Time of death: linking differential equations and linear regression

Mark Nelson

1School of Mathematics and Applied Statistics
University of Wollongong, Wollongong, AUSTRALIA

SIMIODE EXPO 2022
An individual says that they left for a business meeting at 2 pm and returned at 8 pm to find their partner dead. The first temperature measurement of the dead body was made at 9pm. The individual says that they were home all morning and that their partner was alive and well when they left.
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1. Analyse the body temperature measurements reported in table 1 and estimate the time-of-death. Identify the uncertainty in your estimate.
An individual says that they left for a business meeting at 2 pm and returned at 8 pm to find their partner dead. The first temperature measurement of the dead body was made at 9pm. The individual says that they were home all morning and that their partner was alive and well when they left.

1. Analyse the body temperature measurements reported in table 1 and estimate the time-of-death. Identify the uncertainty in your estimate.
2. To what extent is your predicted time-of-death consistent with information provided by the individual?
### Table 1: Body temperature of a dead individual as a function of the time after the first body temperature was made. The temperature of the room in which the body was found is 16°C.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>body temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>35.4</td>
</tr>
<tr>
<td>60</td>
<td>35.3</td>
</tr>
<tr>
<td>120</td>
<td>35.2</td>
</tr>
<tr>
<td>180</td>
<td>35.0</td>
</tr>
<tr>
<td>240</td>
<td>34.8</td>
</tr>
<tr>
<td>300</td>
<td>34.5</td>
</tr>
</tbody>
</table>
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In answering this question you may reuse any relevant information contained in Question 3 from the week 9 assignment, or from your answer to this question, or from the solutions provided to this question. However, you must carefully explain what information you are using, how you are using it, and where it came from.
Heat-balance around the dead-body leads to the ODE

\[
\frac{dT}{dt} = -\lambda (T - T_a), \quad T(0) = T_0, \quad (1)
\]
Background information (from assignment)

1. Heat-balance around the dead-body leads to the ODE

\[
\frac{dT}{dt} = -\lambda \left(T - T_a\right), \quad T(0) = T_0, \quad (1)
\]

2. Solution

\[
T = T_a + (T_0 - T_a) \exp \left[-\lambda \cdot t\right].
\]
Heat-balance around the dead-body leads to the ODE

\[
\frac{dT}{dt} = -\lambda (T - T_a), \quad T(0) = T_0, \tag{1}
\]

Solution

\[
T = T_a + (T_0 - T_a) \exp[-\lambda \cdot t].
\]

Re-arrange.

\[
\ln Z = -\lambda \cdot t,
\]
Heat-balance around the dead-body leads to the ODE
\[ \frac{dT}{dt} = -\lambda (T - T_a), \quad T(0) = T_0, \] (1)

Solution
\[ T = T_a + (T_0 - T_a) \exp [-\lambda \cdot t]. \]

Re-arrange.
\[ \ln Z = -\lambda \cdot t, \]
\[ Z(t) = \frac{T(t) - T_a}{T_0 - T_a} \]
Heat-balance around the dead-body leads to the ODE
\[
\frac{dT}{dt} = -\lambda (T - T_a), \quad T(0) = T_0,
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Solution
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Re-arrange.
\[
\ln Z = -\lambda \cdot t,
\]
\[
Z(t) = \frac{T(t) - T_a}{T_0 - T_a}
\]

\[
t_d = -\frac{1}{\lambda} \log \left[ \frac{T_b - T_a}{T_0 - T_a} \right].
\] (2)
Heat-balance around the dead-body leads to the ODE
\[ \frac{dT}{dt} = -\lambda (T - T_a), \quad T(0) = T_0, \quad (1) \]

Solution
\[ T = T_a + (T_0 - T_a) \exp [-\lambda \cdot t]. \]

Re-arrange.
\[ \ln Z = -\lambda \cdot t, \]
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Assignment ques (different data): find \( t_d \), no interpretation.
Student background

- MATH212: “Mathematical Modelling”.

