www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

SCUDEM III 2018

Problem A - Conflict Between Patrilineal Clans

Roughly 7,000 years ago the genetic record for humans indicates that there was a dramatic decrease in the variation in Y chromosomes [2]. It appears that the number of people carrying the Y chromosome decreased to one twentieth of their previous number. There is not an indication for a corresponding drop in the number of people who do not carry the Y chromosomes.

A group of undergraduate students at Stanford University recently [1,2] hypothesized that the reason for the sudden decrease in males is due to the development of patrilineal clans among humans, and the resulting strife and wars between the clans impacted males at a much higher rate than females. The group developed a mathematical model describing the genetic interactions that result from the hypothesized situation and showed that the resulting genetic trends are consistent with the current genetic variations seen in both male and female populations.

Assuming that the hypothesis is correct, develop a mathematical model describing conflict between neighboring patrilineal clans that can be used to predict the resulting population dynamics including the distribution of males and females in the human population. Based on your models under what conditions are conflicts most intense? Was the decline of patrilineal clans inevitable or is it possible to reach an equilibrium under such a social tradition?

One model for a group of two sets of males is given in the original paper [2]. The model assumes uniform mixing of the different groups of males and only one group of females. Your model should not assume a uniform pool of females, but include different groups of females associated with the clans. Additional discussion on how to extend the model to more than two groups is expected.

References

[1] Collins, Nathan. 2018. Wars and clan structure may explain a strange biological event 7,000 years ago, Stanford researchers find. Stanford University News Service. 30 May. **https://news.stanford.edu/press-releases/2018/05/30/war-clan-structubiological-event/**. Accessed 3 September 2018.

[2] Zeng, Tian Chen, Alan J. Aw, and Marcus W. Feldman. 2018. Cultural hitchhiking and competition between patrilineal kin groups explain the post-Neolithic Y-chromosome bottleneck. *Nature Communications*. Volume 9, Article number: 2077. https://www.nature.com/articles/s41467-018-04375-6. Accessed 3 September 2018. Freely downloadable. This article is licensed under a Creative Commons Attribution 4.0 International License.

www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

SCUDEM III 2018

Additional Issue

Problem A - Conflict Between Patrilineal Clans

What is the role of human mobility in your model? For example, how would the model change if horses were domesticated earlier? Does your model make different predictions for highly mobile groups of people, such as some Pacific Islander groups versus groups whose travels were more limited?

www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

SCUDEM III 2018

Problem B - Swing and a Hit

A device (see the full frontal rendering in Illustration 1) is to be constructed that will be used to permit a metal ball to roll down a ramp as one part of a larger museum exhibit for children. This ball will strike another ball attached to the end of a hanging rod which will cause the device to rotate 180 degrees and lightly strike a domino after the half rotation about the central pole. The device will be built from a vertical, one meter, central pole, and while another rigid half meter pole will extend horizontally from the top of the vertical central pole. At the end of the horizontal pole another rigid rod with a ball at the end of the rod will hang down and will be free to swing in any direction. The hanging rod will have a length of 0.80 meters.

The device will be constructed so that when it is at rest the hanging rod will hang down near the lip of the ramp as show in Illustration 1 below. Separate from the tower, a metal ball whose mass is 600g will roll down a ramp and strike another metal ball attached to the end of the hanging rod. The contraption should initially be in the configuration as shown in Illustration 1, and the moving rolling mass should be moving straight out of the page on the left lower side. The whole contraption will be free to rotate around the central, vertical pole, and it should rotate 180 degrees in order to strike the domino. With respect to the illustration it should rotate out of the page. The ball to be attached to the end of the hanging rod should just lightly strike the top of a domino that is standing up and is 0.3 meters above the floor as shown in the lower right side of Illustration 1.

Your team has been asked to determine how best to accomplish the goal. In particular, to start building the contraption the technicians will need to know the mass of the metal ball to be attached to the bottom of the hanging rod. They will also need to know where to locate the domino. Finally, they need to know at what velocity the original metal ball should be moving when it strikes the ball attached to the hanging rod in order to accomplish the desired result. The primary design constraint is that the domino should be 0.30 meters above the floor. However, if your team decides that the geometry should change then you should provide a clear case for any new recommendations.

The director of the exhibit has indicated that she would like the motion to be as dramatic as possible. In this case that means as wide a swing as possible, but also slow enough so that children can watch in anticipation. At the same time, the motion must be reproducible and reliable.

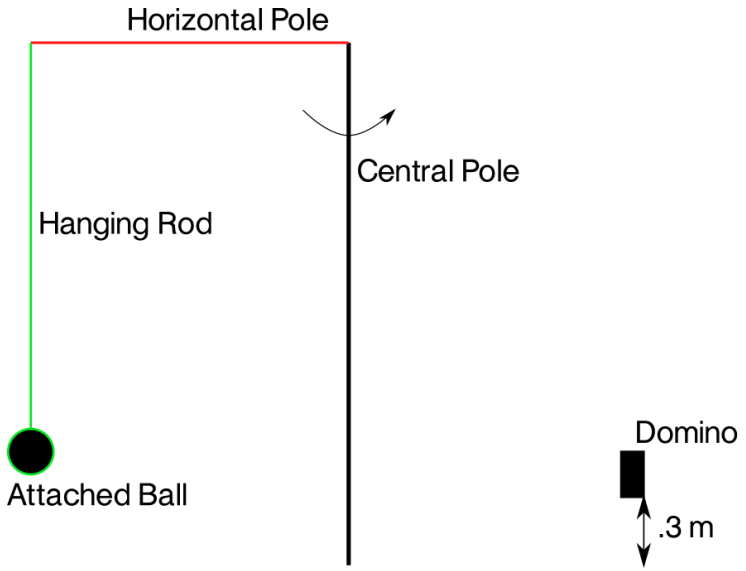

Illustration 1: Side view of the contraption.

www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

SCUDEM III 2018

Additional Issue

Problem B - Swing and a Hit

If you could make one design change for the device what would it be? Explain your choice and why you believe it is the best candidate for a change. What changes would you expect to see in the motion of the new device?

www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

SCUDEM III 2018

Problem C - Snakes in the Long Run

The sex of some reptiles depends on a number of factors including the incubation temperatures of the eggs. The dependence of temperature for Pine Snakes was examined in a paper by Burger and Zappalorti [1]. They found a linear dependence between incubation temperature (in Celsius) and the sex ratio (male/female) for Pine Snakes,

$$SexRatio \approx 0.68 \cdot (Temp) - 0.95 .$$

One question that arises is what might happen if there are rapid changes in environmental temperatures. For example, one prediction about climate change is that there will be larger variations in yearly temperatures.

Assuming this is the case, explore the possible impacts to the Pine Snake population dynamics.

Will the wider variations in temperatures impact the overall population dynamics of the Pine Snake?

How long will it take for impacts, if any, to become noticeable?

Reference

[1] Burger, Joanna and R. T. Zappalorti. 1988. Effects of Incubation Temperature on Sex Ratios in Pine Snakes: Differential Vulnerability of Males and Females. *The American Naturalist*. 132(4): 492-505.

www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

SCUDEM III 2018

Problem C - Snakes in the Long Run

With respect to the evolutionary pressures associated with greater climate variability, describe how your model would change and what changes might you expect to see in how the species will adapt.