WHY DIFFERENTIAL EQUATIONS ON DAY 1 WITH M&MS

SIMIOD EXPO 2022
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INSPIRATION

2016 JMM Meeting in Baltimore

"Creating and Using Online Community Resources for Teaching Differential Equations with Modeling and Technology" by Brian J. Winkel

ALL THIS IN JUST ONE CLASS SESSION??

M & M ® Population modeling – death and immigration

Great first day activity

- Introduces students to "Why Diff Eq?"
- Demonstrates software such as MATLAB
- Illustrates concepts such as Initial Value, Rate of Change, Family of Solutions, Equilibrium Point, Discrete vs Continuous Solutions, Transient Terms, and Steady State Solutions
- Who doesn't like candy on Day 1?

Resources: Modeling Scenarios

Type: Modeling Scenarios ▼ Go

Tag

- [All] ›
- absorption (2) ›
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- air management (1) ›
- Akaike Information Criterion (4) ›
- algae (2) ›
- amplitude (2) ›
- analysis (1) ›

Resources

Sort by Title ▼

- 1-1-S-MandMDeathAndImmigration ›
- 1-1-T-MandMDeathAndImmigration ›
- 1-10-S-LSDAndProblemSolving ›
- 1-10-T-LSDAndProblemSolving ›
- 1-101-S-ClassM&MDeathAndImmigration ›
- 1-101-T-ClassM&MDeathAndImmigration ›
- 1-11-S-Kinetics ›
- 1-11-T-Kinetics ›
- 1-110-S-SnailsInaTidePool ›
- 1-110-T-SnailsInaTidePool ›
- 1-11A-S-Kinetics ›

1-1-T-MandMDeathAndImmigration

We describe a classroom activity in which students use M&M candies to simulate death and immigration. Students build a mathematical model, usually a linear first order, difference or differential equation, collect data, estimate parameters, and compare their model prediction with their actual data. We also present a very helpful narrative about experience in using this material... Learn more ›

Download (PDF)

M&M Population Simulation without Immigration

Directions:

Your cup originally has 50 m&m's.

1. Toss the m&m's gently onto the table.
2. Remove m&m's with the 'm' facing up – they die. Place in the 'X' cup.
3. Count the remaining m&m's from that generation. Record the data.
4. Go to Step 1 and repeat. Keep track of time (generation number) and number of m&m's each time.

Do a rough graph of the population (count) of m&m's versus time (generation number) below:

Using MATLAB to solve

Command Window

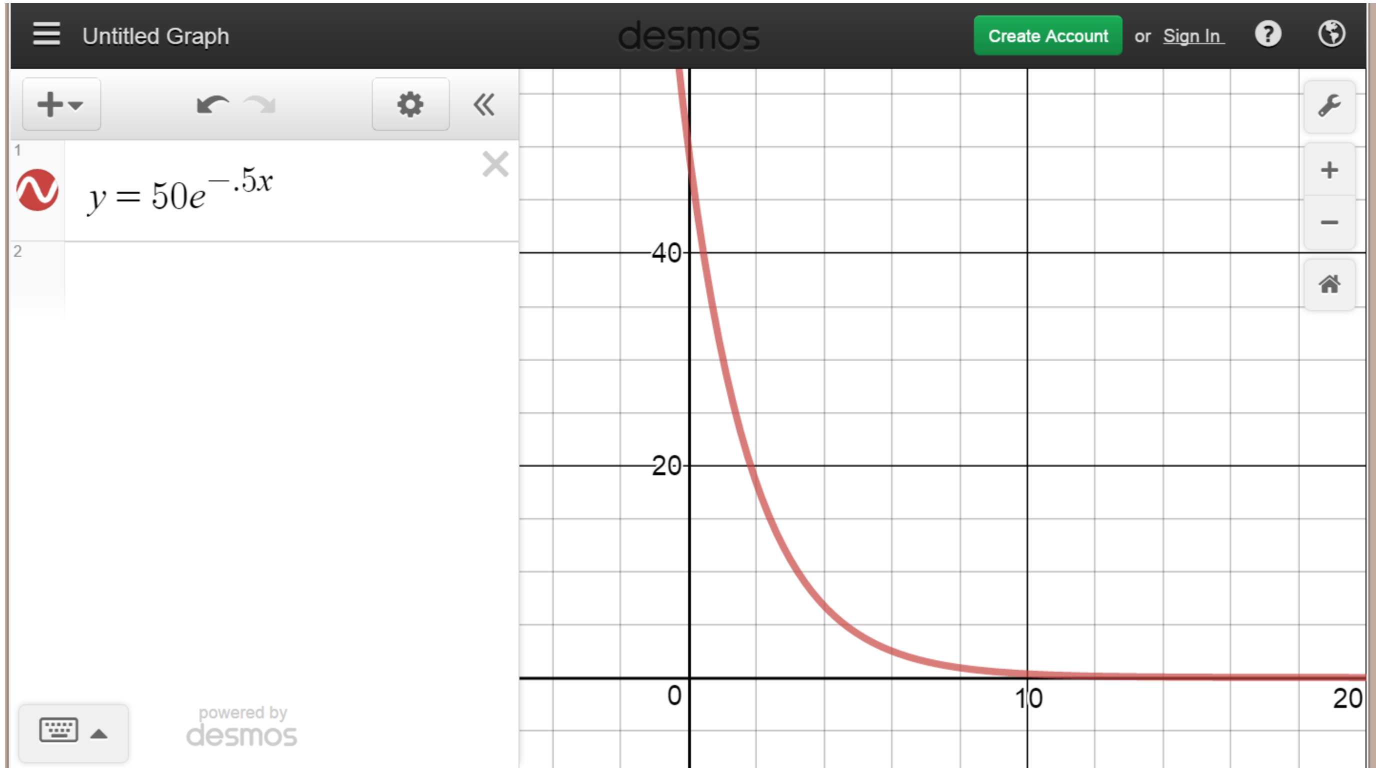
```
>> dsolve('Dy=-0.5*y','y(0)=50')

ans =

50*exp(-t/2)

>>
```

Using DESMOS to graph solution



m+m's

discrete data not continuous data

$y[0] = 20$ ← initial condition

$y[1] = 0.5\, y[0]$

$y[n+1] = 0.5\, y[n]$

$y[n]$ number of "living" m+m's in generation n.

$y[n+1] = y[n] - 0.5 y[n]$

$y[n+1] - y[n] = -0.5\, y[n]$

$y[n+\Delta n] - y[n] = -0.5 y[n]\, \Delta n$

$$\lim_{\Delta n \to 0} \frac{y[n+\Delta n] - y[n]}{\Delta n} = -0.5 y[n]$$

$$\boxed{y'(t) = -0.5\, y(t)}$$ prime notation

(Separable D.E.) $\frac{dy}{dt} = -0.5 y$ Leibniz notation

M&M Population Simulation with Immigration

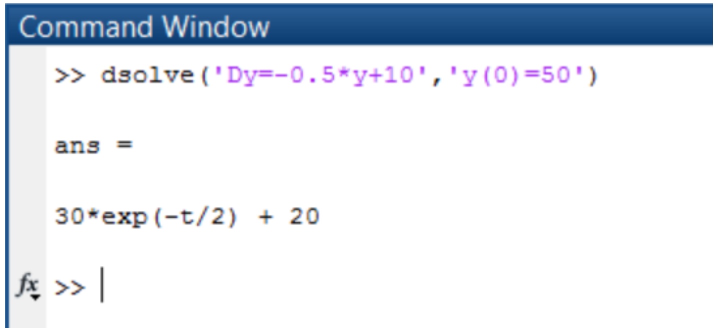
Directions:

Your cup originally has 50 m&m's.

