Time of death: linking differential equations and linear regression

Mark Nelson[1]

[1]School of Mathematics and Applied Statistics
University of Wollongong, Wollongong, AUSTRALIA

SIMIODE EXPO 2022

MATH212 Exam Question: 2020 & 2021 (open book)

An individual says that they left for a business meeting at 2 pm and returned at 8 pm to find their partner dead. The first temperature measurement of the dead body was made at 9pm. The individual says that they were home all morning and that their partner was alive and well when they left.

MATH212 Exam Question: 2020 & 2021 (open book)

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1. *Analyse the body temperature measurements reported in table 1 and estimate the time-of-death. Identify the uncertainty in your estimate.*

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1. *Analyse the body temperature measurements reported in table 1 and estimate the time-of-death. Identify the uncertainty in your estimate.*
2. *To what extent is your predicted time-of-death consistent with information provided by the individual?*

MATH212 Exam Question: 2020 & 2021 (open book)

Time (minutes)	body temperature (^oC)
0	35.4
60	35.3
120	35.2
180	35.0
240	34.8
300	34.5

Table 1: Body temperature of a dead individual as a function of the time after the first body temperature was made. The temperature of the room in which the body was found is 16^oC.

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In answering this question you may reuse any relevant information contained in Question 3 from the week 9 assignment, or from your answer to this question, or from the solutions provided to this question. However, you must carefully explain what information you are using, how you are using it, and where it came from.

Background information (from assignment)

1. Heat-balance around the dead-body leads to the ODE

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -\lambda\left(T - T_a\right), \quad T\left(0\right) = T_0, \tag{1}$$

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$$T = T_a + \left(T_0 - T_a\right)\exp\left[-\lambda \cdot t\right].$$

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$$\ln Z = -\lambda \cdot t,$$

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$$t_d = -\frac{1}{\lambda}\log\left[\frac{T_b - T_a}{T_0 - T_a}\right]. \tag{2}$$

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5. Assignment ques (different data): find t_d, no interpretation.

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- 2020+2021: 36 students.

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- 2020+2021: 36 students.
- 34 students answered the question.

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- What can wrong? (assignment question is template)

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- How did they do?

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- (Spoiler. I did change things from 2020 to 2021...)

Solution

- $t_d = -509\,\text{min}$ (12.21pm)

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- Limits: $t = -571\,\text{min}$ (11.29am) and $t = -459\,\text{min}$ (1.21pm).

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- Inconsistent with information provided ("left at 2pm"). More to follow!

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- Limits: $t = -571\,\text{min}$ (11.29am) and $t = -459\,\text{min}$ (1.21pm).
- Inconsistent with information provided ("left at 2pm"). More to follow!
- What can go wrong?

Student answers ($t = -509\,\text{min}$)

Time of death (t_d)	
-240 -245, -253	
-291	
-396, -396 -396, -396, -396	
-495, -495, -495	
-507, -508, -508, -508, -508, -508	
-509, -509, -509, -509, -509, -509	(6)
-511, -517	
-578, -579	
-642	
-686, -686	
-695	
-792	
-1738	

Student answers ($t = -509$ min)

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-509, -509, -509, -509, -509, -509	(6)
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What can go wrong?

What can go wrong? Out of order!

What can go wrong? Introducing...