Almost all students on a mathematics degree (some STEM). STAT101: includes simple regression. STAT101 not a pre-requisite but a small number of students have not taken STAT101. 2020+2021: 36 students. 34 students answered the question. What can go wrong? (assignment question is template) How did they do? (Spoiler. I did change things from 2020 to 2021...)
Student background

- MATH212: “Mathematical Modelling”.
- Almost all students on a mathematics degree (some flavour).
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- STAT101 not a pre-requisite but...
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- What can wrong? (assignment question is template)
- How did they do?
- (Spoiler. I did change things from 2020 to 2021...)
Solution

- \( t_d = -509 \text{ min (12.21pm)} \)

Inconsistent with information provided ("left at 2pm"). More to follow!
Solution

- \( t_d = -509 \text{ min (12.21pm)} \)
- Limits: \( t = -571 \text{ min (11.29am)} \) and \( t = -459 \text{ min (1.21pm)} \)
Solution

- $t_d = -509 \text{ min (12.21pm)}$
- Limits: $t = -571 \text{ min (11.29am)}$ and $t = -459 \text{ min (1.21pm)}$.
- Inconsistent with information provided (“left at 2pm”). More to follow!
Solution

- $t_d = -509 \text{ min (12.21pm)}$
- Limits: $t = -571 \text{ min (11.29am)}$ and $t = -459 \text{ min (1.21pm)}$.
- Inconsistent with information provided ("left at 2pm"). More to follow!
- What can go wrong?
## Student answers \((t = \text{-509 min})\)

<table>
<thead>
<tr>
<th>Time of death ((t_d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-240, -253</td>
</tr>
<tr>
<td>-291</td>
</tr>
<tr>
<td>-396, -396, -396, -396, -396</td>
</tr>
<tr>
<td>-495, -495, -495</td>
</tr>
<tr>
<td>-507, -508, -508, -508, -508, -508</td>
</tr>
<tr>
<td>-509, -509, -509, -509, -509, -509 (\text{(6)})</td>
</tr>
<tr>
<td>-511, -517</td>
</tr>
<tr>
<td>-578, -579</td>
</tr>
<tr>
<td>-642</td>
</tr>
<tr>
<td>-686, -686</td>
</tr>
<tr>
<td>-695</td>
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Student answers \(( t = -509 \text{ min})\) What can go wrong?

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</tr>
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What can go wrong?
What can go wrong? Out of order!

\[ y = -0.0002x + 0.0041 \]

\[ R^2 = 0.9552 \]
What can go wrong?  Introducing...
What can go wrong? Introducing... Microsoft Excel trend line
What can go wrong? Introducing... Microsoft Excel trend line

\[ y = -0.0002x + 0.0041 \]

\[ R^2 = 0.9552 \]
What went wrong (Part One)

We are going to use linear regression (3 students didn't)

We need to calculate $Z(t)$ correctly (2 students didn't)

We plot $\log Z$ against $t$. (4 students didn't)

Human rounding error (1 student)

Incorrect calculation of $t_d$ (2 students)

Other errors (1 student)

One student found correct regression line

\[ \log Z = -1.5558 \times 10^{-4} t + 0.0041 \]

but incorrect answer (how?): $t_d = -245$ min.
What went wrong (Part One)

1. We are going to use linear regression (3 students didn’t)

We need to calculate $Z(t)$ correctly (2 students didn’t)

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Human rounding error (1 student)

Incorrect calculation of $t_d$ (2 students)

Other errors (1 student)

One student found correct regression line $\log Z = -1.5558 \times 10^{-4} t + 0.0041$ but incorrect answer (how?): $t_d = -245$ min.
What went wrong (Part One)

1. **We are going to use linear regression**  
   *Two students*  
   
   \[ \lambda = -\frac{\log Z(t)}{t}. \]

   *(3 students didn’t)*
What went wrong (Part One)

1. We are going to use linear regression

   Two students

   \[ \lambda = -\frac{\log Z(t)}{t}. \]

   Use 5 non-zero data points: 5 values \( \lambda_i \). Average gives:
   \( t_d = -686 \text{ min} \)
What went wrong (Part One)

1. We are going to use linear regression (3 students didn’t)
2. We need to calculate $Z(t)$ correctly (2 students didn’t)

