**A Direct Experience Study of Resource-Limited Population Growth**

**Materials:**

Paper with a checkerboard printed on it

4 6-sided dice

**Overview:**

Each of the 64 squares on the checkerboard represents a territory that can be occupied by a member of a population. Four squares are initially “occupied,” and the 8 squares that are adjacent to an occupied square are “available.” The experiment consists of a number of time steps in which the population spreads from occupied squares into some of the available squares (based on die rolls). Occupied squares remain occupied for the whole experiment, while available squares remain available until eventually becoming occupied. The data for the experiment is a list of population counts at the end of each time step. This data is used to prepare various graphs to be used as guides for developing verbal and symbolic representations of a resource-limited population growth model.

**Background:**

Either do an actual experiment of growing bacteria or other microorganisms in a culture medium or watch https://youtu.be/hbNp9DBbTkU.

**Experiment 1 / 2 Setup:**

Mark the four center squares with an “X” to indicate them as occupied, and record the initial population as 4. Starting with the central squares simulates a bacterial population seeded into a nutrient environment. Alternatively, one could start with the four corners occupied to simulate spread of an invasive species.

**Experiment 1 progress:**

Each turn proceeds with the following sequence of steps:

1. Use a large dot to mark all unoccupied squares that share a side with an occupied square. (There will be eight such “available” squares in the first turn.)

*Example: Suppose squares a1, a2, b1, and c1 are occupied (using algebraic chess notation). Then a3, b2, c2, and d1 are available.*

1. Determine newly occupied squares:
   1. For each available square, roll one die for each adjacent occupied square; Mark the available square with a slash (“/”) if any of the rolls are 5 or 6.
   2. After testing all available squares, turn the slashes into “X”s to mark as occupied.

*Example: With a1, a2, b1, and c1 occupied, you roll 1 die for a3, 2 for b2, 1 for c2, and 1 for d1. Say you roll 2, 3&5, 4, and 5. As you see the die rolls, you record the results by marking b2 and d1 with slashes while leaving a3 and c2 with dots. After all die rolls, you change the two slashes into “X”s. Note that the slash placed in b2 does not change the number of dice you roll for c2 in this turn. It will count as occupied for the next turn.*

1. Record the population of occupied squares and the increase in population for the current time step.

Stop when nearly all squares are occupied.

**Experiment 2:** Repeat experiment 1, but with a 4, 5, or 6 required for population growth.

**Questions About the Physical Simulation:**

1. The number of occupied squares increases over time. How does the population increase change over time? (Look for trends that persist over several steps rather than any details you might see in one time step.)
2. If you see a board in the middle of the experiment, can you predict if the next population increase will be relatively large or relatively small? How can you tell?
3. Suggest explanations for any persistent increases or decreases in the population increases.
4. Of course the “population” in this experiment is not a real biological population, but the simulation is designed to be as realistic as possible.
   1. Why are some squares unavailable?
   2. Why is it more realistic to roll 2 dice for an available square with 2 occupied neighboring squares, as opposed to rolling just one die?
   3. What assumptions are built into the scenario that are not as realistic?

**Data Analysis:**

Prepare 4 graphs for each experiment, being careful to label the x and y axes for each graph and to identify each as for success probability 1/3 (experiment 1) or 1/2 (experiment 2):

1. Plot population (y) versus time (x).
2. Plot population increase (y) versus time (x).
3. Plot population increase (y) versus population at start of turn (x).
4. Plot relative population increase (population increase / population at start of turn) (y) versus population at start of turn (x).

**Computer Simulations in NetLogo and R:**

While it is important to observe the physical simulation several times, it is cumbersome to collect enough data to represent the model well. This issue can be addressed by an automated version of the experiment. The current versions are PopGrowth2.R, written and maintained by Glenn Ledder, and LOGGROWTH.nlogo, written and maintained by Drew LaMar. Each program produces the four graphs. Maximum flexibility is achieved through the experimenter’s control of parameters. Note that LOGGROWTH can be run by downloading NetLogo version 6.0.1 from <https://ccl.northwestern.edu/netlogo/download.shtml> or using the web server at <http://www.netlogoweb.org/launch>. The screen is better with 6.0.1.

**NetLogo Simulation Details:**

* Sliders allow the experimenter to set the arena size and birth probability.
* A button determines whether the initial population is in the four center squares or the four corners.

**R Simulation details:**

The parameters are defined at the top of the program:

* s is the size of the experiment arena. This should be in the range 8-60, with a default of 50. A larger size gives a longer period of accelerating population growth.
* b is the birth probability. This should be in the range 0.1-0.8, with a default of 0.5.
* d is the death probability. This should be in the range 0-b/4, with a default of 0 (the physical simulation does not include deaths).
* N is the number of trials, with larger numbers generating more data at the cost of more runtime; 4 trials seems to be optimal.
* startup is a binary variable, with 1 for an initial setup of the four center squares and 2 for an initial setup of the four corners.
* curvefit is a binary variable that determines whether to fit the data to the logistic growth model, with 1 or 2 for yes and 0 for no. curvefit=2 is the standard 2-parameter fit for a quadratic function that passes through the origin, while curvefit=1 uses a simpler 1-parameter fit with theoretical maximum population.

**Questions About the Graphs:**

1. Compare graph 2 from the two experiments.
   1. What are the key similarities?
   2. What are the key differences?
2. Compare graph 3 from the two experiments.
   1. What are the key similarities?
   2. What are the key differences?
3. Which of graphs 2 and 3 seems to be more meaningful? Explain.
4. Which of the graphs 1, 2, 3, and 4 is closest to a straight line?

**A Symbolic Mathematical Representation from Empirical Arguments:**

1. Sketch the graph of the equation y=3x-12. Then sketch the graph of the equation

y=3(x-4). Conclude that the formula y=m(x-a) is an acceptable alternative to the formula y=mx+b as the generic formula for a straight line. Describe the graphical meanings of the parameters m, b, and a.

1. Write down a symbolic mathematical representation of a model for resource-dependent growth using the graph that is closest to a straight line (see question 8). Use the formula y=m(x-a), but replace the generic variables x and y with the variables on the horizontal and vertical axes of the appropriate graph.
2. Determine the biological significance of the graphical parameters m and a:
   1. One of the graphical parameters is connected to the success probability. Compare the appropriate graphs from experiments 1 and 2 to identify which one and describe how that graphical parameter and the success probability are related.
   2. Identify what simulation parameter is most closely connected to the other graphical parameter. Hint: Compare the appropriate graphs from your physical simulation and the computer simulation.

**Verbal and Symbolic Mathematical Representations from Biological Arguments:**

1. Recall what you saw in the early stages of the physical simulation and look at the points at the extreme left of graph 3. Fill in the details in the following verbal statement: “When the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is small, the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is approximately proportional to the number of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ spaces.” Explain.
2. Now consider the late stages of the physical simulation. Fill in the details: “When the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is small, the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is approximately proportional to the number of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ spaces.” Explain. Hint: Two of the three blanks in the statement should be the same as above.
3. Determine the connection between the quantities in the final blanks for questions 12 and 13: “The \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ minus the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.” The first blank here is the third blank from question 13, the third blank here is the third blank from question 12, and the second blank here is a parameter in the simulation. Explain.
4. A dependent variable can be proportional to two different quantities x1 and x2 using a formula y=m x1 x2. Use this idea to create a mathematical formula to combine the verbal statements in questions 12 and 13, with the answer to 14 used to express x2 in terms of x1.
5. Compare the formulas in questions 15 and 10. Explain why these are the same mathematical model.