#### <span id="page-0-0"></span>www.simiode.org **SIMIODE** A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

# Fall 2024 FYMSiC Online Teaching Meetup 24 September 2024, 5:00 PM - 6:00 PM EDT

# Modeling Makes Mathematics Fun and Real For Real!!

Thanks for the invitation to share with you.

Brian Winkel, SIMIODE, Chardon OH USA Director@simiode.org.

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## Personal Career Path

- ▶ Undergraduate mathematics degree in 1964 food service, office work, tutor, organist, Math Club.
- ▶ Discovery of Modeling with Mathematics at initial teaching position after PhD in Noetherian Ring Theory, 1971.
- ▶ Teaching with modeling at liberal arts colleges, research university, engineering schools, military academies.
- ▶ Diversion Founded (1977) and edited for 30 years Cryptologia, devoted to all aspects of cryptology.
- ▶ Consolidation Founded (1990) and edited for 20 years PRIMUS - Problems, Resources, and Issues in Mathematics undergraduate Studies.
- ▶ Fully on track Founded (2013) and directs SIMIODE -Systemic Initiative for Modeling investigations and Opportunities with Differential Equatio[ns](#page-0-0).

Emphases - Modeling with Differential Equations Differential Equations Invented to Study Change

All Models discussed are FREEly available in SIMIODE. Student Version to the public and Teacher Version for teachers at <https://qubeshub.org/community/groups/simiode>. Just Google SIMIODE QUBES.

- ▶ What can you see from the other side of the valley?
- ▶ Getting malled
- ▶ m&m modeling of death and immigration yummy
- ▶ Torricelli's Law for Falling Column of Water

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- ▶ LSD Drug Model highlights
- ▶ Sources and Opportunities

What can you see from the other side of the valley?



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- ▶ How would you get started?
- ▶ What kinds of mathematical issues might you embrace/invent/need?
- $\triangleright$  What misconceptions might we have? E.g., do you think we can see the top of the mountain on the other side of the valley?
- ▶ Does this problem look like it will permit a "closed form" solution or require some numerical efforts?
- $\blacktriangleright$  How realistic is this situation and who might be interested in such a problem?

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## **Extensions**

- $\blacktriangleright$  Find the point on the viewing mountain from which we can see the most area on the mountain on the other side of the valley?
- $\triangleright$  For a set of strategic points on the viewing mountain on the other side of the valley what is the highest (lowest) point from which we can see these points?
- $\triangleright$  What is the highest point p9the lowest point) on the mountain on the other side of the valley we can see - from any point on the viewing mountain?

## Getting Malled

The average driving speed to reach a shopping mall is uniformly 30 mph. People are willing to spend no more than 1 hour driving to reach the mall. H Suppose a new improved East-West road is built, passing 10 miles due North of the shopping mall and that the speed limit on the new road is 55 mph. Determine the new neighborhood for the shopping mall.



- $\triangleright$  What reasonable and simplifying assumptions can we make to get started? Why are they so?
- ▶ How could we begin?
- $\triangleright$  Can we identify some points which are in the new neighborhood which were not in the old neighborhood?

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▶ Who would be interested in our result? ...

# Getting On and Getting Off Highway Ideas



Sketch with hints.K ロ ▶ K @ ▶ K 할 ▶ K 할 ▶ ... 할 ... 900



# Death and immigration

### From 1-001-MMDeathImmigration-Modeling Scenario

Here is the picture of the material needed ...



Equipment:

- ▶ 1 small bag regular m&m's
- $\blacktriangleright$  1 small cup
- $\blacktriangleright$  1 paper plate

Start with initial population, say about 25 m&m's. Keep all others in storage cup. At each generation place "dead" m&m's in cup.

- 1. Toss the m&m's gently onto the plate.
- 2. Remove m&m's with the 'm' facing  $up$  they die.
- 3. Add 10 m&m immigrants from the cup.
- 4. Count remaining m&m's from that generation. Record data.
- 5. Go to Step 1 and repeat. Record time (generation number) and number of m&m's each time.

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## Original Data One Parameter Model Data in **Blue** and Model in Red



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## m&m Spread of Disease

## From 1-017-DiseaseSpread-Modeling Scenario



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# Modeling a Falling Column of Water - Torricelli's Law



We use data taken from video at **[SIMIODE YouTube Channel](https://www.youtube.com/watch?v=NBr4DOj4OTE&list=PLHUyzRr_S0TtP0gAUffhAS8FDccmLh0m8&index=14)**. Cylindrical column (radius  $= 4.17$  cm) of water empties through a bore hole (diameter  $= 11/16" = 0.218281$  cm) in bottom of column. Exit hole at bottom of column - height is 0 cm.

