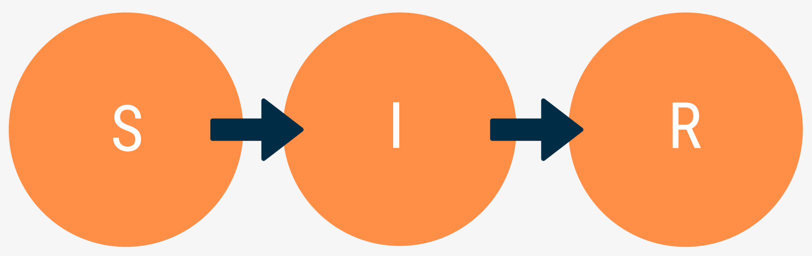
MALARIA PROPAGATION : A FACTOR SIGNIFICANCE ANALYSIS

INTRODUCTION

- Malaria is a potentially life-threatening parasitic disease caused by infection with Plasmodium parasite transmitted by an infective female Anopheles mosquito. It is preventable and curable but could however be deadly if left untreated.
- Malaria is widespread in the tropical and subtropical regions that exist in a broad band around the equator. This includes much of sub-Saharan Africa, Asia, and Latin America.
- In 2019, there were an estimated 229 million cases of malaria worldwide with an estimated 409,000 deaths. Children aged under 5 years are the most vulnerable group affected by malaria; in 2019, they accounted for 67% (274 000) of all malaria deaths worldwide.
- The paper 'Analysis of ODE Models for Malaria Propagation' by Fanni Dorner and Rahele Mosleh analysed the existing primitive model for Malaria named the Ross Model as well as the more effective Extended Ross Model which was a great improvement upon the original Ross model as it fixed the shortcomings of the original model.
- The paper highlighted the role of mosquitoes in the spread of malaria and introduced a differential equation system to describe the dynamics of Malaria at the population level.
- The Ross Model was constructed on the basis of the SIR model created by Kermack and McKendrick in 1927.
- The SIR model has three compartments which are the Susceptibles (S) - who have yet to contract the disease and become infectious, Infectious (I) - who can pass on the disease to others and Recovered (R)who can not transmit the disease. These compartments were used to form a system of ordinary differential equations which was used to model epidemics of a disease. The SIR model consists of the following system.



$$\begin{cases} \dfrac{dS(t)}{dt} = -aS(t)I(t) \\ \dfrac{dI(t)}{dt} = aS(t)I(t) - bI(t) \\ \dfrac{dR(t)}{dt} = bI(t), \end{cases}$$

a = Contact Rate
b = Recovery Rate
t = Time
S(t) = Number of Susceptible Individuals as a function of t (time)
I(t) = Number of Infectious Individuals as a function of t (time)
R(t) = Number of Recovered Individuals as a function of t (time)

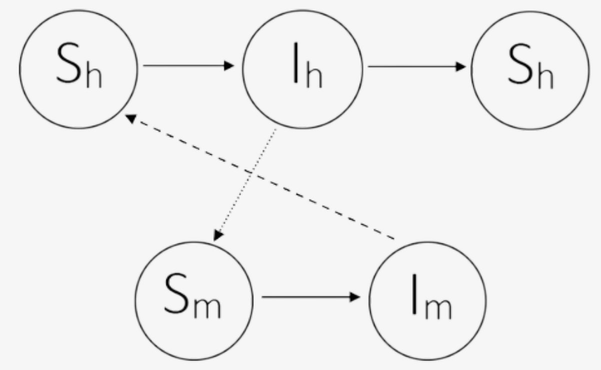
MALARIA PROPAGATION MODELS

THE ROSS MODEL

Based on the SIR model, Ronald Ross developed his own model regarding Malaria propagation.In his model he made some assumptions.He assumed:

1. The human population under consideration remained constant.
2. The mosquito population under consideration remained constant.