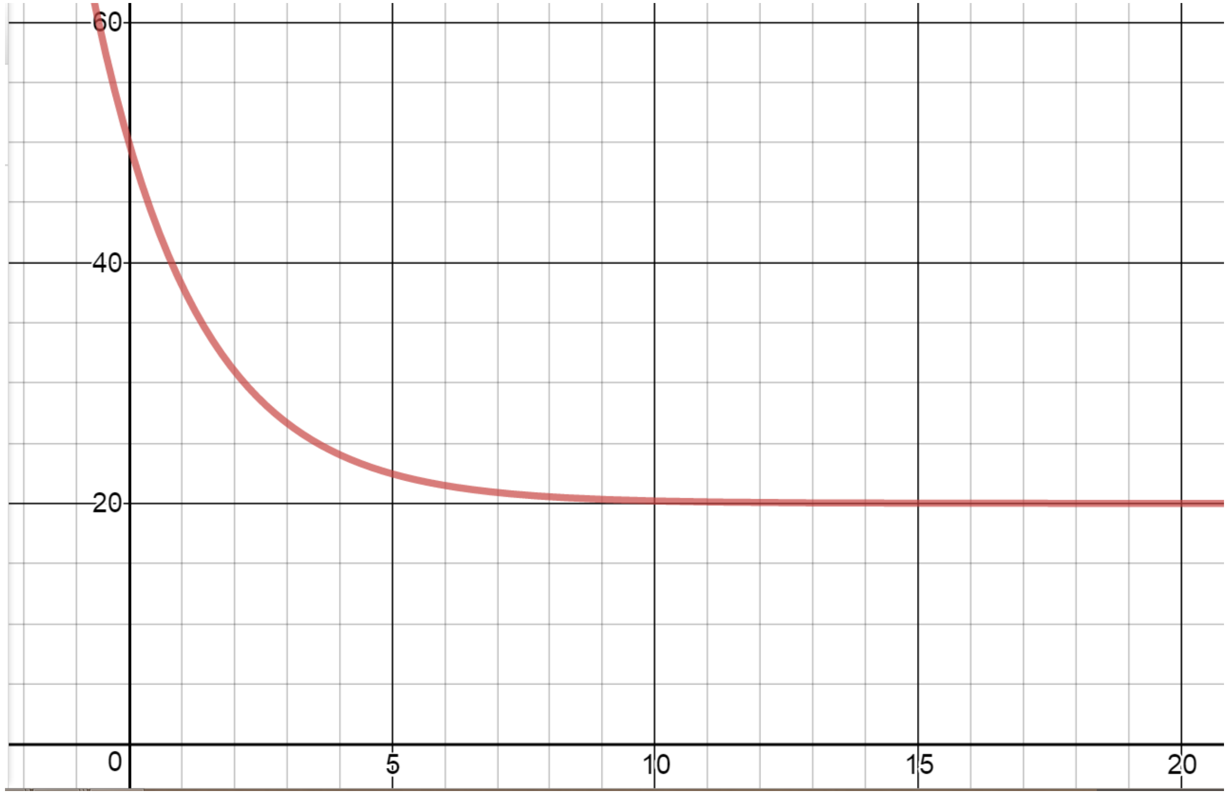
1. Toss the m&m's gently onto the table.
2. Remove m&m's with the 'm' facing up – they die. Place in the 'X' cup.
3. Add 10 m&m immigrants from the 'X' cup.
4. Count the remaining m&m's from that generation. Record the data.
5. Go to Step 1 and repeat. Keep track of time (generation number) and number of m&m's each time.

Do a rough graph of the population (count) of m&m's versus time (generation number) below:

Using MATLAB to solve



Using DESMOS to graph solution

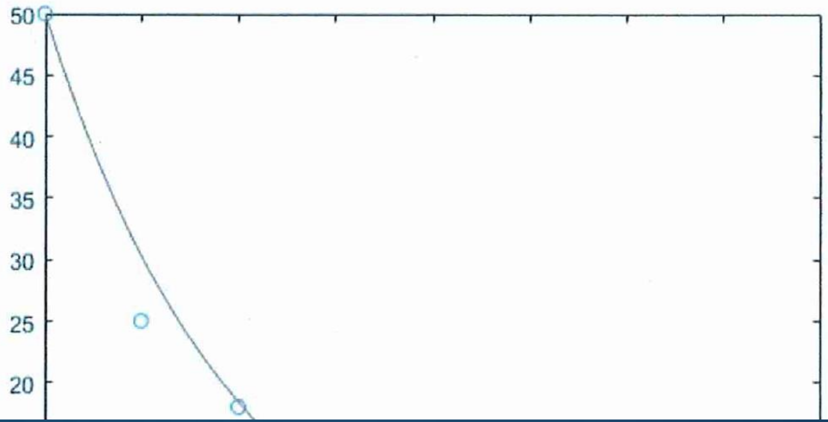


Part 1: Without Immigration

- For this activity, we upended a cup of 50 M&M's and removed all of those which showed the 'M' side facing up. We recorded the number of remaining M&M's, and repeated this until no M&M's were left in the cup
- Clearly, after enough trials, there will be no M&M's left.
- The number of M&M's after each trial looked like this:
  1. 25
  2. 18
  3. 10
  4. 4
  5. 3
  6. 2
  7. 1
  8. 0
- Our original differential equation was y' = -0.5y, and the solution to that equation is y = 50e^(-t/2).

Number of Trials

Number of M&M's Remaining



M&M Population Simulation with Immigration

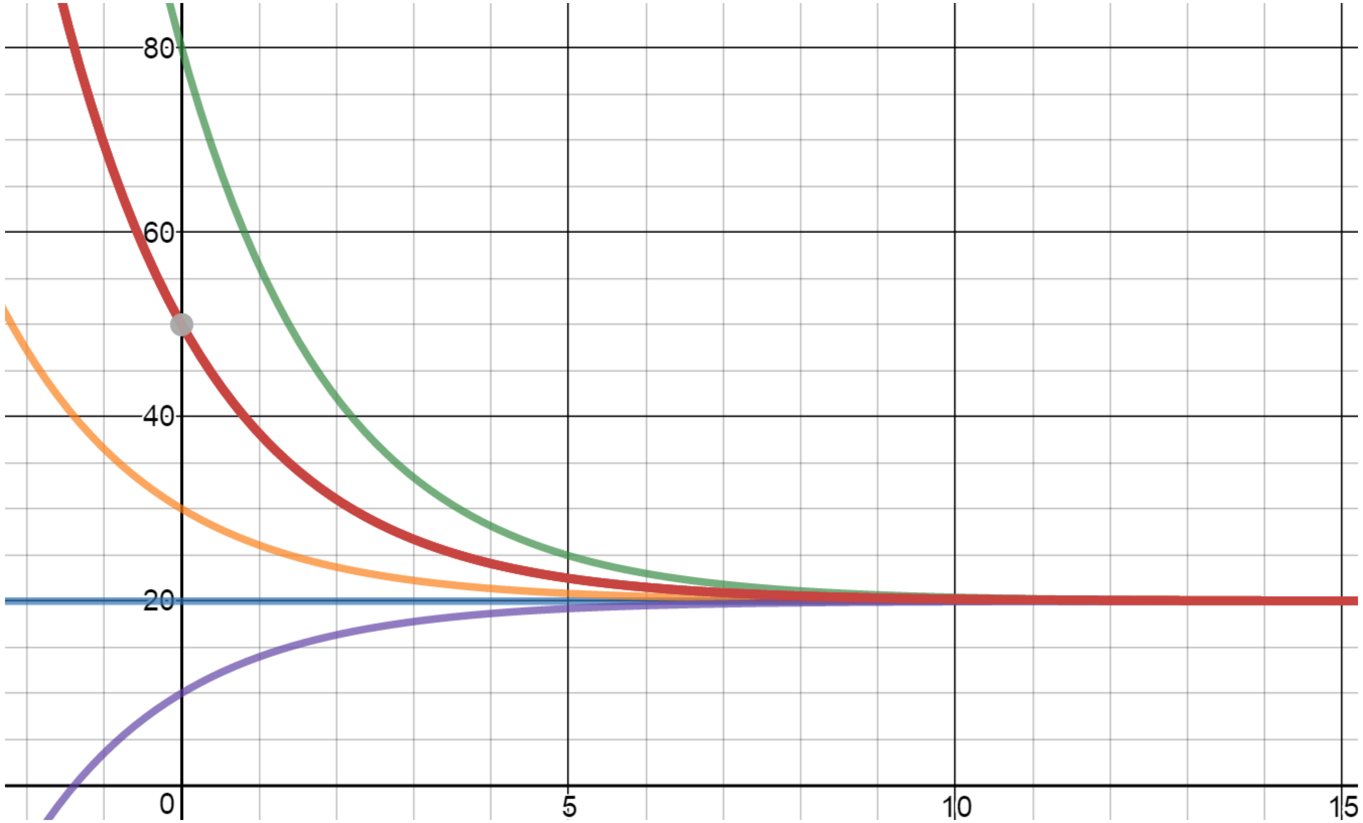
Directions:

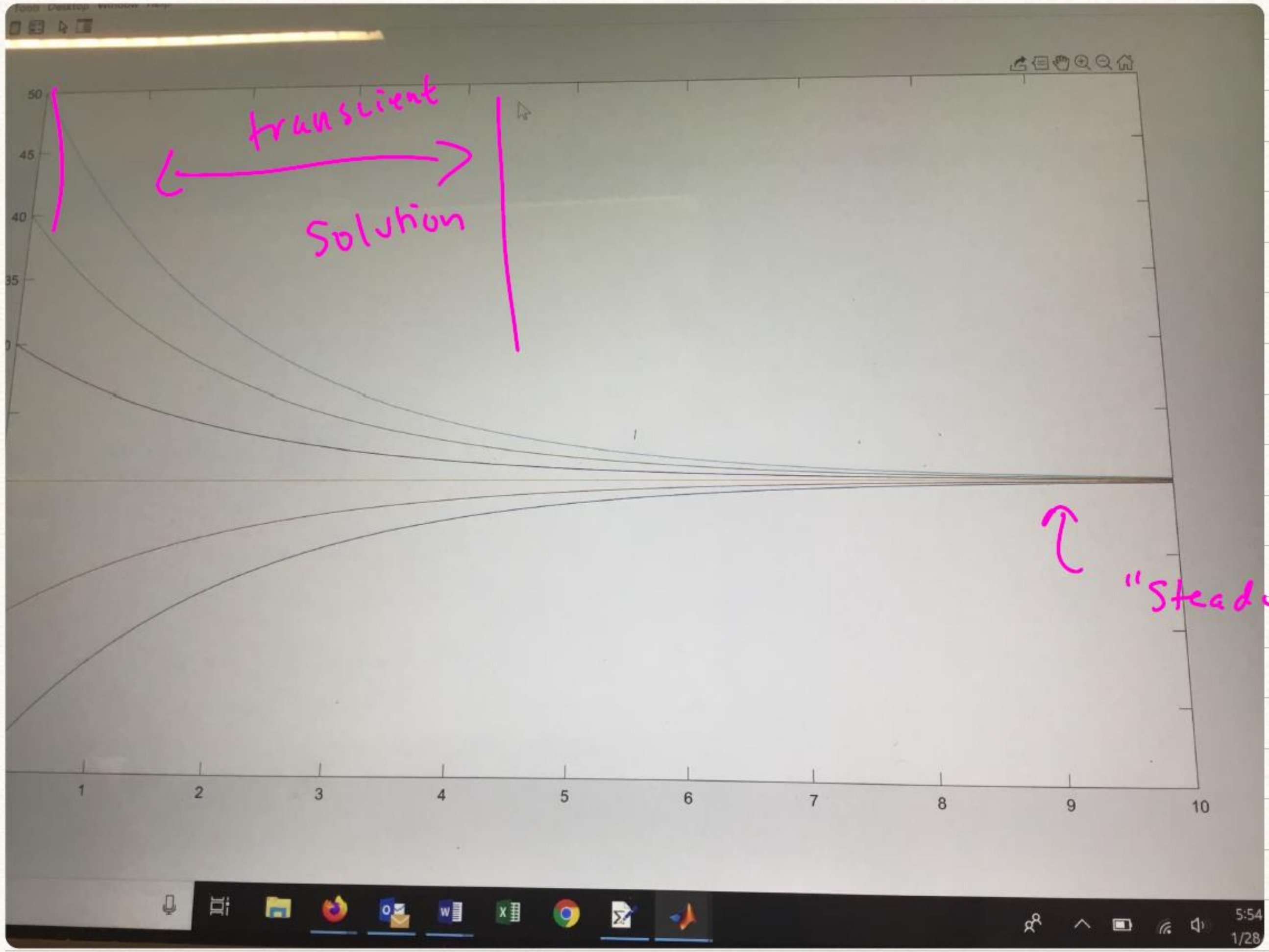
Your cup originally has _____ m&m's.

