CONNECTING MODELS TO DATA THROUGH CASE STUDIES AND WET LABS

SIMIODE EXPO
February 13, 2022

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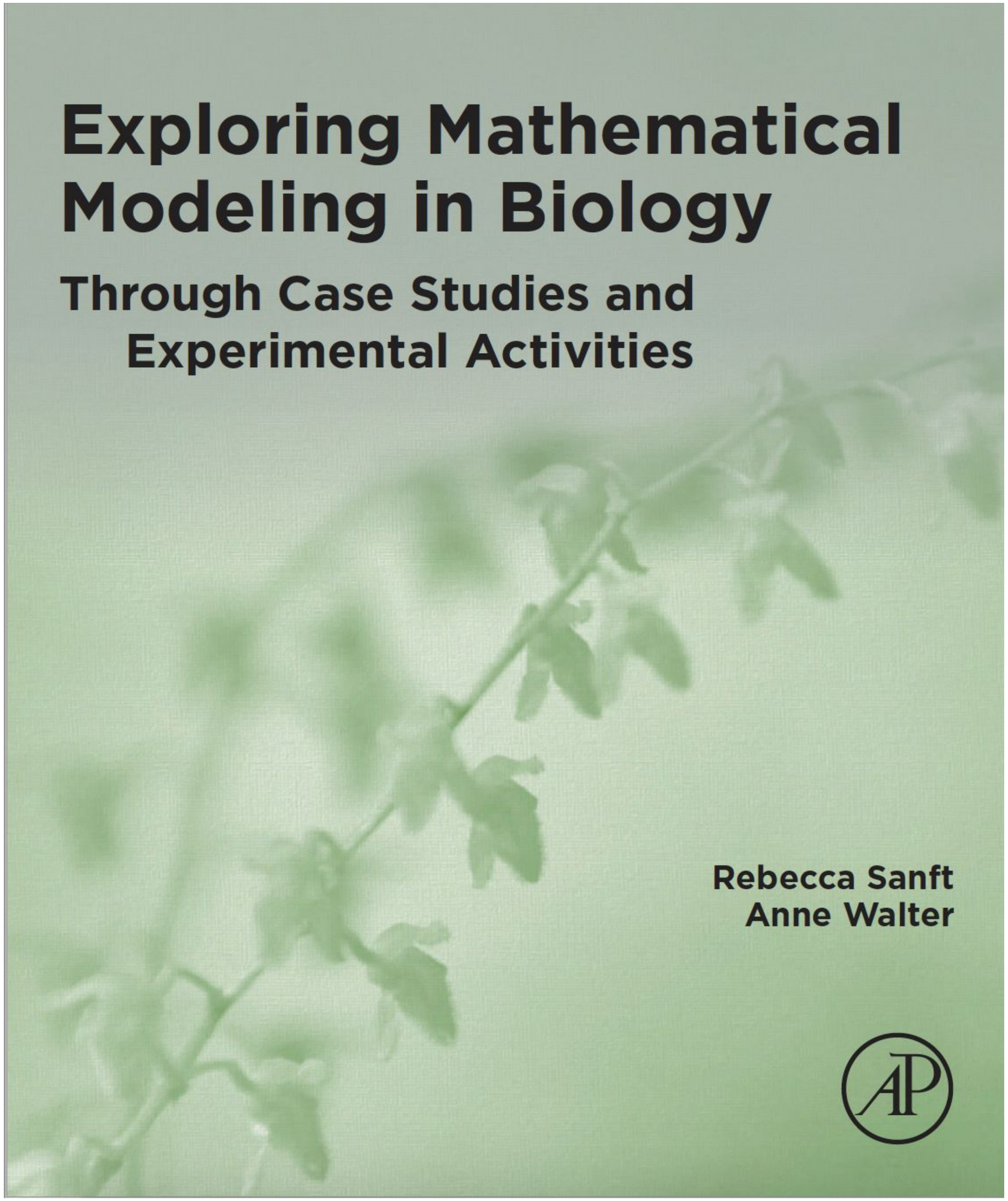
Exploring Mathematical Modeling in Biology
Through Case Studies and Experimental Activities
Rebecca Sanft
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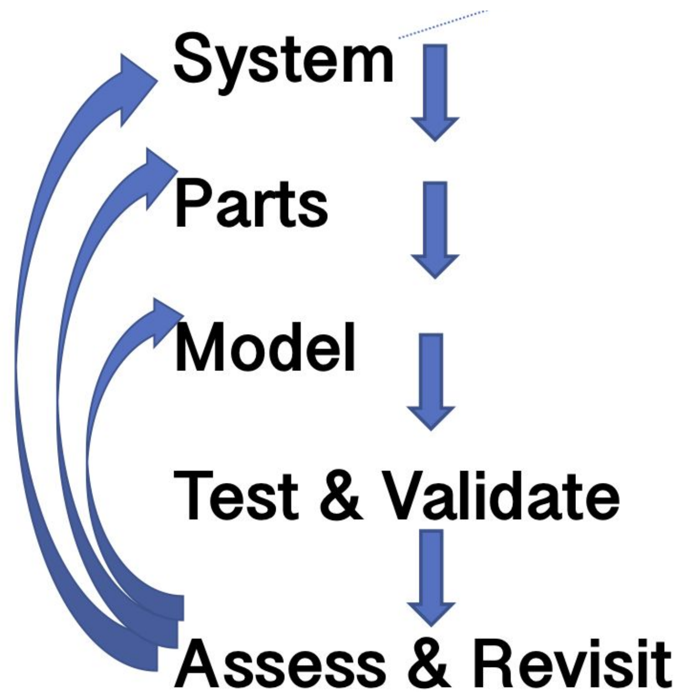
Breeze2


