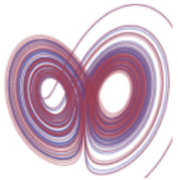


CODEE : the other, more senior, differential equations group

Beverly West, Cornell University,
for SIMIODE EXPO 2021



CODEE Community of Ordinary Differential Equations Educators

codee.org

CODEE has a very long history:

The seeds were planted in 1976 when Bob Borrelli and Courtney Coleman traveled



(Harvey Mudd College)



from Claremont, CA to Ithaca, NY. The occasion was an NSF summer workshop, run by Bill Lucas at Cornell University, on creating Modules of Applicable Mathematics for undergraduate math courses; I also attended.

1976, with Borrelli & Coleman

Bob, Court, and I worked together on differential equations.
Two of their memorable contributions were

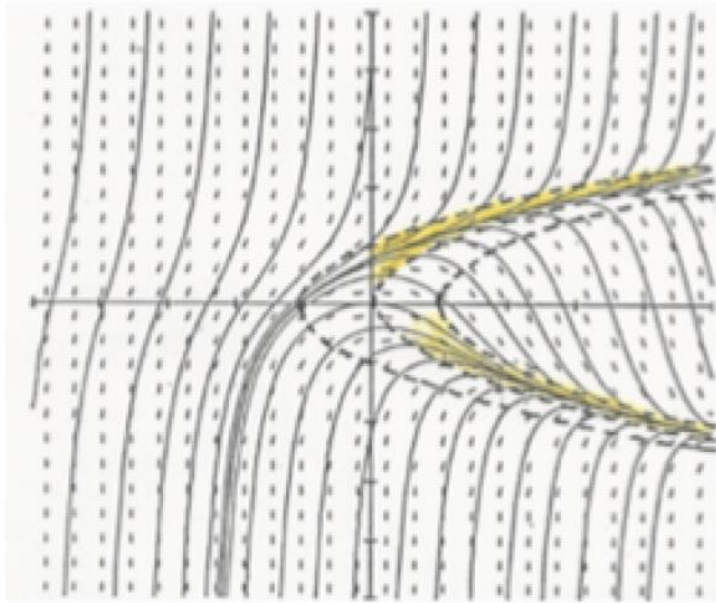
"Shaking a String to Rest", and
"Tossing a Tennis Racquet",

and one of mine (long before computer-drawn solutions) was
"Sketching Solutions to ODEs".

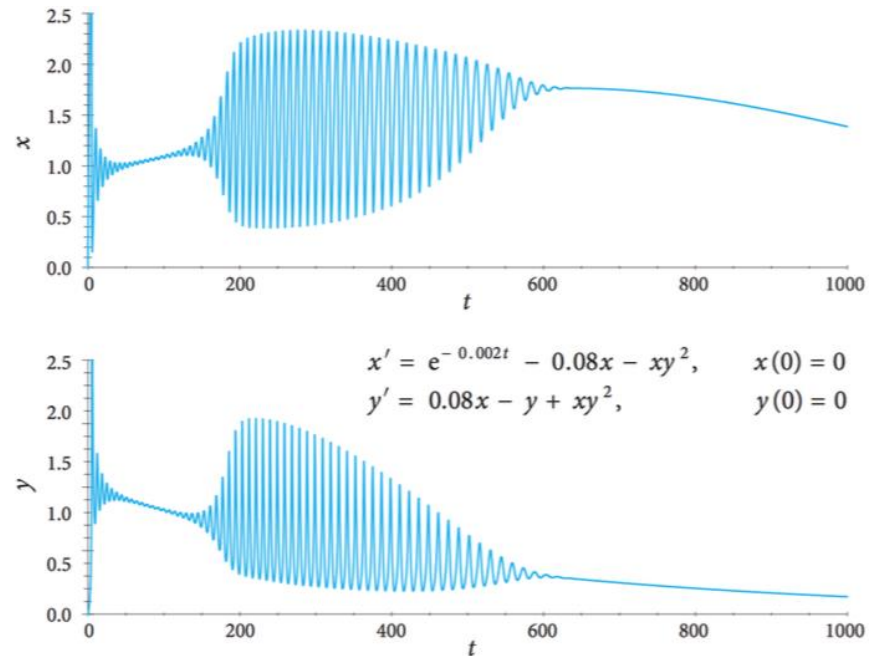
We had a wonderful time, and became long term colleagues.