What can go wrong? Introducing... Microsoft Excel trend line

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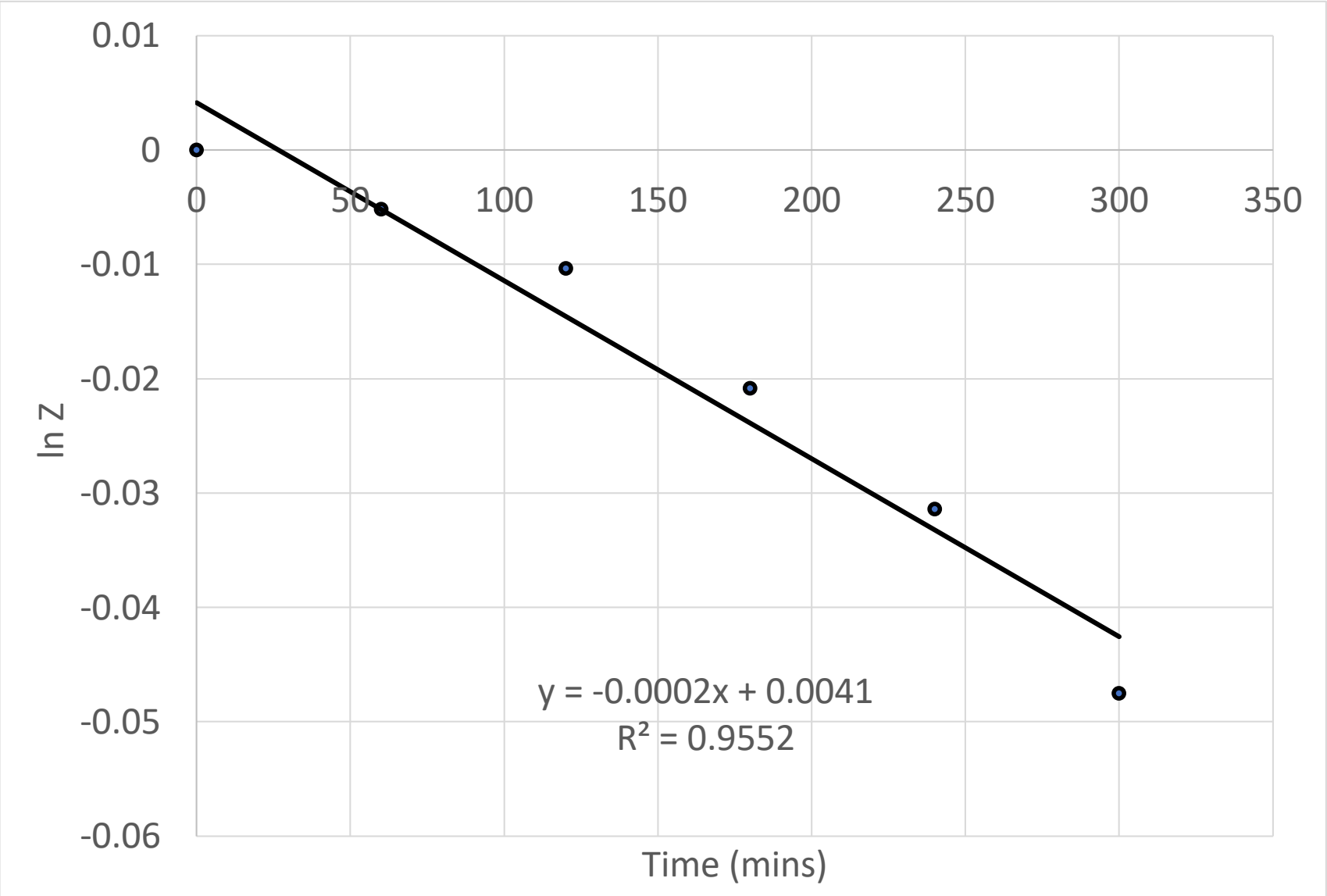

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$$\lambda = -\frac{\log Z(t)}{t}.$$

Use 5 non-zero data points: 5 values λ_i. Average gives:
$t_d = -686\,\text{min}$

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   Four students plotted T as a function of t.
   Answers: $t_d = -507, 508, 508, 517$ (minutes)

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1. **We are going to use linear regression** (3 students didn't)
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   Four students plotted T as a function of t.
   Answers: $t_d = -507, 508, 508, 517$ (minutes)
   (differences due to rounding of m/b by the *regression package*)

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1. **We are going to use linear regression** (3 students didn't)
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4. **Human rounding error** (1 student)
   - Calculated: $\lambda = -1.5558 \times 10^{-4}\,\mathrm{min}^{-1}$. (would give $t_d = -509\,\mathrm{min}$)
   - Rounded: $\lambda = -1.6 \times 10^{-4}\,\mathrm{min}^{-1}$. (gives $t_d = -495\,\mathrm{min}$)

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5. **Incorrect calculation of t_d** (2 students)

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5. **Incorrect calculation of t_d** (2 students)
6. **Other errors** (1 student)

   One student found correct regression line

$$\log Z = -1.5558 \times 10^{-4} t + 0.0041$$

but incorrect answer (how?): $t_d = -245\,\text{min}$.

Quick recap

- 36 students

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- 2 did not submit answers

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- 13 students made errors along the way

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- 13 students made errors along the way
- 21 students answered correctly...