System
Parts
Model
Test & Validate
Assess & Revisit

Teaching Challenges
- what we observe and try to address

1. Articulating the system components & relationships.
2. Identifying units & assumptions
3. Translating into mathematical terms & making mathematical adjustments given the data at hand.
4. Finding reasonable parameter values.
5. Coding – stepwise process.
6. Assessing the meaning of the results (aka sense of failure when model doesn't fit vs. sense of enlightenment).

Our strategy
– example building to the SIR Model

Introduce modeling skills using our "touchstone" logistic growth model:

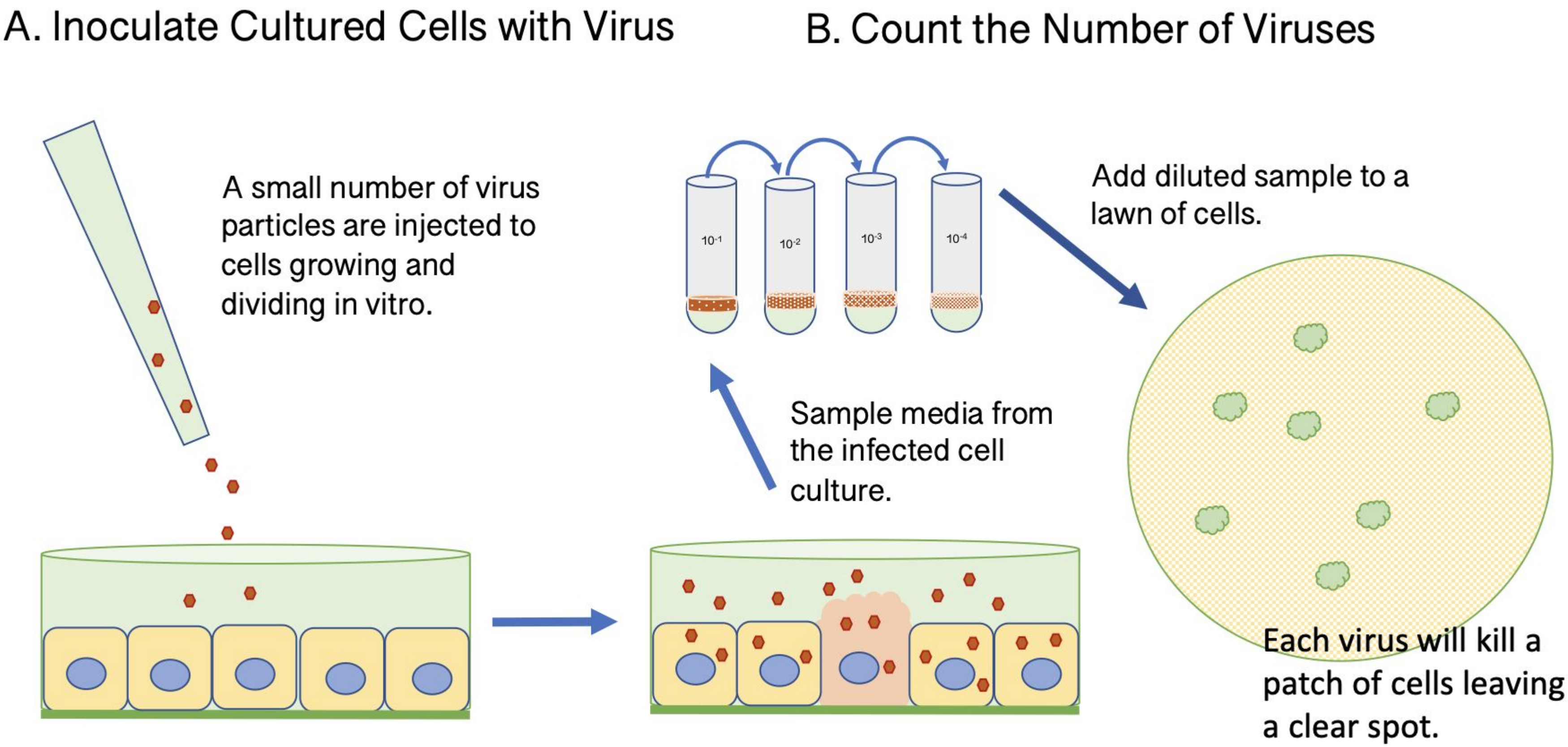
- Numerical solutions to systems of differential equations
- Parameter estimation via least squares regression (modFit)
- Sensitivity Analysis

Apply these skills in a guided example:

- Virus infection in a cell culture

Revisit a similar problem as a case study for in depth exploration.

