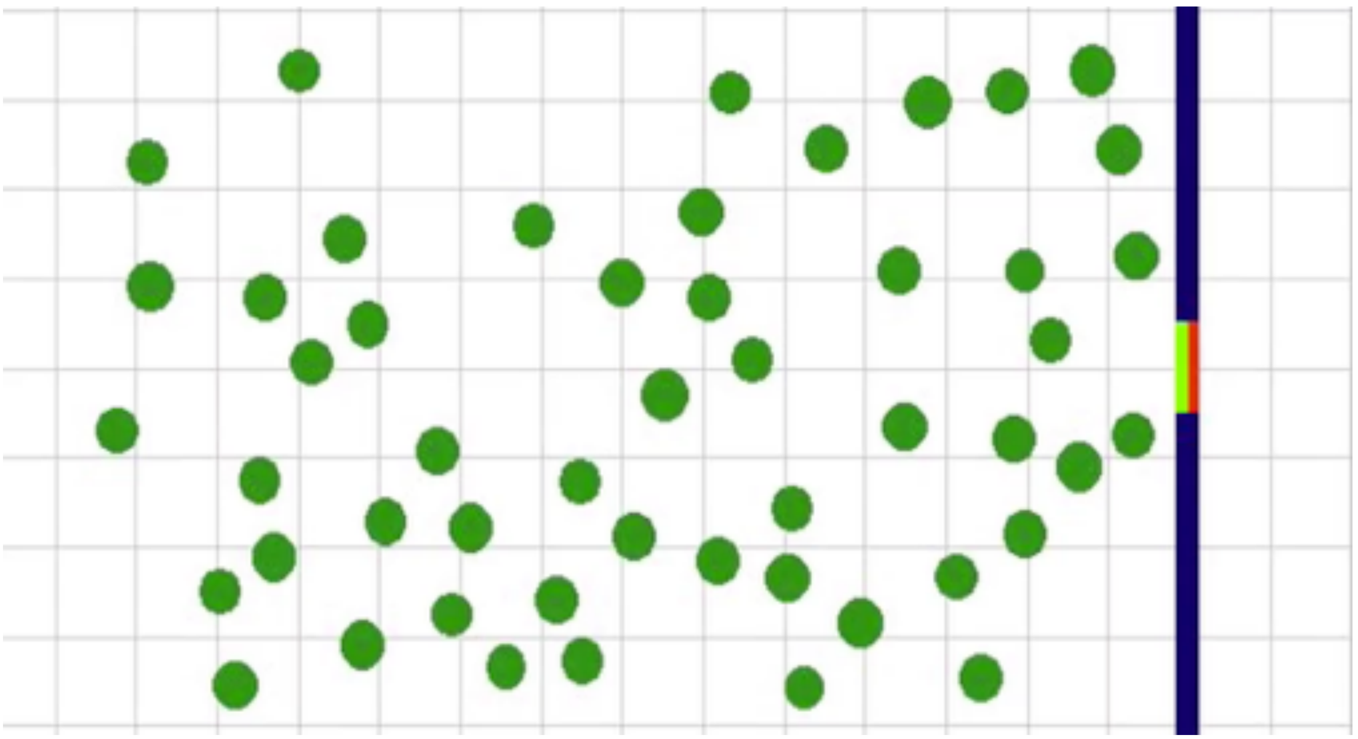
Assumptions:

1. Substitute each trade organizations with its member countries
2. Countries in same trade organization are connected
3. All trade agreements are bilateral
4. All free trade agreements treated equally

Research Project

When	What
1/3 of way through term	Students propose projects/groups. Instructor responds. Students revise.
Second half of term	Students work in and out of class on research project.
Last two days of class	Students present their projects live. Peers and instructor give feedback.

- Modeling city expansion as network growth
- Dynamics of crime and recidivism
- How do crowds become dangerous? Modeling human crowd disasters subject to social contagion
- Resilience of random and small-world networks
- A simple surf wave model



Surfable ocean waves

Open ocean wave length
Open ocean wave height
Bathymetry

Assumptions
Computational model

Peel angle
Iribarren number

Rincon Beach, CA
Wave length 155 ft. (Google Maps)
Wave height 8 ft. (picture of truck)
Reef slope (crude USGS data)

Peel angle 53° (actual 54°, Google Maps)
Iribarren number 0.69 (plunging, consistent)

Some metrics

According to student responses, by the end of the course,

- 84% of the assignments in this course involved collaboration
- 77% of the hours spent working on this course were collaborative
- Each student presented 11 times
- 87% of students view the world more quantitatively
- 100% of students find mathematical modeling more useful
- 100% of students feel more comfortable with open-ended problems

The End