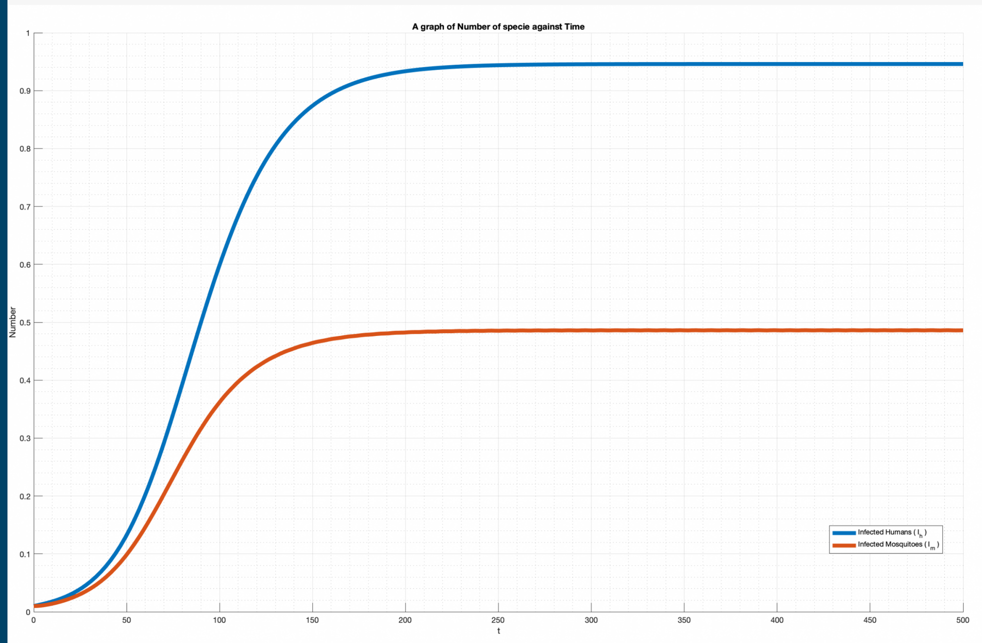
In his model , he determined that the Recovered human population would return to the Susceptible group due to the lack of development of significant immunity and the short lifespan of mosquitoes would prevent them from entering the Recovered group.
Armed with this information Ross developed the following model and systems of equations:



$$\begin{cases} \dfrac{dS_h}{dt} = rI_h(t) - abmI_m(t)(1 - I_h(t)) \\ \dfrac{dI_h}{dt} = abmI_m(t)(1 - I_h(t)) - rI_h(t) \\ \dfrac{dS_m}{dt} = \mu I_m(t) - acI_h(t)(1 - I_m(t)) \\ \dfrac{dI_m}{dt} = acI_h(t)(1 - I_m(t)) - \mu I_m(t) \end{cases}$$

Simplified

$$\begin{cases} \dfrac{dI_h}{dt} = abmI_m(t)(1 - I_h(t)) - rI_h(t) \\ \dfrac{dI_m}{dt} = acI_h(t)(1 - I_m(t)) - \mu I_m(t) \end{cases}$$