Demonstrate "articulating the problem" before developing equations



Guided Example

Uninfected cells:

$$\frac{dU}{dt} = \lambda - \alpha U - \beta U V$$

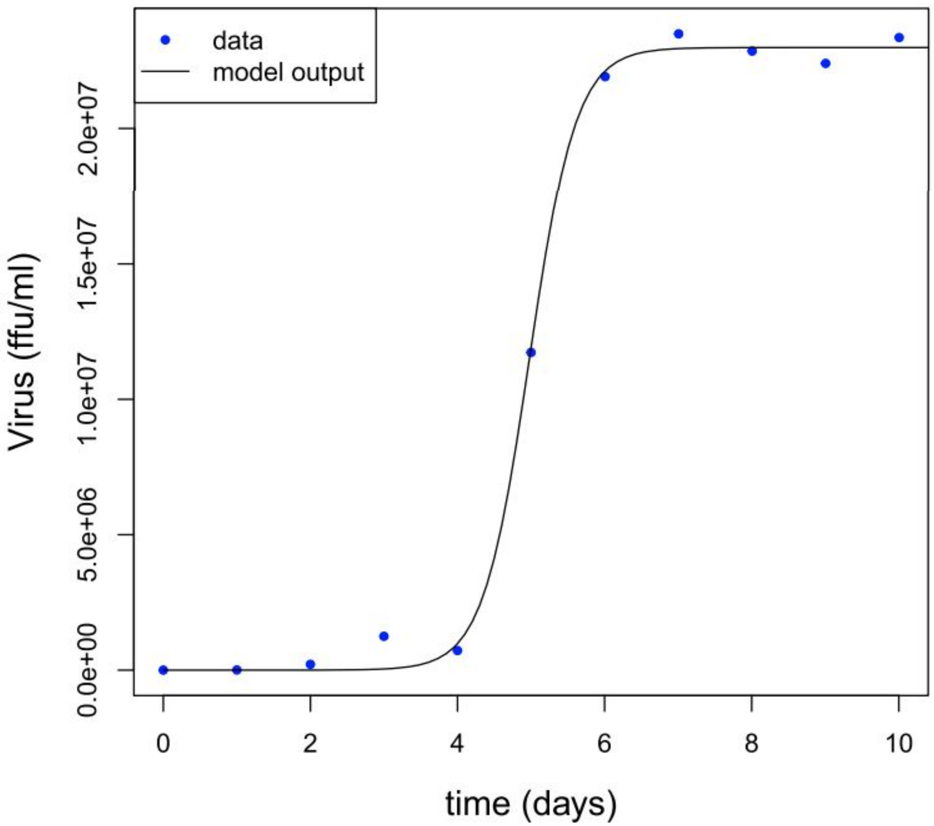
Infected cells:

$$\frac{dI}{dt} = \beta U V - \delta I$$

Virus particles:

$$\frac{dV}{dt} = pI - cV$$

Model Calibration



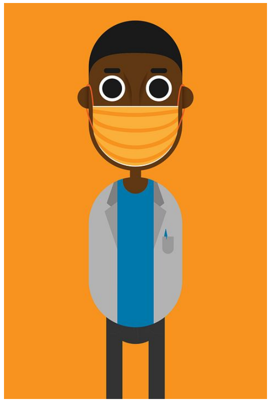
Case Study: Modeling the 2009 Influenza Pandemic

- Articulate and then introduce the model equations.
- Explore assumptions
- Explore modifications
- Play with the model to understand each part.
- Introduce a new term: cumulative number of cases to access to available data.
- Explore concept of epidemic threshold & effective reproduction number.
- Identify how public health interventions are modeled.

**Now: Model influenza in Australia & use parameters to:

a) model the first wave

b) propose realistic & effective intervention strategies before second wave



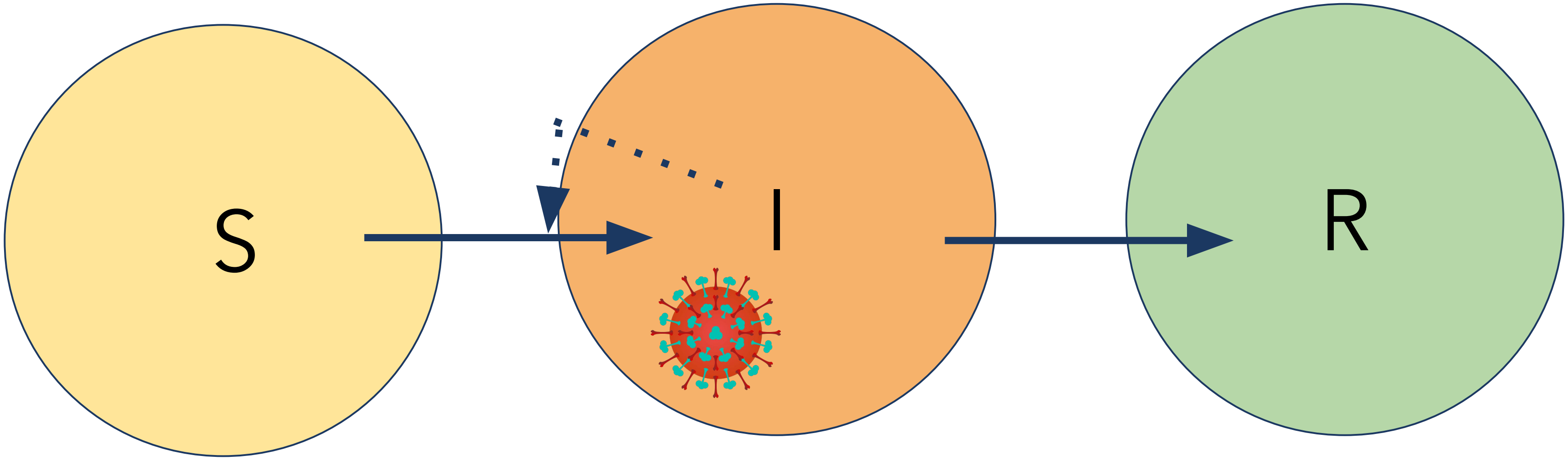
1. Articulating the system components SIR model

What will happen next?