1. Toss the m&m's gently onto the table.
2. Remove m&m's with the 'm' facing up – they die. Place in the 'X' cup.
3. Add 10 m&m immigrants from the 'X' cup.
4. Count the remaining m&m's from that generation. Record the data.
5. Go to Step 1 and repeat. Keep track of time (generation number) and number of m&m's each time.

Do a rough graph of the population (count) of m&m's versus time (generation number) below:

Using DESMOS to graph solutions





Family of curves

"Steady state solution"

Part 2: With Immigration

- Here we repeated the steps from Part 1, however, after removing the M&M's but before recording the remainder, we added 10 M&M's back into the remaining population.
- Since on average half of the M&M's are being removed each trial, and we are adding in 10 before recording the results, it makes sense that the population would reach the steady state solution at 20 M&M's. On average with 20 M&M's, 10 will be removed from the population, and 10 M&M's will be added back to the population which brings the total back to 20.
  1. 21
  2. 20
  3. 19
  4. 22
  5. 23
  6. 24
  7. 22
  8. 22
  9. 26
  10. 23
  11. 22
  12. …
- The original differential equation was $y' = -0.5y + 10$, and the solution to that equation is $y = e^{(-t/2)} + 20$.

MOVING ONLINE

- More detailed instructions necessary
- Students must buy their own m&m's
- Follow up video (unlisted) posted after the due date
  - https://youtu.be/Ji8a_Elkj08

M&M Population Simulation

For these simulations, you will need a single serving bag of regular M&M's (not peanut, mini, etc.) and two cups (one will be a discard cup).

Simulation 1:
Verify all candies have a blank side and an 'm' side. Discard (or eat) any M&M's that do not have a blank and an 'm' side. Record the number of M&M's and note it in the table below. We'll call this the Initial Value.

Read the directions below. Before conducting the experiment, answer the following questions:

- How many M&M's do you think will be left if you ran this experiment an infinite number of times? Call this the Steady State solution. Type your answer here.
- How many generations do you think it will take to get to the steady state solution? Type your answer here.

Directions:

1. Toss the M&M's gently onto the table.
2. Remove M&M's with the 'm' facing up – they die. Place in the discard cup.
3. Count the remaining M&M's from that generation. Record the data in the table below.
4. Go to Step 1 and repeat. Keep track of time (generation number) and number of M&M's each time. Add rows to the table as needed.

Generation	"living" M&M's
0	(put initial value here)
1	
2	

When you are satisfied that you have reached the true steady state solution, enter the data into MATLAB and graph. Paste the graph below: (Note – there is a separate document under "MATLAB help" on Blackboard titled "scatter plot in MATLAB" if you need assistance). What type of function does this graph appear to be?

RESULTS

Student Evaluation at the end of the semester

> *Believe it or not, the m&m activity on the first day of class really stood out to me. It helped to show the real-world applications of differential equations and I thought it was amazing that we could build an equation using real-world data.*

Two and half years later, one of the students from that class stopped by my office to say:

> *"That m&m population modeling we did really made me understand what Diff Eq is all about."*