Borrelli and Coleman, continued

About a decade later I travelled to a JMM hauling a set of computer pictures that John Hubbard at Cornell had initiated. I ran into Bob and told him with great excitement that I had something to show him -- could we find a place to talk? Bob patted his briefcase and said *he* had something to show *me*. We *both* had the same news:



$x' = x^2 - t$
(John Hubbard)



chemistry oscillations
(Borrelli and Coleman)

Computer graphics would revolutionize the study of ODEs.

This means that ...

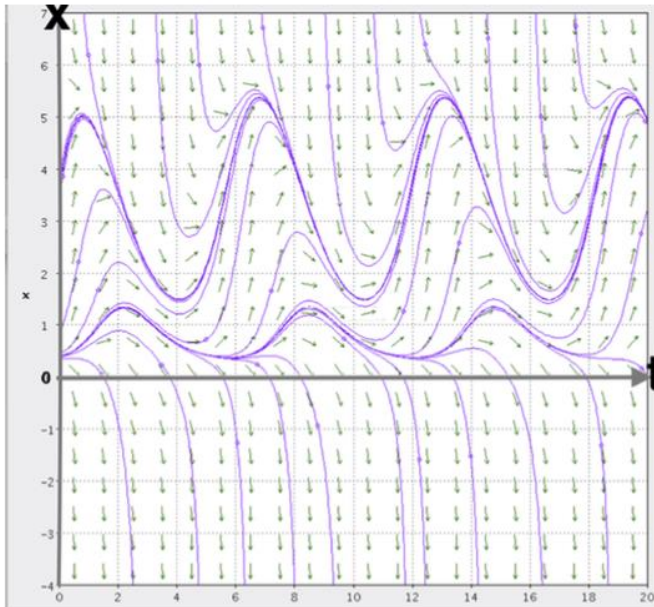
- We are no longer restricted to the minority of ODEs that have analytic solutions
- We can see the *behaviors* of solutions to any ODE

$$x' = \underbrace{(2 + \cos t)x}_{\text{Natural, positive rate of growth, with seasonal fluctuation}} - \underbrace{0.5x^2}_{\text{Population decrease due to crowding}} - \underbrace{C}_{\text{Constant rate of decrease due to hunting}}$$

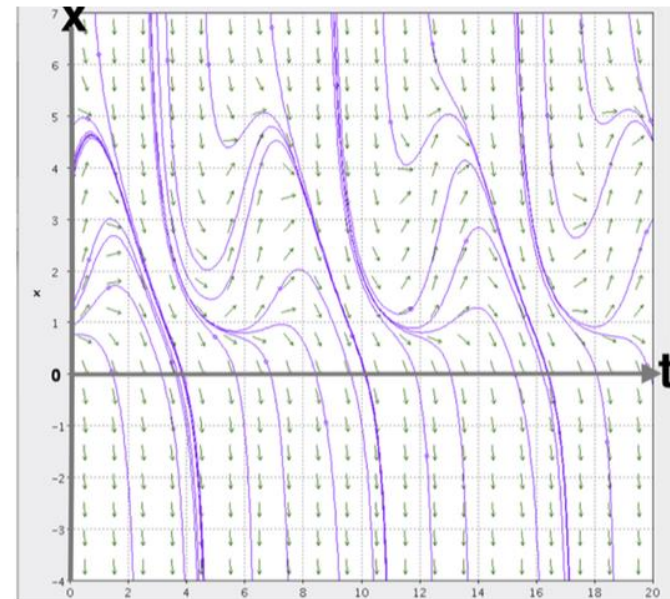
Natural, positive rate of growth, with seasonal fluctuation