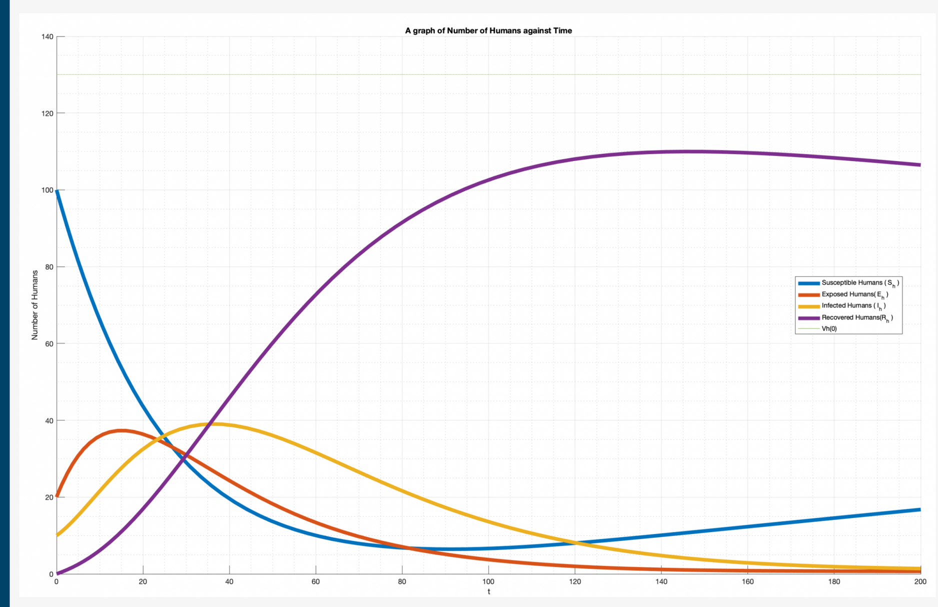


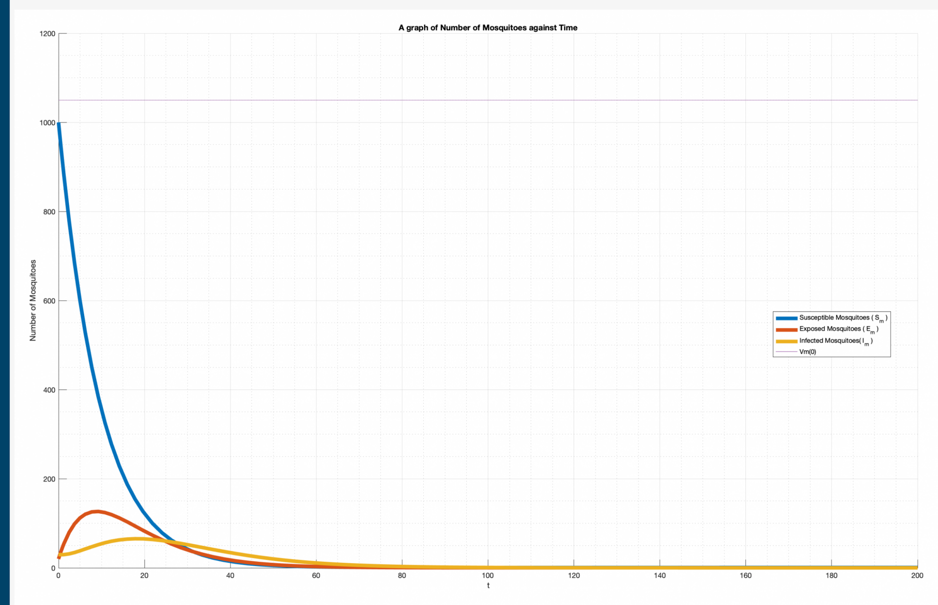
The Ross Model has several short comings. The shortcomings include:
1. The model assumed constant total population by considering birth and mortality to be equal leading to a lack of vital dynamics.
2. There was no latency period. The transition from susceptible state to infected state is not immediate. The model did not account for a gestation period or latency period.
3. The model did not include a recovery rate.

To eliminate shortcomings in the Ross Model, the Extended Ross Model was introduced. The Extended Model introduced class of exposed (E) specie, those individuals whom are infected but not able to pass on the infection to others to cater for a latency period.

THE EXTENDED ROSS MODEL

To eliminate shortcomings in the Ross Model, the Extended Ross Model was introduced. The Extended Model introduced class of exposed (E) specie, those individuals whom are infected but not able to pass on the infection to others to cater for a latency period.