What determines how fast it will happen?

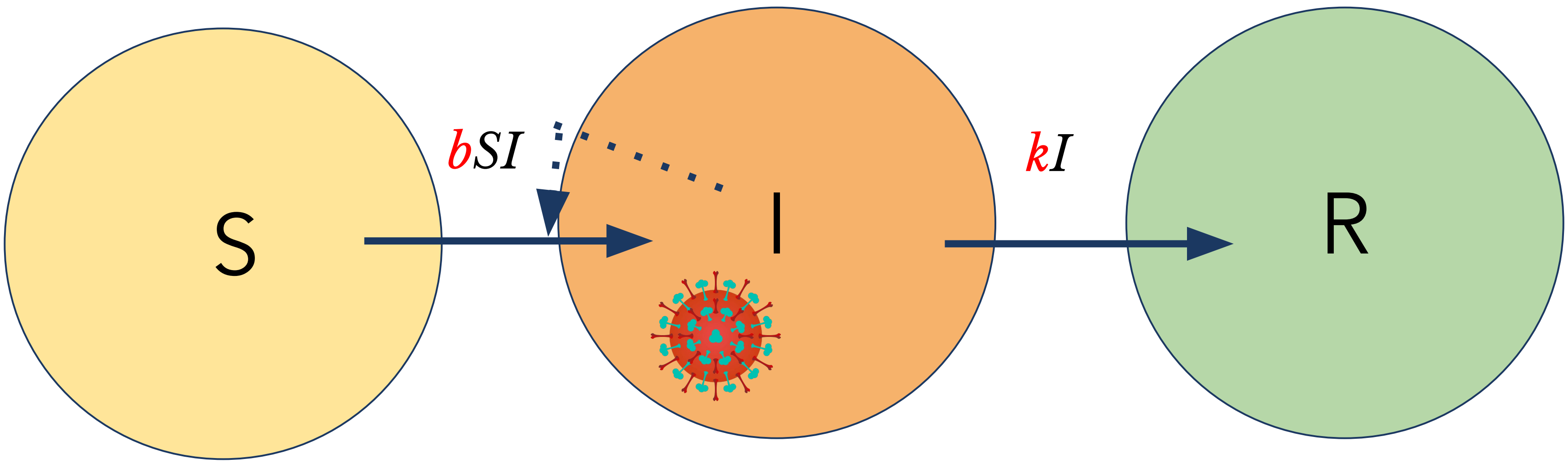
What happens after people are infected?

Building the Model - Diagram and Assumptions



- What are the key populations (variables)?
- How do individuals move among compartments?
- Mass action (*S* to *I*) and first order (*I* to *R*) rates
- What are the assumptions of the model?

Building the Model - Parameters



$$\frac{dS}{dt} = -bSI, \qquad \frac{dI}{dt} = bSI - kI, \qquad \frac{dR}{dt} = kI,$$