Population decrease due to crowding

Constant rate of decrease due to hunting



$C = 1$: Many sustainable solutions



$C = 2$: NO sustainable solution

Bob ran with this idea; he and Court applied to the NSF for a grant to set up a **Consortium for Ordinary Differential Equations with Experiments (CODEE)**. (1991)

- Harvey Mudd College,
- Cornell University,
- Rensselaer Polytechnic Institute,
- Washington State University,
- St. Olaf College,
- West Valley Community College,
- Stetson University

Our mission: To inform and help our colleagues teaching ODEs how to take advantage of the sudden amazing new opportunities.

CODEE: Consortium for ODE Experiments Community of ODE Educators

First grant 1991 (to inform and help our colleagues teaching ODEs how to take advantage of the sudden amazing new opportunities)

- Summer workshops
- Contributed paper sessions at JMM, ICTCM
- Newsletter

Workshops were wildly successful; talks engendered much excitement and progress.

(International Conference for Technology in Collegiate Mathematics)

Second grant 1995 (to continue our mission)

- *ODE Architect* software package (for Windows)
- Continued Newsletter and Sessions at meetings

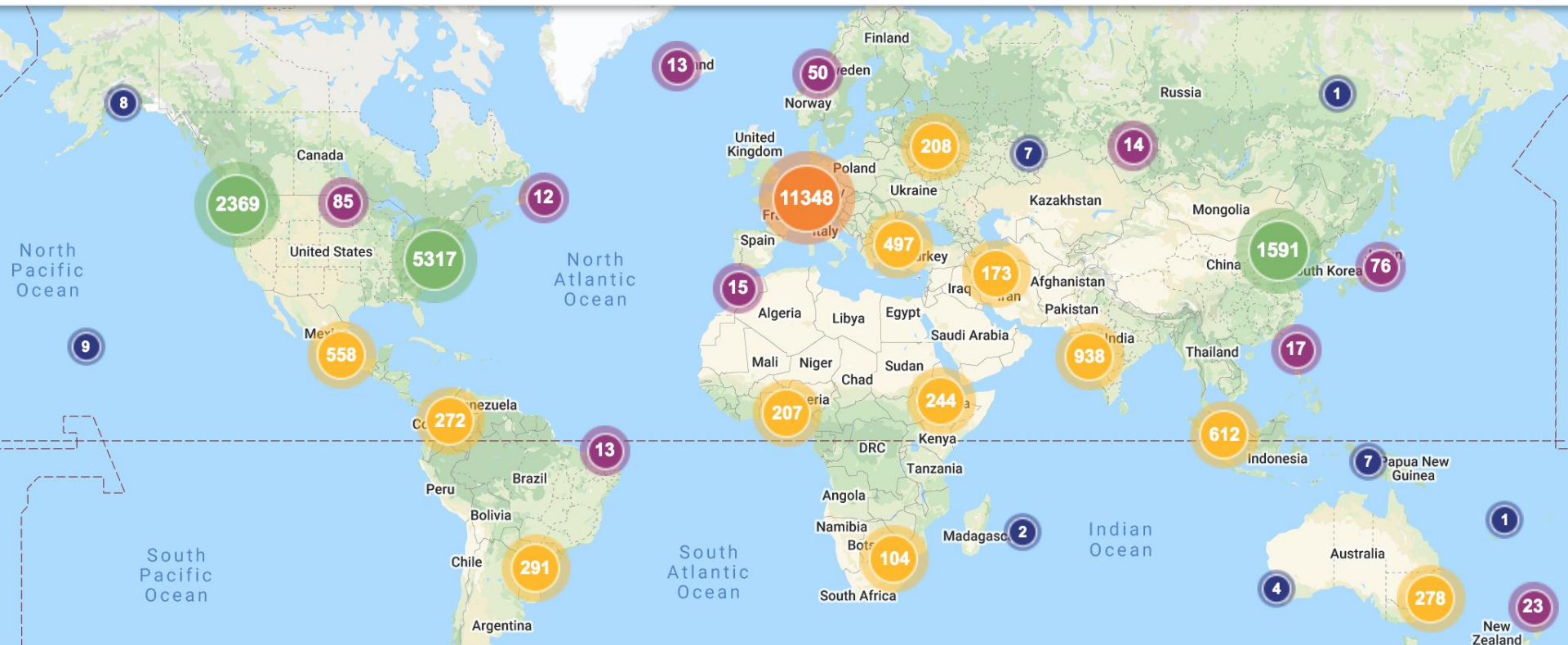
ODE Architect won the distinction of being named by Forbes Magazine as one of the “nine best digital projects on the planet” in December 1998 (one of 9 out of more than 1000 entries, not limited to academia, and the only one in math).