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- 2 did not submit answers
- 13 students made errors along the way
- 21 students answered correctly... oh no they didn't

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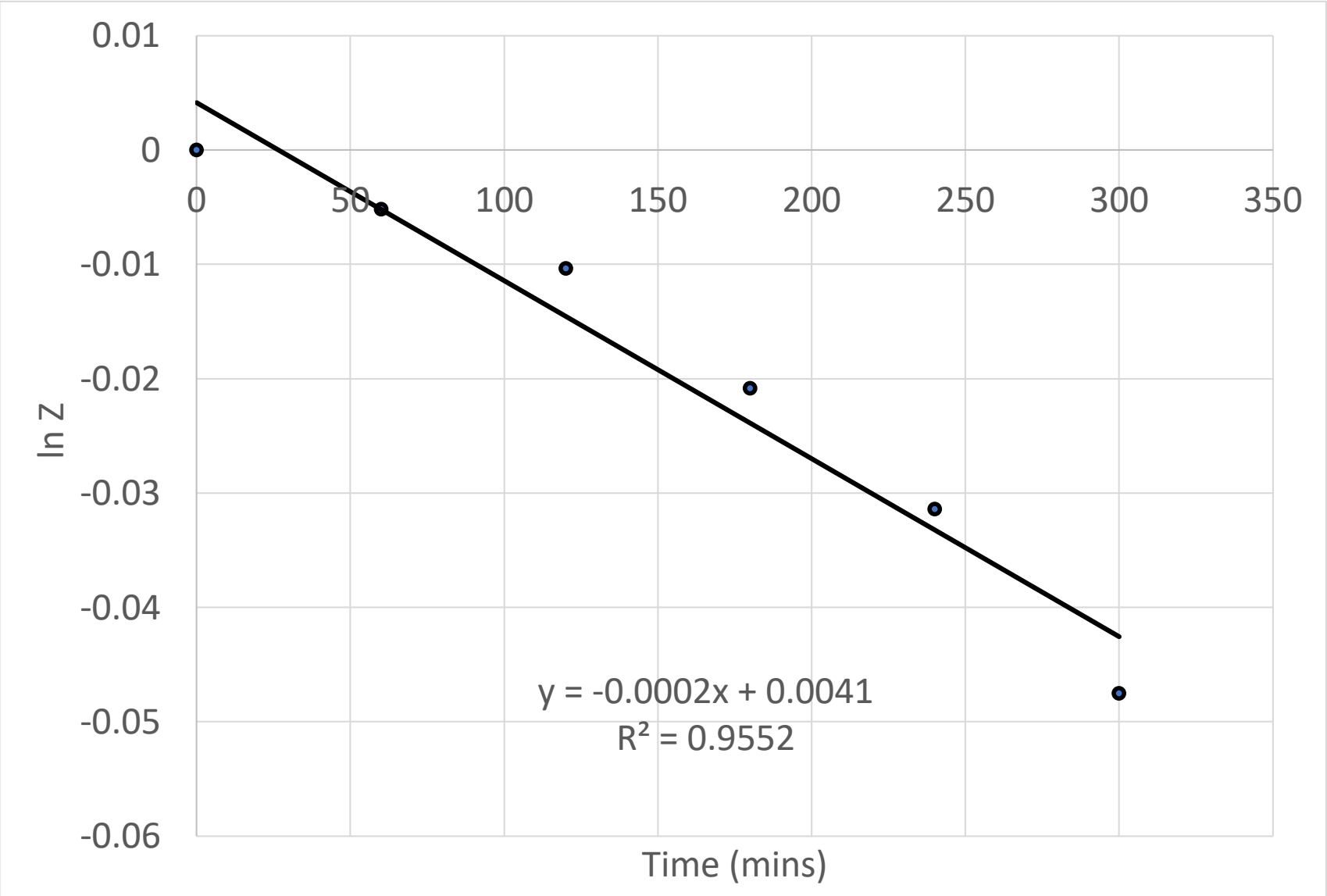

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  $\lambda = -1.56 \times 10^{-4}\,\mathrm{min}^{-1}$,
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- Three students (2 Excel LINEST, 1 www.socscistatistics.com)
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- Five students (Excel trend line)
  $\lambda = -2 \times 10^{-4}\,\mathrm{min}^{-1}$
  Four students: $t_d = -396\,\mathrm{min}$
  One student : $t_d = -253\,\mathrm{min}$

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  Four students: $t_d = -396\,\mathrm{min}$
  One student : $t_d = -253\,\mathrm{min}$
- One student (Excel but unit of time is hours)
  $\lambda = -0.0093\,\mathrm{h}^{-1}$
  $t_d = 511\,\mathrm{min}$.

Conclusion

- Evidence inconsistent with statement.

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  - ‘statement seems suspicious’
  - ‘untrue’
  - ‘likely lying’
  - ‘the individual is lying’ (2)
  - ‘invalidates their alibi’
  - ‘they are implicated in the death’

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- Significant figures produced by packages: wtf?

If you only remember one thing remember...

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