One student found correct regression line
\[ \log Z = -1.5558 \times 10^{-4} t + 0.0041 \]
but incorrect answer (how?):
\[ t_d = -245 \text{ min.} \]
What went wrong (Part One)

1. **We are going to use linear regression** (3 students didn’t)
2. **We need to calculate** $Z(t)$ **correctly** (2 students didn’t)
3. **We plot** $\log Z$ **against** $t.$ (4 students didn’t)
What went wrong (Part One)

1. **We are going to use linear regression**
   (3 students didn’t)

2. **We need to calculate** \( Z(t) \) **correctly**
   (2 students didn’t)

3. **We plot** \( \log Z \) **against** \( t \).
   (4 students didn’t)
   Four students plotted \( T \) as a function of \( t \).

One student found correct regression line:
\[
\log Z = -1.5558 \times 10^{-4} t + 0.0041
\]
but incorrect answer (how?):
\[
t_d = -245 \text{ min.}
\]
What went wrong (Part One)

1. We are going to use linear regression (3 students didn’t)
2. We need to calculate $Z(t)$ correctly (2 students didn’t)
3. We plot $\log Z$ against $t$. (4 students didn’t)
   Four students plotted $T$ as a function of $t$.
   Answers: $t_d = -507, 508, 508, 517$ (minutes)

   - Human rounding error (1 student)
   - Incorrect calculation of $t_d$ (2 students)
   - Other errors (1 student)

One student found correct regression line
$\log Z = -1.5558 \times 10^{-4} t + 0.0041$ but incorrect answer (how?): $t_d = -245$ min.
What went wrong (Part One)

1. We are going to use linear regression  
   (3 students didn’t)
2. We need to calculate $Z(t)$ correctly  
   (2 students didn’t)
3. We plot $\log Z$ against $t$.  
   Four students plotted $T$ as a function of $t$.  
   Answers: $t_d = -507, 508, 508, 517$ (minutes)  
   (differences due to rounding of $m/b$ by the regression package)
What went wrong (Part One)

1. We are going to use linear regression (3 students didn’t)
2. We need to calculate $Z(t)$ correctly (2 students didn’t)
3. We plot $\log Z$ against $t.$ (4 students didn’t)
4. Human rounding error (1 student)

One student found correct regression line

$$\log Z = -1.5558 \times 10^{-4} t + 0.0041$$

but incorrect answer (how?):

$$t_d = -245 \text{ min.}$$
What went wrong (Part One)

1. We are going to use linear regression (3 students didn’t)
2. We need to calculate $Z(t)$ correctly (2 students didn’t)
3. We plot $\log Z$ against $t.$ (4 students didn’t)
4. Human rounding error (1 student)
   - Calculated: $\lambda = -1.5558 \times 10^{-4} \text{ min}^{-1}.$ (would give $t_d = -509 \text{ min}$)
   - Rounded: $\lambda = -1.6 \times 10^{-4} \text{ min}^{-1}.$ (gives $t_d = -495 \text{ min}$)
What went wrong (Part One)

1. We are going to use linear regression
   (3 students didn’t)
2. We need to calculate $Z(t)$ correctly
   (2 students didn’t)
3. We plot $\log Z$ against $t$.
   (4 students didn’t)
4. Human rounding error
   (1 student)
5. Incorrect calculation of $t_d$
   (2 students)
What went wrong (Part One)

1. We are going to use linear regression
   (3 students didn’t)
2. We need to calculate $Z(t)$ correctly
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3. We plot $\log Z$ against $t$.
   (4 students didn’t)
4. Human rounding error
   (1 student)
5. Incorrect calculation of $t_d$
   (2 students)
6. Other errors
   (1 student)
What went wrong (Part One)

1. We are going to use linear regression
   (3 students didn’t)
2. We need to calculate $Z(t)$ correctly
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3. We plot $\log Z$ against $t$.
   (4 students didn’t)
4. Human rounding error
   (1 student)
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   (2 students)
6. Other errors
   (1 student)

One student found correct regression line

$$\log Z = -1.5558 \times 10^{-4} t + 0.0041$$

but incorrect answer (how?): $t_d = -245 \text{ min.}$
Quick recap

- 36 students
Quick recap

- 36 students
- 2 did not submit answers
Quick recap

- 36 students
- 2 did not submit answers
- 13 students made errors along the way
Quick recap