Third grant 2010 (to set up a way to continue our mission, without funding. In cooperation with the Claremont Colleges Library, we could do this!)

- CODEE website with refereed, electronic, CODEE Journal and other resources for teaching differential equations.

Symbiosis with SIMIODE

- CODEE is delighted that SIMIODE, founded in 2013, has taken up the workshop/meetings role that excites and energizes those of us who are teaching ODEs.
- CODEE is happy to focus on maintaining the global reach of the online Journal.



The Global Mission of the CODEE Journal

Downloads in over 130 of 190 countries in the world, fully two thirds of them, from the beginning, are outside the United States. We feel a real mission.

- *Exotic locations include Nepal, Cambodia, Vietnam, Dominica, Mauritius, Lesotho, Kazakhstan, Kyrgistan, Tajikistan, Turkmenistan, Uzbekistan, ...*

So, let's talk about the CODEE Journal.

The CODEE Online Journal

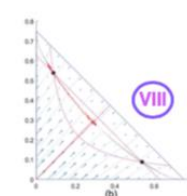
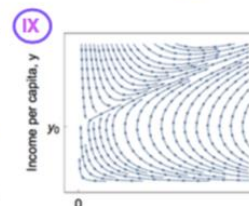
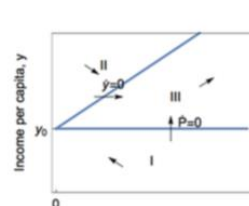
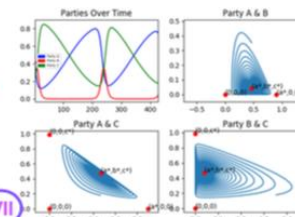
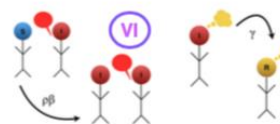
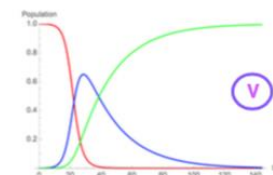
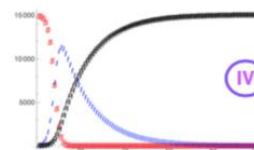
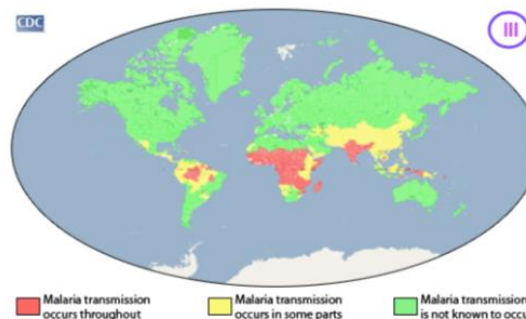
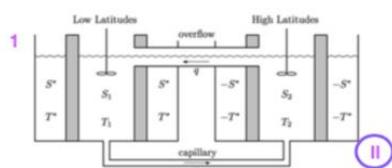
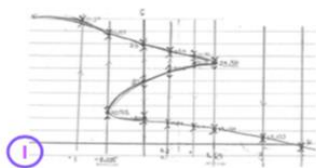
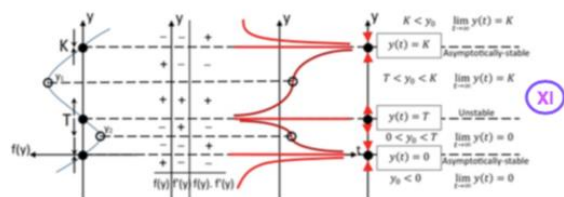
- The Journal is a peer-reviewed, open-access publication, distributed by the CODEE (Community of Ordinary Differential Equations Educators)
 - The Community seeks to improve the teaching and learning of ordinary differential equations (ODEs) through the dissemination of materials that are useful to both educators and education researchers.
 - Articles: Expository in nature with an aim to advance the art and practice of the teaching and learning of ODEs.
 - Projects: Units that instructors of ODEs can use in their classrooms. Projects that involve modeling and/or computer experimentation are especially welcome. Interdisciplinary projects are also highly desirable.
 - Resources: Links to many relevant text and software sites.
 - Special Issues: Two, so far, **Volumes 12 (from 2018) and 14 (from 2020)**.
-