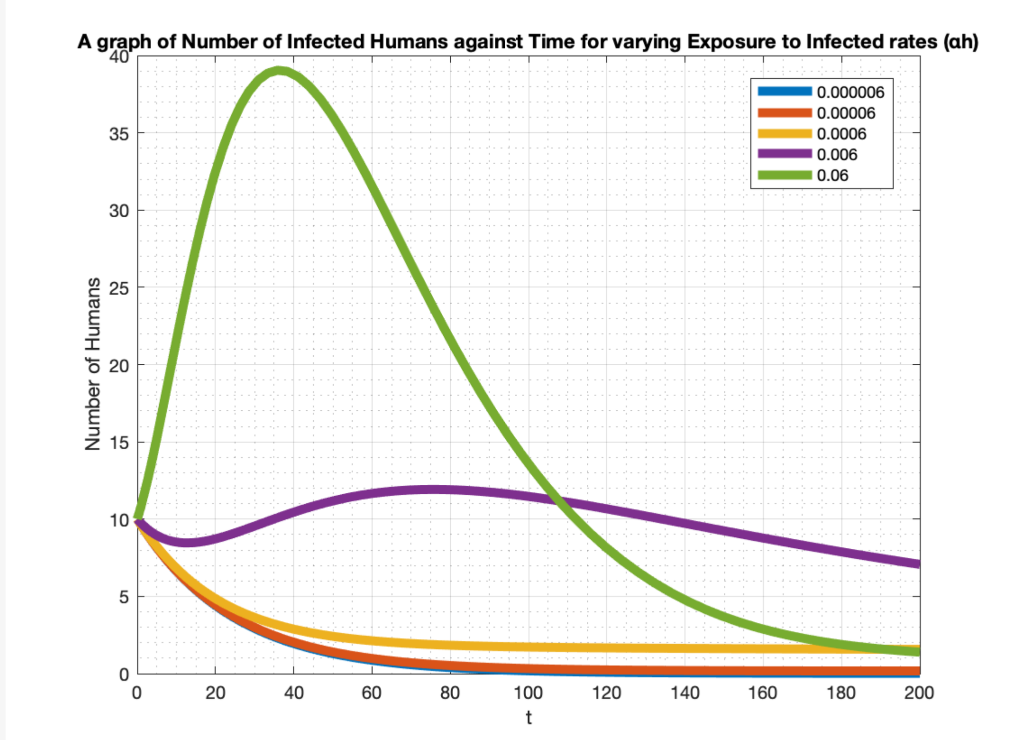
$$\begin{cases} \dfrac{dS_h(t)}{dt} = \Lambda_h - \dfrac{b\beta_h S_h(t)I_m(t)}{1+\nu_h I_m(t)} - \mu_h S_h(t) + \omega R_h(t) \\ \dfrac{dE_h(t)}{dt} = \dfrac{b\beta_h S_h(t)I_m(t)}{1+\nu_h I_m(t)} - (\alpha_h + \mu_h)E_h(t) \\ \dfrac{dI_h(t)}{dt} = \alpha_h E_h(t) - (r + \mu_h + \delta_h)I_h(t) \\ \dfrac{dR_h(t)}{dt} = rI_h(t) - (\mu_h + \omega)R_h(t) \\ \dfrac{dS_m(t)}{dt} = \Lambda_m - \dfrac{b\beta_m S_m(t)I_h(t)}{1+\nu_m I_h(t)} - \mu_m S_m(t) \\ \dfrac{dE_m(t)}{dt} = \dfrac{b\beta_m S_m(t)I_h(t)}{1+\nu_m I_h(t)} - (\alpha_m + \mu_m)E_m(t) \\ \dfrac{dI_m(t)}{dt} = \alpha_m E_m(t) - (\mu_m + \delta_m)I_h(t), \end{cases}$$

