Games are Mathematical Models

Teaching with Kerbal Space Program

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Outline

Preparing a lesson in navigation on the Mun in KSP

- Games as mathematical models
- *Kerbal Space Program* is a model solar system
- NASA KSP Math & Physics Lab *Grant*
- MAT199 NASA KSP Math & Physics Lab *Course* (Fall 2020-Spring 2021)
- Examples from the Kerbal Math & Physics Lab online *Workbook*

Games as Mathematical Models

Interesting parallels between doing mathematics and playing games:

- In both, we simulate a real-life activity or other phenomenon with a set of rules and assumptions.
- In math: What can we discover or achieve from a given set of axioms, while playing by the rules of math and logic?
- In a game: What can we discover or achieve while following the rules of the game?

Discovering the Armstrong Memorial on the Mun in KSP

Kerbal Space Program (KSP)

Computer game (PC, Mac, Xbox and PS4)

by Jamie Logan at KerbalX.com

- Available at *KerbalSpaceProgram.com* and *Steam.com*
- Model solar system illustrates *universality* of math and physics
- Multidisciplinary environment
- Models textbook Newtonian physics with patched conic trajectories
- Multiple planets and moons, with and without atmosphere, create a virtual lab environment
- Players build rockets, satellites, landers, rovers, spaceplanes, jets, airplanes, and …
- Large active community of players, extensions and mods add options and realism

NASA KSP Math & Physics Lab Grant
Fall 2020 and Spring 2021

https://youtu.be/GC1jZ3qIVdQ

Goals:
- Create MAT 199 NASA KSP Math & Physics Lab course
- Promote math and physics literacy
- Promote STEM majors and careers
- Students complete algebra and pre-calculus chapters of the Kerbal Math & Physics Lab online workbook
- Study enrollment and demographic data in STEM at MCCC

Total enrolled in Fall 2020 and Spring 2021: 30 students
Total successfully completed: 23 students

Advertised to students at Intermediate Algebra level

STEM majors enrolled in course: 20
Non-STEM majors enrolled in course: 10

MAT199 NASA KSP Math & Physics Lab Course

- Offered Fall 2020 and Spring 2021 at Montgomery County Community College in Blue Bell, PA
- Cost sharing with MCCC and NASA Pennsylvania Space Grant Consortium
- Students completed exercises and lab activities from the *Kerbal Math & Physics Lab* workbook
- 1 elective credit in math, meeting 2 hours/week for 15 weeks via Zoom
- Guest speakers:
  - Lockheed Martin engineers and programmers
  - Private Division game developers
  - Scott Manley and Mike Aben (YouTube creators)
  - Cody Short, orbital modeling engineer at Analytic Graphics, Inc
  - Nicholas Seiberlich, former MCCC and Temple student, now professional engineer.
  - Patrick Watt, Fall 2020 NASA KSP student, accepted into NASA Experience online program