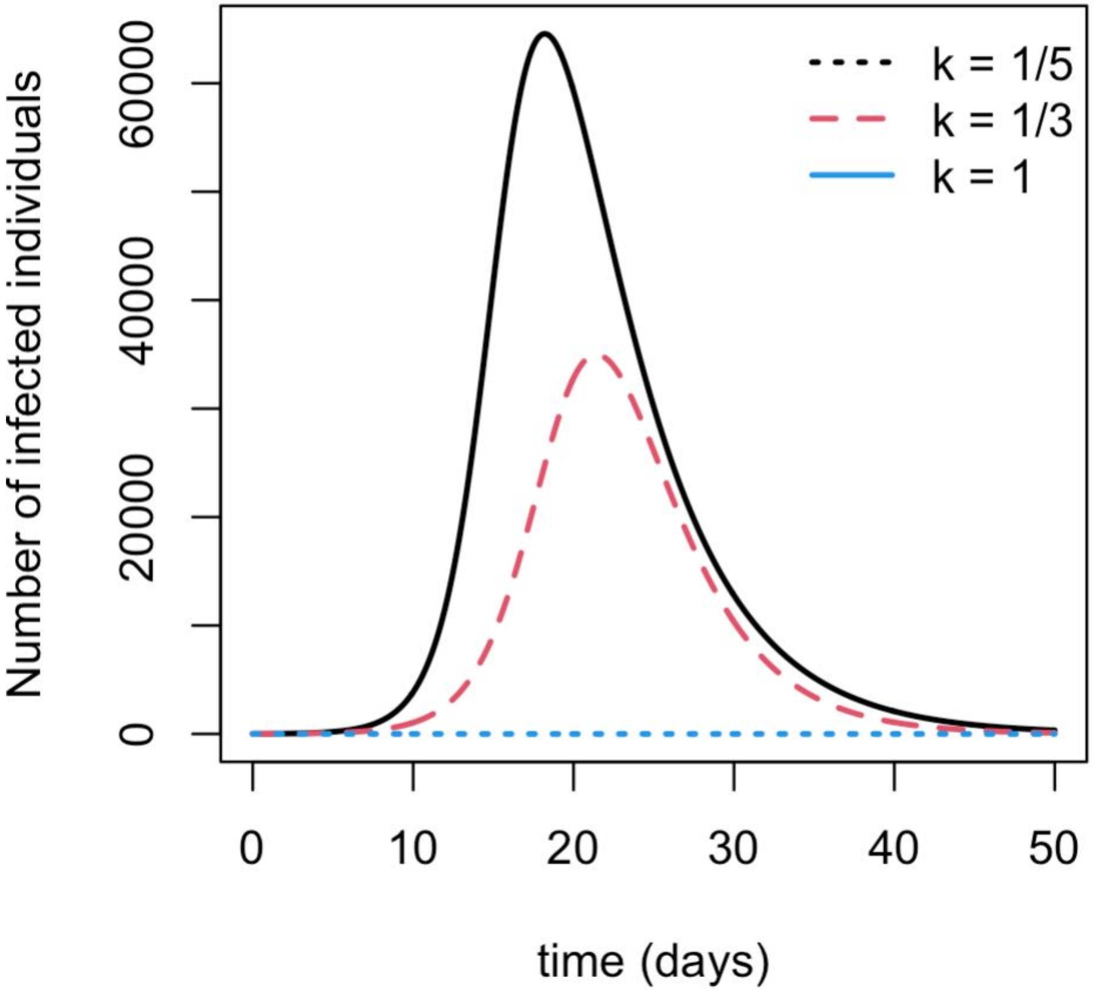
- What are the units of each parameter?
- What is the interpretation of each parameter?
- How do we obtain values of these parameter?

Exploration leads to understanding

How would $I(t)$ change as parameters vary?

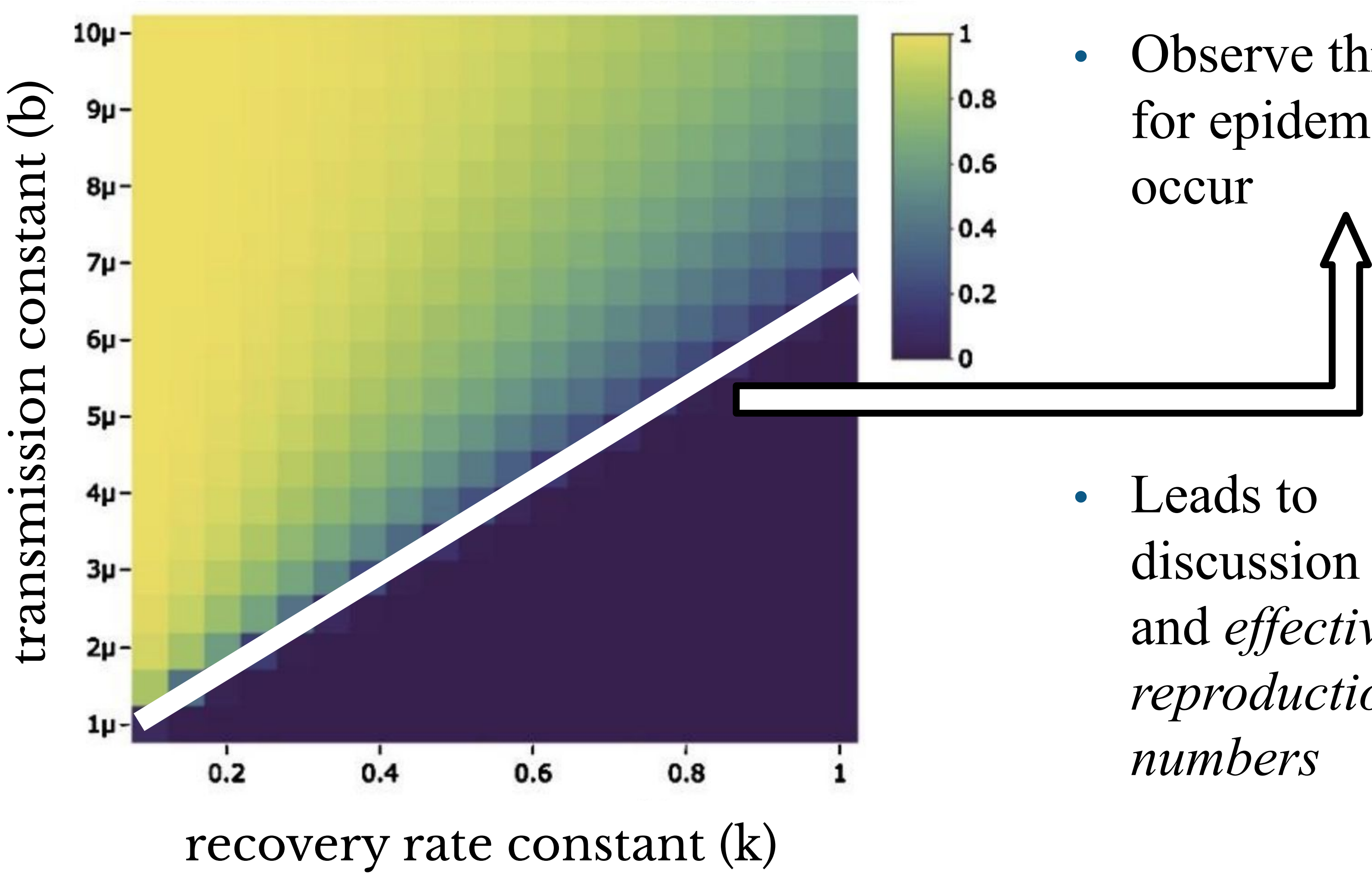
Simulate the model to confirm predictions.