Parameters and Implications for the Model

Λh = Birth rate of humans - assumption that all children are born healthy so all births join susceptible class
Λm = Birth rate of mosquitoes - assumption that all mosquitoes are born uninfected so all births join susceptible class
βh = Probability of presence of parasite in mosquito bite
βm = Probability of presence of parasite in blood sucked by mosquito
αh = Rate at which exposed humans moves to the Infected state
αm = Rate at which exposed mosquitoes moves to the Infected state
μh = Natural human death rate - reduces entire population
μm = Natural mosquito death rate - reduces entire population
δh = Disease induced human death rate - Only infectious class is reduced by this class
δm = Disease induced mosquito death rate - Only infectious class is reduced by this class
ω = Rate at which recovered human loses immunity and rejoins susceptible population
r = Average recovery rate of humans
b = Proportion of bites that produce infection in humans
vh = Proportion of antibodies produced by humans in response to antigens produced by infectious mosquito bite
vm = Proportion of antibodies produced by mosquitoes in response to antigens produced from infectious human blood
1/(1+vhIm(t)) = Saturating feature that inhibits force of infection from infectious mosquitoes to susceptible humans
t = Time
Sh(t) = Number of Susceptible humans at t
Eh(t) = Number of Exposed humans at t - Exposed humans cannot transmit disease as parasite is in asexual stage
Ih(t) = Number of Infectious humans at t - Individuals here will either die or move to recovered class
Rh(t) = Number of Recovered humans at t
Sm(t) = Number of Susceptible mosquitoes at t
Em(t) = Number of Exposed mosquitoes at t
Im(t) = Number of Infectious mosquitoes at t

PARAMETER SENSITIVITY ANALYSIS AND MODIFICATIONS

Sensitivity analysis is a study to measure the impacts of fluctuations in parameters of a mathematical model or system on the outputs or performance of the system.
Varying post-exposure infection rate attempted to model a situation where a vaccine for Malaria was in use. In the parameter sensitivity analysis, the values of (αh) were obtained by consistently reducing the paper supplied value of 0.06 by dividing by 10 to study the resulting effect on infection rate of humans (Ih). This resulted in the following set of values of αh for the analysis and corresponding maximum human infection values.



From the results, it is observed that a decrease in post-exposure infection rates led to a simultaneous drastic reduction in the number of infected humans. This is an indication that if a vaccine was introduced to decrease the rate of post-exposure infections, there would be a corresponding decrease in human infection rates.

Paper Modifications

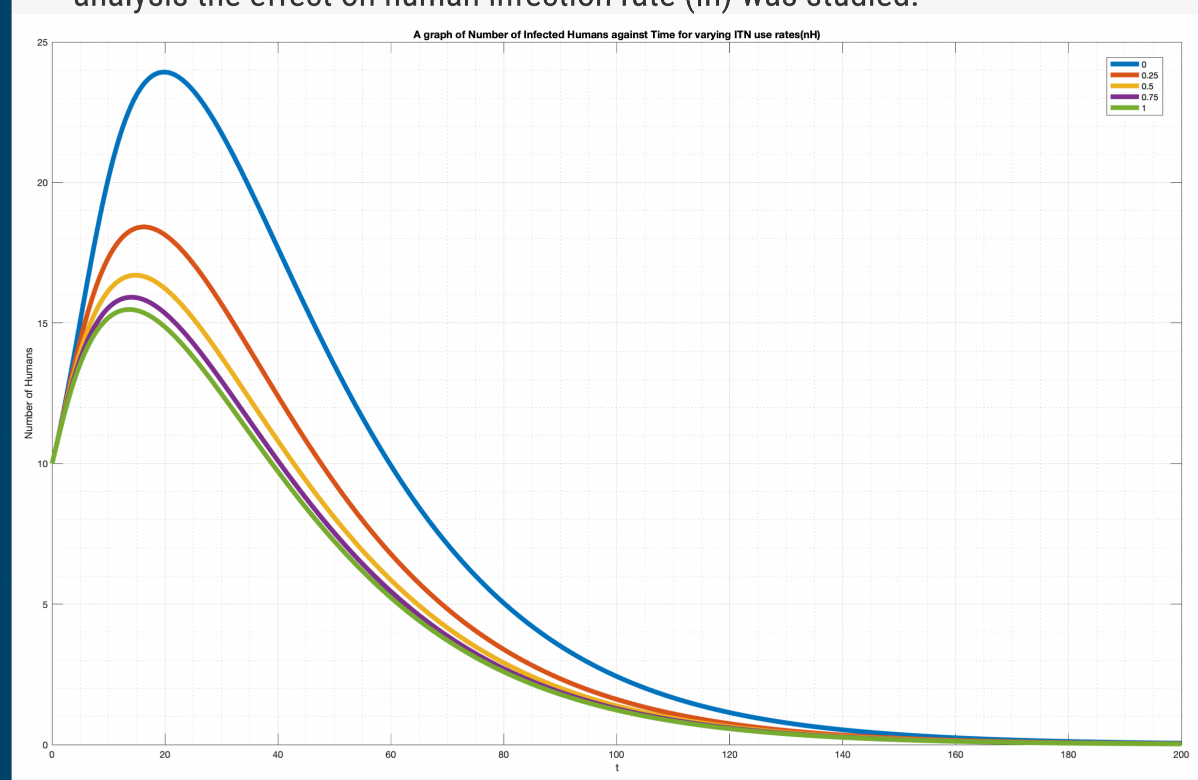
- Mosquito Nets (ITNs) are a form of personal protection that has been shown to reduce malaria illness, severe disease, and death due to malaria in endemic regions.
- This modification involved the subtraction of the term involving the product of the proportion of the population utilizing ITNs (nH), the efficacy of ITNs (pM) and the susceptible population (Sh) [the resulting term is (- nH*pM*Sh)] from the susceptible class of humans (Sh) and the addition of this same factor (+nH*pM*Sh) to the recovered class of the population (Rh). This is done to move all members to who are protected by the ITNs to the recovered population as they are no longer a part of the susceptible population.