2018 Special Issue of the CODEE Journal



Linking Differential Equations to Social Justice and Environmental Concerns

A Special Issue of the CODEE Online Journal
Edited by Samer Habre of the CODEE Editorial Board



2018 Special Issue of the CODEE Journal

- Climate Change:

Bifurcation Diagrams Explore Small Changes with Big Effects

Stommel's Conceptual Model, applied to Oceans

- Spread and Prevention of Malaria in Central America

- Transmission of Cholera in a Population with Contaminated Water

- Adaptation of SIR Models to

Scenarios that Support the Common Good

Mathematics of Gossip

Three Political Parties

*Sustainable agriculture,
Drug & alcohol use,
Internet spread of violent ideologies,
Criminal activity,
Health issues: bulimia, obesity.*

- Consensus Building by Committed Agents

- Kremer's Model Relating Population Growth to Changes in Income & Technology

- MIT Haiti initiative

- Equity by Visualization (Iran)

codee.org, Volume 12

2020 Special Issue of the CODEE Journal

Engaging Learners: Differential Equations in Today's World

I. *Engaging Students by Internationalizing the Undergraduate Calculus Course* Chinenye Ofodile (Albany State University)

Student papers

- i. Refugee crisis in Greece, by Robert Lavender, James Hawkins
- ii. Unemployment in Nigeria, by Tavis Jackson
- iii. Cardiovascular Disease in U.S., by AunJrae Barnes
- iv. HIV/Aids in Georgia, USA, by Consuela Blue
- v. Abortion in Georgia, USA, by Jazlyn Meeks
- vi. Coronavirus Outbreaks in Last Twenty Years, by Mohammed Najeeb
- vii. Home Runs in Major League Baseball, by Pierce Thomas

Driven by *data* that seems to show exponential growth, fitting solutions to $y' = ky$.

This first paper for the new special issue really caught our attention. It was about a Calc II class that hit their first DE, $y' = ky$, at the end of the course.

2020 Special Issue of the CODEE Journal

Engaging Learners: Differential Equations in Today's World

Details for

I. *Engaging Students by Internationalizing the Undergraduate Calculus Course* Chinenye Ofodile (Albany State University)

The students in the internationalized section, working either as individuals or in pairs, were asked at the beginning of the course to pick a real world topic where a population seemed to be increasing (or decreasing) at an ever greater (or slower) rate. They were assigned to:

- *Research the culture and background for that topic.*

When they reached $y' = ky$ at the end of the course, they were then asked to

-
- *Solve the DE and find parameters to fit the data found in their research and predict future behavior; and finally,*
 - *Suggest interventions that might diminish the rate of change,*
 - *Write up their results as a proper research report, and give presentations, both in class and at regional meetings.*

2020 Special Issue of the CODEE Journal

Engaging Learners: Differential Equations in Today's World

Results:

- I. *Engaging Students by Internationalizing the Undergraduate Calculus Course*
Chinenye Ofodile (Albany State University)

Students really dug into their topics, did the math, and thought hard specifically about the intervention requirement.