Here recovery rate in 1/days is explored.



Exploration leads to discovery

Fraction infected throughout outbreak



- Observe threshold for epidemic to occur
- Leads to discussion of *basic* and *effective reproduction numbers*

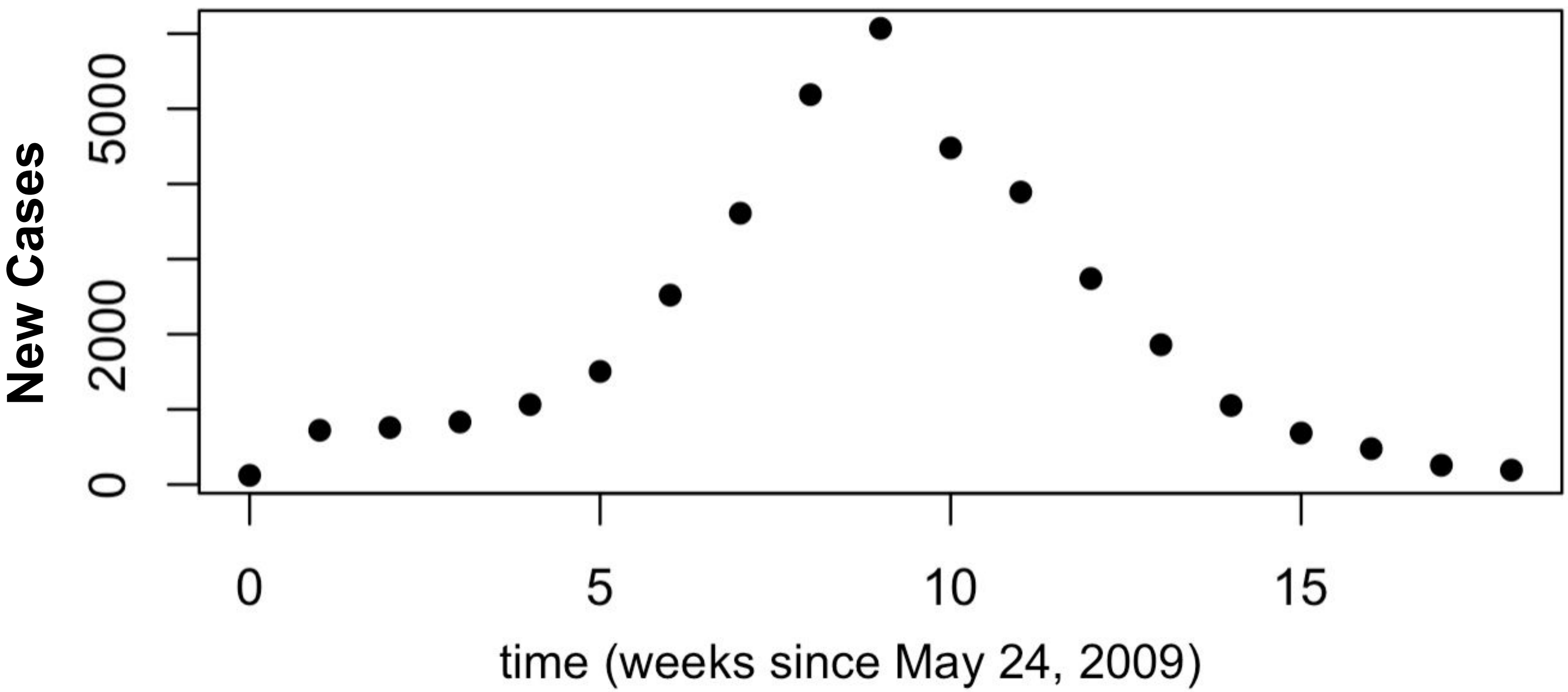
Public Health Interventions

- Flu vaccine
- Antivirals
- Self-isolation
- Public health campaigns to encourage frequent hand washing and face masks