This creates the following modified equations:

Modified Extended Ross Model

$$\begin{cases} \dfrac{dS_h(t)}{dt} = \Lambda_h - \dfrac{b\beta_h S_h(t)I_m(t)}{1+\nu_h I_m(t)} - \mu_h S_h(t) + \omega R_h(t) \quad -n_h p_m S_{h(t)} \\ \dfrac{dE_h(t)}{dt} = \dfrac{b\beta_h S_h(t)I_m(t)}{1+\nu_h I_m(t)} - (\alpha_h + \mu_h)E_h(t) \\ \dfrac{dI_h(t)}{dt} = \alpha_h E_h(t) - (r + \mu_h + \delta_h)I_h(t) \\ \dfrac{dR_h(t)}{dt} = rI_h(t) - (\mu_h + \omega)R_h(t) \quad +n_h p_m S_{h(t)} \\ \dfrac{dS_m(t)}{dt} = \Lambda_m - \dfrac{b\beta_m S_m(t)I_h(t)}{1+\nu_m I_h(t)} - \mu_m S_m(t) \quad -p_m S_{m(t)} \\ \dfrac{dE_m(t)}{dt} = \dfrac{b\beta_m S_m(t)I_h(t)}{1+\nu_m I_h(t)} - (\alpha_m + \mu_m)E_m(t) \quad -p_m E_{m(t)} \\ \dfrac{dI_m(t)}{dt} = \alpha_m E_m(t) - (\mu_m + \delta_m)I_m(t) \quad -p_m I_{m(t)} \end{cases}$$

- Comparing the original Extended Ross model to the modified model, it is observed that factoring in ITN use resulted in a substantial decrease in the susceptible population and an increase in the recovered population. This is a seeming indication that ITN use can make a significant dent in the fight for malaria eradication.
- Varying proportion of population utilizing ITNs (nH) was an attempt to analyse the effect of large scale ITN use on malaria propagation. In the parameter sensitivity analysis, the values of (nH) were obtained by both consistently increasing the proportion of the population by 0.25 between the values of 0 and 1 as 0 indicated no ITN use and 1 indicate acrossboard ITN usage, during the analysis the effect on human infection rate (Ih) was studied.



An increase in the proportion of the population utilizing ITNs resulted in a corresponding decrease in infection rates. These results indicate that widespread ITNs use would result in a significant decrease in Malaria cases.

By Deborah Enya Esi Debre