We seek to model  $h(t)$ , the height of the column of water.

#### What Students Can Accomplish

Outline of modeling process

- $\blacktriangleright$  seeing and collecting data,
- $\triangleright$  conjecturing empirical models,
- $\triangleright$  building an analytical model from scientific first principles,

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- $\triangleright$  creating a differential equation model,
- $\triangleright$  solving of the differential equation,
- $\blacktriangleright$  estimating parameter,
- $\triangleright$  comparing model with the actual data.

IDEA - have students collect data from different configurations, i.e. different bore hole sizes at bottom of column of water and compare models and parameters.

Here is data we collected. What do you see or notice?

Make some observations now.

What about the shape of the data plot? Does this fit your reality?





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### Linear Fit? Easy to do with Excel's TrendLine capabilities.





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Exponential Decay Fit? Also, easy to do with Excel's TrendLine capabilities.





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All are empirical fits with no understanding. They just fit a function to data.

Neither line nor exponential are good.

Can we articulate why neither is that great?

What happens to height  $h(t)$ ?

How fast is column of water falling? Early and later?

For large  $h(t)$  the column of water falls faster.

For small  $h(t)$  falls slower.



Ideas about  $\frac{dh(t)}{dt}$  - rate of change in height  $h(t)$ ?



Check out the average rate of falling of the height of the column of water in several intervals, say, [0, 2.187],

$$
\frac{10.6-11.1}{2.187-0} = -0.2286
$$

or in the interval [39.503, 47.663],

$$
\frac{2.0 - 3.0}{47.663 - 39.503} = -0.122549.
$$

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What do you see? What can you say about  $h'(t)$ ?

Let's find a model from some first principles. This would be an analytic model.

NOT just fit a function to data. NOT just "it looks like it falls faster or slower."





 $(1, 1)$   $(1, 1)$   $(1, 1)$   $(1, 1)$   $(1, 1)$   $(1, 1)$   $(1, 1)$   $(1, 1)$ 

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Enter Evangelista Torricelli 1608–1647, an Italian physicist and mathematician, and a student of Galileo. Best known for his invention of the barometer. Obviously, also known for his wicked mustache!



Torricelli's Law to the rescue!

$$
\frac{dh(t)}{dt}=-b\sqrt{g\cdot h(t)}, \quad h(0)=h_0 \quad b>0.
$$

Say it out loud in sentence form.

Explain to yourself what it means, what it implies.

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Does Torricelli's Law agree with observations?

For large  $h(t)$  the column of water DOES fall faster. For small  $h(t)$  the column of water DOES fall slower.

We build the model that IS Torricelli's Law from First Principles. This will be an analytic model.

#### Basically, The Law of Conservation of Energy says that

.

#### Total Energy is conserved

We will apply it to a slab of water, first at the surface of the column of water and then at the bottom of the column ( $h = 0$ )

> Total Energy is the the sum of the potential energy and the **kinetic energy** of a particle of mass  $m$  and this sum is constant at each instance in time, t.

> > **KORKAR KERKER SAGA**

Now by The Law of Conservation of Energy - Initial Total Energy equals Final Total Energy.

$$
TE_i = \frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mg \cdot 0 = \frac{1}{2}mv_f^2 = TE_f,
$$

Divide both sides by m and multiply by 2 - to solve for  $v_f$ :

$$
v_f=\sqrt{2gh+v_i^2}\,.
$$

Since  $v_i = 0$  we have one classical form of Torricelli's Law

$$
v_f=\sqrt{2gh}\,,
$$

where  $v_f$  is the speed of the water as it leaves the exit or bore hole.

You are helping students realize notions from the "S" of STEM . . . while doing the "M" of STEM.

So, for a cylinder of constant cross sectional area we have an analytic model (differential equation!) for  $h(t)$ .

$$
\frac{dh(t)}{dt}=-b\sqrt{g\cdot h(t)},\quad h(0)=h_0.
$$

We solve this differential equation for  $h(t)$  to realize a model. What strategy/technique can we employ? What technology?

We use this solution and our data to estimate parameter b and validate our model by comparing model predictions to data.

$$
\frac{dh(t)}{dt}=-b\sqrt{g\cdot h(t)}=-b\sqrt{g}\cdot (h(t))^{1/2}.
$$

KEY POINT: In traditional courses the differential equation is presented and students work on techniques with no knowledge of where the differential equation comes from or what its purpose is.

SIMIODE motivates with modeling.