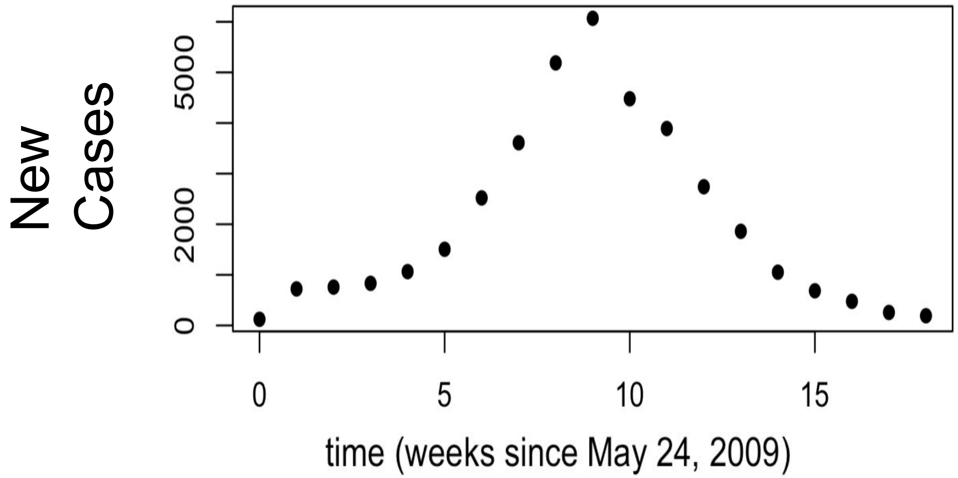
How do these strategies affect the model parameters? How do changes in the parameters affect the effective reproduction number?

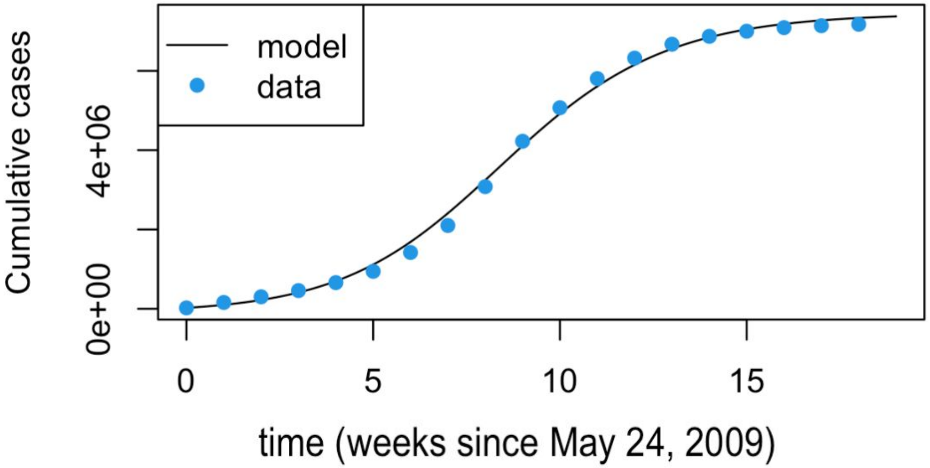
2009 H1N1 First Wave in Australia

Goals: Use data from the 2009 influenza pandemic to estimate parameters and test public health interventions, which could be used to prevent a second wave.



Solve a practical problem: Match data to model





Model: $I(t)$ = number infected on a given day, BUT

Reported data is in new cases per day.

Take into account cases were underreported.

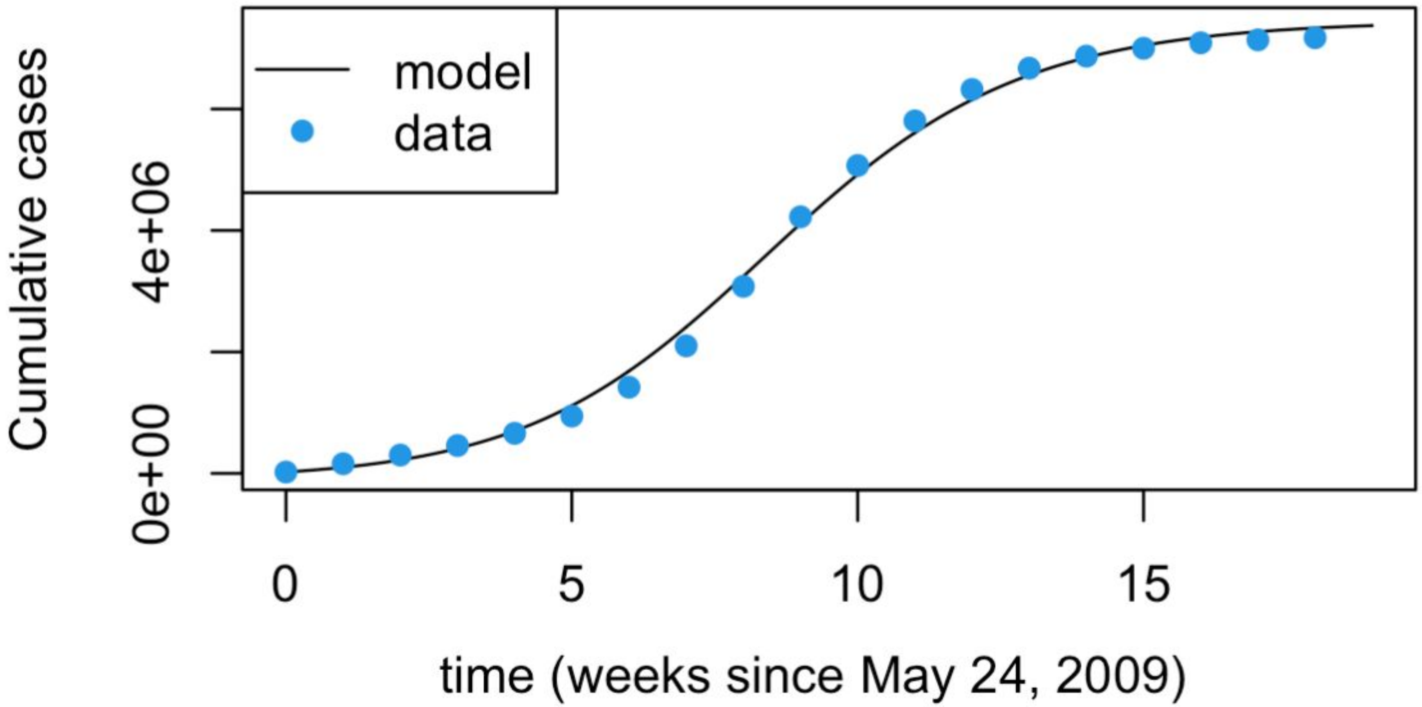
Regroup to use cumulative number of cases vs. time.