The Kerbal Math & Physics Lab Online Workbook

by Christopher S. Vaughen

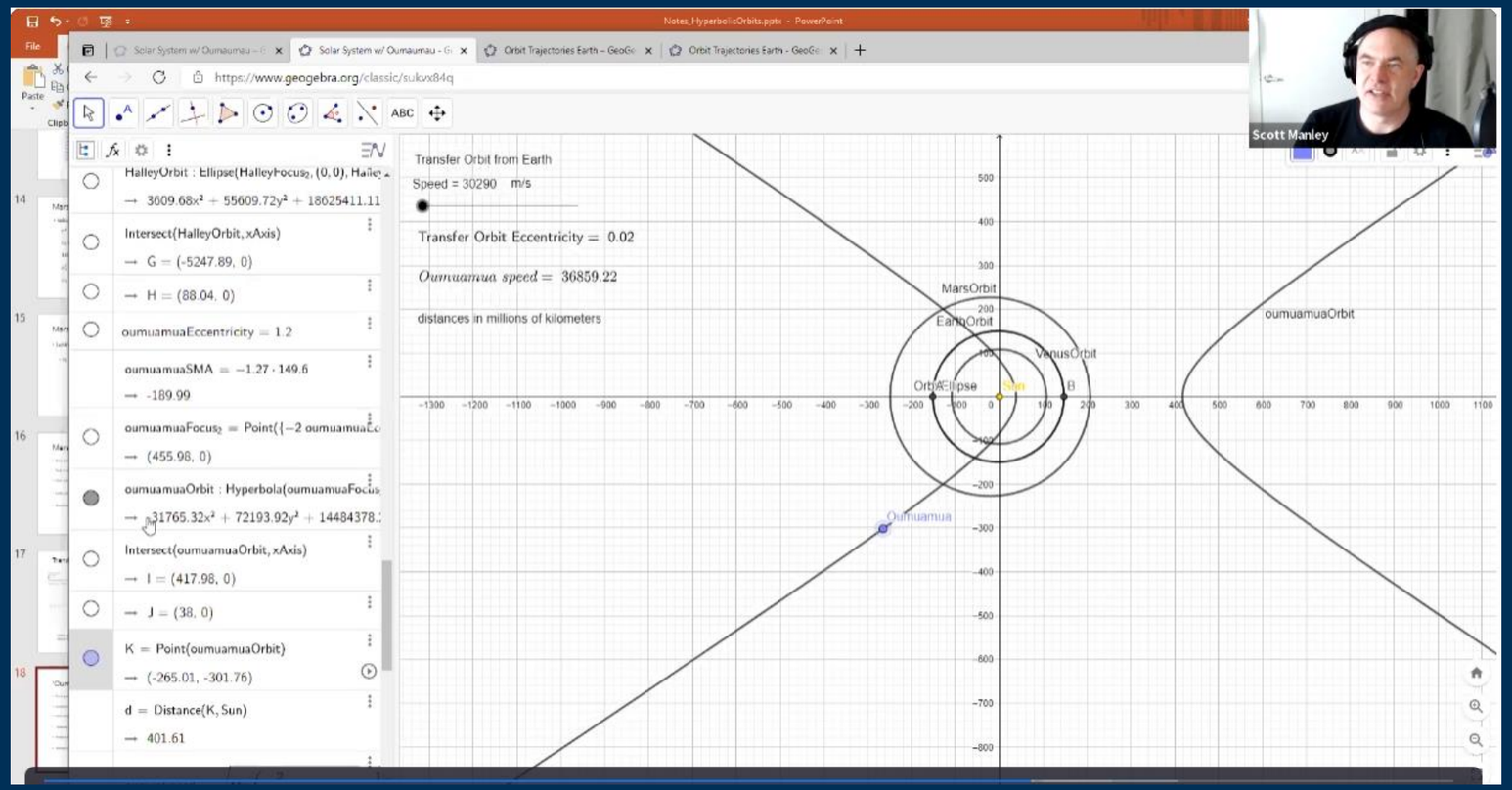
- Available online: *https://sites.google.com/view/kspmath*
- Math and physics questions and solutions, lab activities
- Many questions don’t require access to the game and provide examples for traditional lectures, assignments, exams.
- Based on both KSP and real world examples
- Useful for traditional math/physics courses, lab activities, project based learning, math, physics and engineering clubs, and summer enrichment programs

Spring 2021 Schedule

MAT 199 - NASA KSP Math & Physics Lab
Schedule Spring 2021
Section OSK - TR 12:45-1:45pm