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Separate the variables (Done to introduce technique or practice.)

$$
(h(t))^{-1/2}\cdot\frac{dh(t)}{dt}=-b\sqrt{g}.
$$

OR

$$
(h(t))^{-1/2} \cdot dh = -b\sqrt{g} \cdot dt.
$$

Integrate both sides. (What is C?)

$$
\int (h(t))^{-1/2} \cdot \frac{dh(t)}{dt} dt = \int -b\sqrt{g} dt + C,
$$

$$
2(h(t))^{1/2}=-b\sqrt{g}\cdot t+C.
$$

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Now to find C using Initial Conditions:

$$
2(h(t))^{1/2}=-b\sqrt{g}\cdot t+C.
$$

$$
2(h(0))^{1/2} = -b\sqrt{g} \cdot 0 + C = C.
$$

Thus we have

$$
2(h(t))^{1/2} = -b\sqrt{g} \cdot t + 2(h(0))^{1/2}.
$$

Divide both sides by 2 and then square both sides yields:

<span id="page-32-0"></span>
$$
h(t) = \left(-\frac{b\sqrt{g}}{2} \cdot t + (h(0))^{1/2}\right)^2.
$$
 (1)

This is model for height of the column of water,  $h(t)$ , at time t. What do we know and what do we need to estimate  $b$  in  $(??)$  $(??)$  $(??)$ ?  $h(0) = 11.1$  cm and  $g = 980$  cm/s<sup>2</sup> KID KA KERKER KID KO Thus from  $h(0) = 11.1$  cm and  $g = 980$  cm/s<sup>2</sup>

$$
h(t) = \left(-\frac{b\sqrt{g}}{2} \cdot t + (h(0))^{1/2}\right)^2
$$

becomes

$$
h(t) = \left(-\frac{b\sqrt{980}}{2}\cdot t + (11.1)^{1/2}\right)^2,
$$

and expanded in decimals we have

$$
h(t) = (-15.6525 \cdot b \cdot t + 3.33166)^{2}.
$$
 (2)

We have arrived at our model and now we seek to determine b and validate our model and predict our data.

How might we do this?

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We turn to our Excel spreadsheet and seek to determine the parameter b which minimizes the sum of the squared errors between our data  $(h_i)$  and our model  $(h(t_i))$  over our data points.

$$
SSE(b) = \sum_{i=1}^{9} (h_i - h(t_i))^2.
$$

Minimize as a function of the parameter  $b$ :

$$
SSE(b) = \sum_{i=1}^{9} (h_i - h(t_i))^2 .
$$

where

- $\blacktriangleright$   $t_i$  is the *i*<sup>th</sup> time observation,
- $\blacktriangleright$   $h_i$  is the observed height at time  $t_i$ ,
- $\blacktriangleright$   $h(t_i)$  is our model's prediction of the height at time  $t_i$ , and

 $\blacktriangleright$   $n = 9$  is the number of data points we have.

#### Model Analysis in Excel Using Solver



Torricelli Modeling Scenario [1-015-Torricell-ModelingScenarioi.](https://qubeshub.org/community/groups/simiode/publications?id=3305&v=2)

Solver can minimize the TOTAL SEE or SSE which is currently 7.679799546 with parameter  $b = 0.002$  by asking Solver to minimize SEE or SSE as a function of h cell.

#### Parameter Estimation with Excel Solver - Results



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We offer a model for LSD flow between plasma and tissue compartments in the human body.



$$
V_P = \underbrace{(0.163M)}_{\text{kg of plasma liter/kg}} \cdot \underbrace{(1)}_{\text{filter/kg mill/iter}} \cdot \underbrace{(1000)}_{\text{ml/liter}} = \underbrace{(163M)}_{\text{ml of plasma}}
$$
(3)

Using the notion of "simple change in something," in this case amount of LSD in each compartment we can produce the system of differential equations in (2). We discourage them from building rate of change models of just concentration as they can be difficult with units. The last term in the equation for  $C_T'(t)$  in (2) reflects the exponential decay of the LSD in the tissue compartment due to excretion.

$$
V_P C'_P(t) = k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t)
$$
  
\n
$$
V_T C'_T(t) = k_b V_P C_P(t) - k_a V_T C_T(t).
$$
\n(4)

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We seek the model built from parameter  $(k_a, k_b,$  and  $k_e)$  estimates using the solution of the system of differential equations and minimization of the sum of square error function

$$
SSE(k_a, k_b, k_e) = \sum_{i=1}^{7} (C_P(t_i) - O_i)^2.
$$

Now with Mathematica's powerful FindMinimum command we can determine the values of the parameters  $k_a$ ,  $k_b$ , and  $k_e$  which minimize this  $SSE(k_a, k_b, k_e)$  function,