In addition to writing up their work as a proper research report, they had to present their work in class, and later at regional meetings.

In 2018 and 2019 they were able in some cases to have the teams report their ideas to appropriate local agencies (yet another skill!) and have agencies respond that they would try some of the interventions.

2020 Special Issue of the CODEE Journal

Engaging Learners: Differential Equations in Today's World

The other papers in this issue:

- II.** *Modeling the Ecological Dynamics of a Three-Species Fish Population in the Chesapeake Bay*
Iordanka Panayotova (Christopher Newport University)
Maila Brucal-Hallare (Norfolk State University)
- III.** *Qualitative Analysis of a Resource Management Model and Its Application to the Past and Future of Endangered Whale Populations*
Glenn Ledder (University of Nebraska-Lincoln)
- IV.** *Origins of the SIR Model and Critical Application to the 2014-2016 Ebola Epidemic*
Francesca Bernardi (Worcester Polytechnic Institute)
Munuchehr Aminian (California Polytechnic University, Pomona)
- V.** *Facing the Pandemic Together: Forming a Collaborative Research Group*
Michael Barg (Niagara University)

This special issue is expected to get up on the website before the end of March

To request preprints, write jmmcodee@gmail.com

My Absolute Favorite Modeling-First Example, from Anaesthesiology

Back in the 1970s (long before the computer graphics revolution), my students had an option to write an extra credit paper explaining any application that we had *not* covered in the course.

One pre-med student had worked over the summer with an anaesthesiologist who noticed that the *more* anaesthetic a patient received, the *faster* they recovered from it!

So anti-intuitive!

Note: this was long long ago, when there were far fewer anaesthetics, procedures, and studies. Presumably this phenomenon was for a particular drug in a particular application.

Anaesthesiology, continued

She, the anaesthesiologist, had looked to mathematics to see if there were an explanation, and there was!

She modeled the situation with a diffusion equation (yes, a PDE, but intro to PDEs *was* in our course, a collection of topics for non-STEM majors, and the story is so worth telling!).

My student was able to explain this, with some hard work writing the DEs in cylindrical coordinates (to model the veins through which the drug is injected and then dissipates).

Anaesthesiology, continued

The basic diffusion equation is

$$D \frac{\partial^2 C}{\partial x^2} = \frac{\partial C}{\partial t}$$

D = diffusivity constant
 C = concentration function

Consider a nerve cell as an infinite cylinder, so we need cylindrical coordinates:

$$\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} = \frac{\partial C}{\partial t}$$

C = concentration r = radius from the central axis of the nerve
 D = diffusivity constant
 t = time r_0 = radius of cylinder

Anaesthesiology, continued

As before, $\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} = \frac{1}{D} \frac{\partial C}{\partial t}$

and assuming $C(r, t) = f(t) C(r)$,

$f(t) = e^{-\lambda^2 t}$ and $\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\lambda^2}{D} C = 0$