$$dC/dt = bSI$$

Use $C(t)$ to calibrate the model.

Calibrate the SIR Model

- Use the estimated reporting rate to obtain actual number of cases
- Compute cumulative number of cases
- Estimate the transmission rate constant (b) and recovery rate constant (k).



Vaccination Strategies to Prevent Second Wave

- Virus is evolving and resulting in higher transmissibility.

  How many people would be infected without vaccines?

- Researchers have estimated vaccine effectiveness to be 56%.

  Simulate different vaccination strategies. Report your findings, and use figures and tables to support your results.

Adapting the exercise

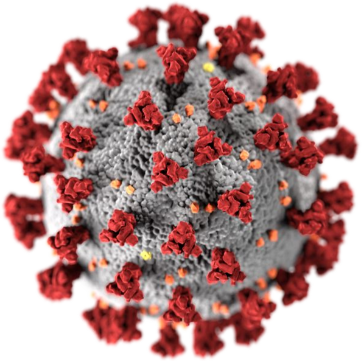
Data sets available on multiple pandemics & locations.

Add complexity with a vector! or by dividing data into cohorts (age, geographic, etc.)

Current covid-19 data are especially rich & timely

to be handled with care.

opportunities to discuss ethics.



SARS-CoV2 data at the virus level is also rich and timely.

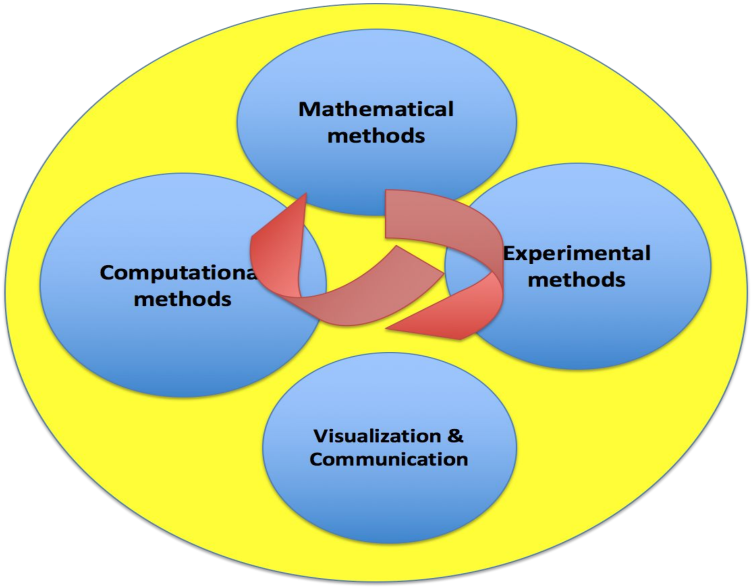
Other multicompartment systems.

Conclude:

Bringing biology and mathematics students together requires clear communication about every step of the modeling process. Make no assumptions about what students know!

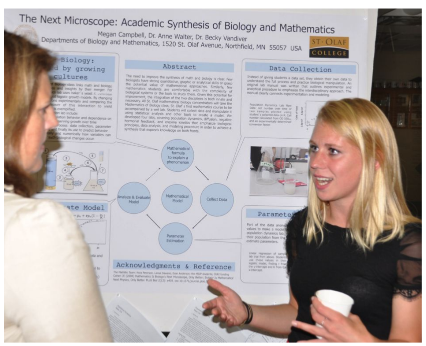
Enhances understanding.

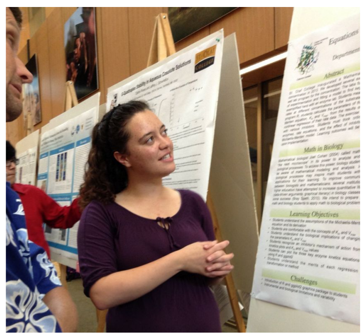
Invites exploration through confidence building, sharing expertise.



Acknowledgements

- Matt Richey (former St. Olaf Associate Dean)
- Nora Peterson, Megan Campbell and Lansa Dawano





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- Sanft, Rebecca, and Anne Walter. *Exploring Mathematical Modeling in Biology Through Case Studies and Experimental Activities.* Academic Press, 2020 (https://beckysanft.com/modeling-book/)

- St. Olaf course: https://wp.stolaf.edu/mathbio/