FindMinimum[SSE[ $k_a$ ,  $k_b$ ,  $k_e$ ],  $\{k_a$ , 1/3},  $\{k_b$ , 1/4},  $\{k_e$ , 1/4}]

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- ▶ Indeed, we obtain the minimum sum of square errors to be 0.080945, when  $k_a = 4.63679$ ,  $k_b = 3.18659$ , and  $k_e = 0.41128$ .
- $\triangleright$  We encourage students to use different initial guesses in the FindMinimum command for each of the three parameters,  $k_a$ ,  $k_b$ , and  $k_e$ , to give them some idea of the robustness of the command itself and confidence that they have a true minimum sum of square errors.
- $\blacktriangleright$  This gives us a final model expression for  $C_P(t)$

$$
C_P(t) = 0.128905 (41.2194e^{-7.99617t} + 53.9669e^{-0.238492t})
$$

.

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Plot of the observed values of the average concentration of LSD  $(ng/ml)$  (squares) and the model built from parameter  $(k_a, k_b,$  and  $k<sub>e</sub>$ ) estimates using the solution of the system of differential equations and minimization of the sum of square error function



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# Plot of LSD (drug) in tissue obtain from data on LSD in plasma, all because of our modeling efforts.



 $\left\{ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$  $\mathbb{R}^{n-1}$  $2990$ 

## Strong correlation between amount of LSD in tissue and performance on simple arithmetic questions.

MathTestScore %



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## SIMIODE has over 400 Project Ideas and Resources

Hundreds of resources, mainly published articles and links to articles, at Project Ideas and Resources can serve as resources for faculty to create class projects or author and publish Modeling Scenarios in SIMIODE.

They can also serve students who are seeking projects for their course work or own edification.

Resources are listed as Year, Author(s), and Title (or descriptive name) with a description, often the abstract, and some narrative.

All have lots of tag words or keywords so the topic areas can be found in the search capability.

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All Models discussed are FREEly available in SIMIODE. Student Version to the public and Teacher Version for teachers - all found at

<https://qubeshub.org/community/groups/simiode>. OR ust Google SIMIODE QUBES.

Students' cognate courses, e.g., chemical kinetics, life science Petri dish growth, economics modeling phenomena, sociology, linguistics.

Get hold of their textbook or article in the field and bring it into your class.

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### **TEACHER VERSION** KINETICS - RATE OF CHEMICAL REACTION

**Brian Winkel** Director SIMIODE Cornwall NY 12518 USA BrianWinkel@simiode.org

**Abstract:** We help students see the connection between college level chemistry course work and their differential equations coursework. We do this through modeling kinetics, or rates of chemical reaction. We study zeroth, first, and second order reactions and offer many opportunities to model these chemical reactions with data, some of which comes from traditional introductory chemistry textbooks. We ask students to verify their model through parameter estimation. We use Excel's Trendline addition to graphs/charts to select the models for the data and transformed data to take advantage of Trendline's set function choices and we also use Mathematica's direct nonlinear fitting capabilities.

Keywords: chemical reaction, kinetics, rate of reaction, order, Law of Mass Action

Tags: first order, differential equation, data, modeling, fitting, parameter estimation, zeroth order, first order, second order, chemistry textbook

#### Decomposition of  $H_2O_2$

5) Consider the following data (Table 5) for the reaction describing the decomposition of  $H_2O_2$ , hydrogen peroxide.

$$
2H_2O_2 \longrightarrow 2H_2O + O_2 \tag{7}
$$

Time t in s	$[H_2O_2]$ mol/L
$\overline{0}$	1
120	0.91
300	0.78
600	0.59
1200	0.37
1800	0.22
2400	0.13
3000	0.082
3600	0.05

Table 5. Data [8, p. 682] for the decomposition of  $H_2O_2$  is given.

- a) Plot the data.
- b) From the plot make a conjecture as to the order  $(m = 0, 1, 2)$  of the reaction.
- c) Conduct a complete analysis, determining the order and the parameters. Plot the data and the model, being sure to defend what the order is and what the order is not vis-á-vis  $m = 0, 1, 2$  orders.