This is Bessel's equation with solution

$$C(r, t) = A + B e^{-\lambda^2 t} J_0(\lambda r)$$

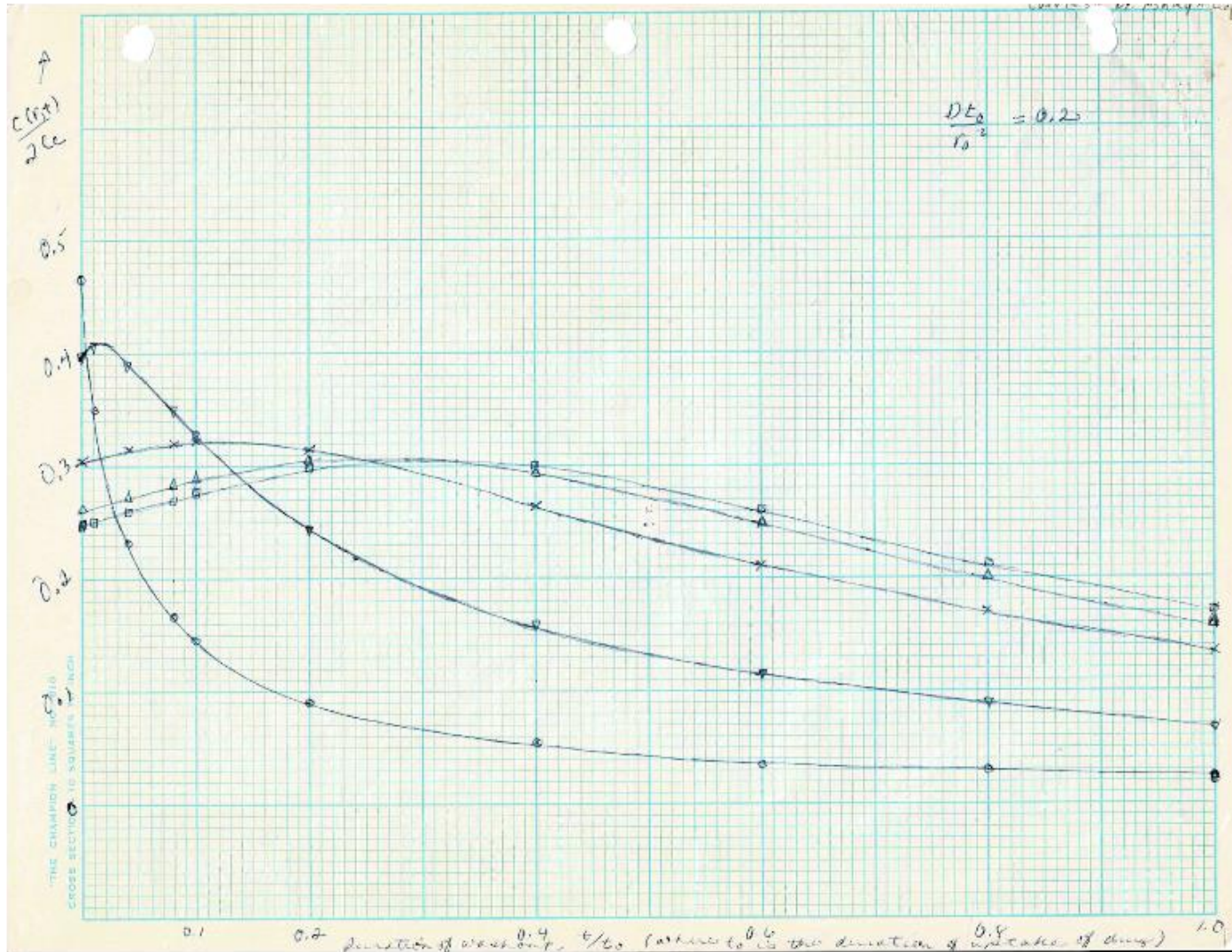
and since $C(r, \infty) = 0$ for all r , $A = 0$

At $r = r_0$, $C(r_0, t) = 0$ for all t .

This implies $C(r, t) = B e^{-\lambda^2 t} J_0\left(\frac{\beta_n r}{r_0}\right)$

Anaesthesiology, continued

The resulting solutions for 5 different initial doses:



Higher dose *does* flow out faster!

May we continue our synergy with

SIMIODE!

It's been wonderful for me, and so fulfilling, to have a life full of differential equations.

Thanks to you all!



codee.org