- 36 students
- 2 did not submit answers
- 13 students made errors along the way
- 21 students answered correctly...
Quick recap

- 36 students
- 2 did not submit answers
- 13 students made errors along the way
- 21 students answered correctly... oh no they didn’t
What can go wrong? Introducing... Microsoft Excel trend line
What can go wrong? Introducing... Microsoft Excel trend line
What can go wrong... your regression package

\[ \lambda = -1.55650 \times 10^{-4} \text{ min}^{-1}, \quad t_d = -509 \text{ min}. \]
What can go wrong... your regression package

\[ \lambda = -1.55650 \times 10^{-4} \text{ min}^{-1}, \quad t_d = -509 \text{ min}. \]

- Two students (Google Sheets):
  \[ \lambda = -1.56 \times 10^{-4} \text{ min}^{-1}, \]
  \[ t_d = -508 \text{ min} \]
What can go wrong... your regression package

\[ \lambda = -1.55650 \times 10^{-4} \text{ min}^{-1}, \quad t_d = -509 \text{ min}. \]

- Two students (Google Sheets):
  \[ \lambda = -1.56 \times 10^{-4} \text{ min}^{-1}, \quad t_d = -508 \text{ min} \]

- Three students (2 Excel LINEST, 1 [www.socscistatistics.com](http://www.socscistatistics.com))
  \[ \lambda = -1.6 \times 10^{-4} \text{ min}^{-1}. \]
  \[ t_d = -495 \text{ min} \]
What can go wrong... your regression package

\[ \lambda = -1.55650 \times 10^{-4} \text{ min}^{-1}, \quad t_d = -509 \text{ min}. \]

- Two students (Google Sheets):
  \[ \lambda = -1.56 \times 10^{-4} \text{ min}^{-1}, \quad t_d = -508 \text{ min} \]

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  \[ \lambda = -1.6 \times 10^{-4} \text{ min}^{-1}. \]
  \[ t_d = -495 \text{ min} \]

- Five students (Excel trend line)
  \[ \lambda = -2 \times 10^{-4} \text{ min}^{-1} \]
  Four students: \[ t_d = -396 \text{ min} \]
  One student: \[ t_d = -253 \text{ min} \]
What can go wrong... your regression package

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  \[ \lambda = -1.6 \times 10^{-4} \text{ min}^{-1}. \]
  \[ t_d = -495 \text{ min} \]

- Five students (Excel trend line)
  \[ \lambda = -2 \times 10^{-4} \text{ min}^{-1} \]
  Four students: \[ t_d = -396 \text{ min} \]
  One student: \[ t_d = -253 \text{ min} \]

- One student (Excel but unit of time is hours)
  \[ \lambda = -0.0093 \text{ h}^{-1} \]
  \[ t_d = 511 \text{ min}. \]
Conclusion

- Evidence inconsistent with statement.
Conclusion

- Evidence inconsistent with statement.
  - ‘statement seems suspicious’
  - ‘untrue’
  - ‘likely lying’
  - ‘the individual is lying’ (2)
  - ‘invalidates their alibi’
  - ‘they are implicated in the death’
Conclusion

- Evidence inconsistent with statement.
- Police procedural: you are providing scientific advice to the legal team representing the defendant. (two variations)
Conclusion

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- Fever?
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- Fever?
- It is a truth universally acknowledged...
Evidence inconsistent with statement.

Police procedural: you are providing scientific advice to the legal team representing the defendant. (two variations)

Fever?

It is a truth universally acknowledged...

Did change something between 2020 and 2021.

Is it fair?
Conclusion

- Evidence inconsistent with statement.
- Police procedural: you are providing scientific advice to the legal team representing the defendant. (two variations)
- Fever?
- It is a truth universally acknowledged...
- Did change something between 2020 and 2021.
- Is it fair?
- Significant figures produced by packages: wtf?
If you only remember one thing remember...
If you only remember one thing remember... Microsoft Excel trend line
If you only remember one thing remember… Microsoft Excel trend line