## **STUDENT VERSION MODELING EVICTIONS**

Mary Vanderschoot Department of Mathematics and Computer Science **Wheaton College** Wheaton IL USA

#### **STATEMENT**

According to the National Law Center on Homelessness and Poverty, unaffordable rents and a lack of legal protections for renters have created a national "eviction epidemic" [4]. Matthew Desmond, author of *Evicted: Poverty and Profit in the American City* and director of the Eviction Lab at Princeton University, estimates that 2.3 million evictions were filed in the U.S. in 2016 (four evictions per minute). Desmond writes, "Eviction is a direct cause of homelessness, but it also is a cause of residential instability, school instability [and] community instability" [1]. In this project you will develop and analyze two mathematical models to study eviction trends in a city using an actual eviction rate.

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## **STUDENT VERSION** PROPAGATION OF THE WORD JUMBO

Rachel L. Bayless Department of Mathematics Agnes Scott College Decatur GA USA

Rachelle C. DeCoste Department of Mathematics **Wheaton College** Norton MA USA

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#### **STATEMENT**

In 1861, the elephant who became known as Jumbo was captured in Ethiopia and purchased by animal collector Johan Schmidt. First known as the "Children's Giant Pet," the largest elephant known at the time was renamed Jumbo, a word believed to have been created from the combination of the Swahili words *jumbe* and *jambo*, meaning "chief" and "hello", respectively. [1]



Figure 1. A scatter plot of percentage of printed words that are the word jumbo versus time for the years 1880 to 1940.

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Set students free to find their own projects.

WARNING: Make sure it is worthy of differential equation structure, not just data fitting with curve.

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PRIMUS, 23(3): 274-290, 2013 Copyright © Taylor & Francis Group, LLC ISSN: 1051-1970 print / 1935-4053 online DOI: 10.1080/10511970.2012.753966



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### **Browsing Your Way to Better Teaching**

#### **Brian Winkel**

**Abstract:** We describe the use of browsing and searching (in libraries, online, inside sources, at meetings, in abstracts, etc.) as a way to stimulate the teacher of undergraduate mathematics, specifically in differential equations. The approach works in all other areas of mathematics. Browsing can help build new and refreshing teaching materials based on how mathematics is used and explored in places other than mathematics. These "other" places are where almost all of our students will be going after they study with us and we should: (i) know about their journey and arrival points; and (ii) understand the disciplinary approaches for those areas which sent these students to us in the first place for their mathematics studies. We describe a personal browsing experience that spanned almost 40 years and proved to be very worthwhile in finding applications of differential agustians to modeline I wonede A sid Disthylamide in the human hady.

Examples of browsing

- ▶ Collegial Conversation
- ▶ Conference and Meeting Presentations
- ▶ Invited Speakers
- ▶ Friends and Colleagues Who Know Your Own Interests
- ▶ Cognate area textbooks and journals
- ▶ The Projects or Starred Exercises in the Text
- ▶ New Issues of Journals in Your Institution's Library

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- ▶ Back Issues of Journals on Dusty Library Shelves
- ▶ The Internet Google Search Doh!
- ▶ Older Books in Your Library and On-Line
- ▶ On-Line Course Descriptions and Courses

More examples of browsing

- ▶ Newspapers and Magazines
- ▶ SIMIODE Doh!
- ▶ Professional Society Websites
- ▶ Blogs
- $\blacktriangleright$  Funded sites government and foundations
- ▶ COMAP Consortium for Mathematics and its Application

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- ▶ General Online Support Materials
- ▶ YouTube Videos
- ▶ Student Assistants
- $\triangleright$  On a walk through campus
- ▶ Online data sets Census, Covid, Malaria, etc.

Work modeling into your classes all of them.

Take your time and . . .

try not to do too much all at once.

Give students something challenging, interesting, real, and with a view to more mathematics to do even better in modeling, e.g., consulting article and writing for audience - client and senior consultant is one framework.

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INT. J. MATH. EDUC. SCI. TECHNOL., 1990, VOL. 21, NO. 3, 363-368

#### First year calculus students as in-class consultants

by BRIAN J. WINKEL

#### Department of Mathematics, Rose-Hulman Institute of Technology, Terre Haute, Indiana 47803, U.S.A.

#### (Received 7 June 1988)

We demonstrate how we use traditional text book calculus problems to involve first year calculus students as consultants. Students are asked to solve a 'real world' problem after interviewing 'guest' client-a printer. Students go through the initial steps of interviewing client, extracting information, ascertaining the problem, formulating the problem, solving the problem, and writing up two 'solutions'—one is a step by step solution for the client and the second is a technical support document for the 'senior consultant'. This description can serve as a model for further student involvement with fresh approaches to applications of the calculus.

Thank you for visiting with me.

# Questions, Comments, Discussion . . .

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Deferential equations.