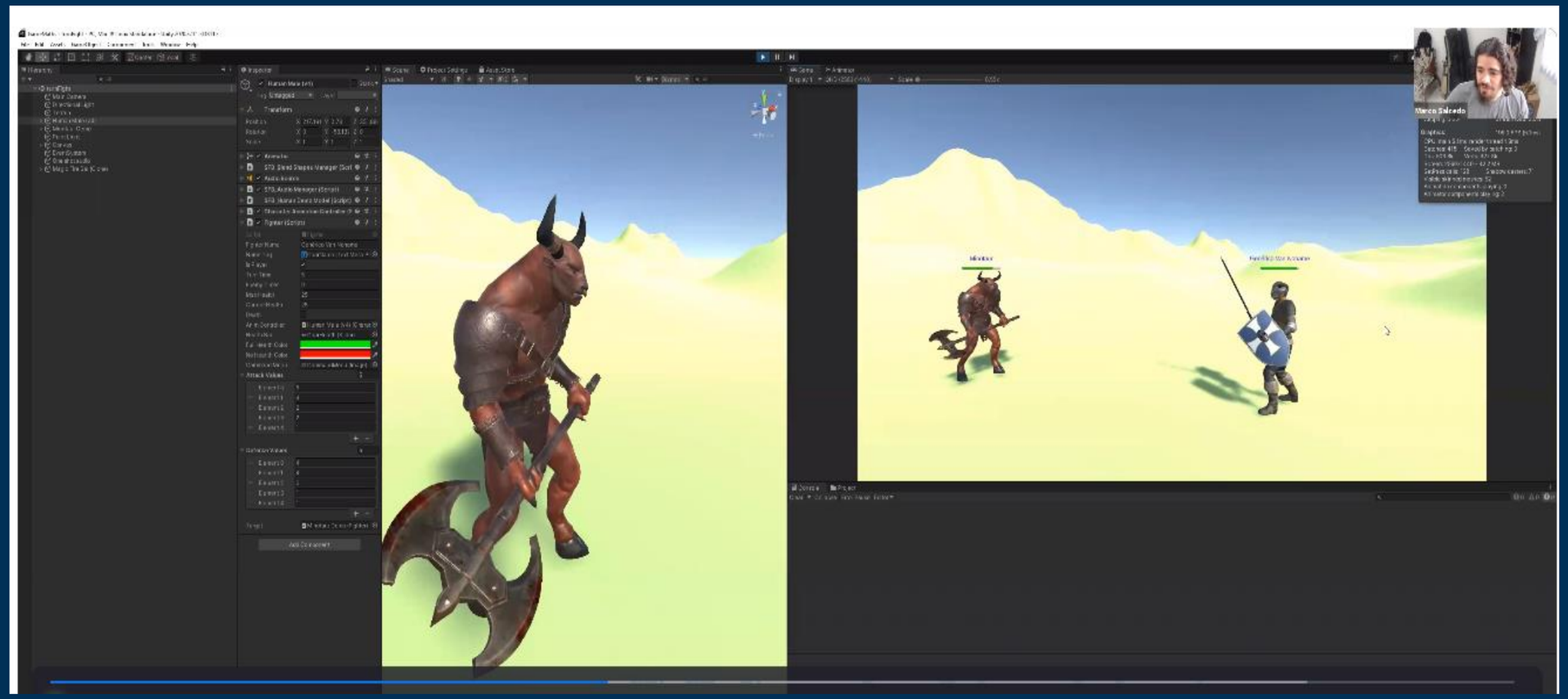
Class	Day	Date	Topic	HW/LAB due date
1	Thursday	**1/21**	Intro, gravity and acceleration	**HW0 due 1/23**
2	Tuesday	**1/26**	Gravity and acceleration	
3	Thursday	**1/28**	Gravity and acceleration	**HW1 due 1/29**
4	Tuesday	**2/2**	Thrust to weight ratio, acceleration and g-force	
5	Thursday	**2/4**	Thrust to weight ratio, acceleration and g-force	**HW2 due 2/5**
6	Tuesday	**2/9**	Acceleration and g-force lab	
7	Thursday	**2/11**	Acceleration and g-force lab, orbital speed	**LAB1 due 2/12**
8	Tuesday	**2/16**	Orbital speed	
9	Thursday	**2/18**	Orbital speed, NASA mission objectives, *Perseverance* landing	**HW3 due 2/19**
10	Tuesday	**2/23**	Orbital period,	
11	Thursday	**2/25**	Orbital period, SpaceX and private industry	**HW4 due 2/26**
12	Tuesday	**3/2**	Computing the mass of a planet or moon, kinetic and potential energy in circular orbits	
13	Thursday	**3/4**	*Presentation by Patrick Watt, former NASA KSP student in Fall 2020, describing his successful completion of the NASA NCAS online program*	
14	Tuesday	**3/9**	Linear and Angular Speed	
15	Thursday	**3/11**	Great circle navigation	**HW5 due 3/12**
			Spring Break - no class 3/15 – 3/19	
16	Tuesday	**3/23**	Great circle, rhumb line navigation	
17	Thursday	**3/25**	*Presentation by Cody Short, Astrodynamics Software Engineer*	
18	Tuesday	**3/30**	*Presentation by Private Division/Squad – Mathematics in Game Programming*	
19	Thursday	**4/1**	Vis-viva lab	
20	Tuesday	**4/6**	*Presentation by Nick Sieberlich, mathematics of control systems, rovers, Ingenuity helicopter*	
21	Thursday	**4/8**	Delta-v for a Hohmann transfer	
22	Tuesday	**4/13**	*Presentation by Scott Manley – Mathematics of hyperbolic orbits, the Netflix movie "Stowaway"*	
23	Thursday	**4/15**	Vis-viva lab, *last day to drop*	**HW6 due 4/16**
24	Tuesday	**4/20**	The Rocket Equation	
	Thursday	**4/22**	The Rocket Equation	
25	Tuesday	**4/27**	Kepler's Laws of Planetary Motion,	**LAB2 due 4/23**
26	Thursday	**4/29**	Launch Azimuth, Latitude and Orbital Inclination	
27	Tuesday	**5/4**	Launch Azimuth, Latitude and Orbital Inclination, last official class meeting	
	Wednesday	**5/5**	*Reading day*	**HW7 due 5/7**
28	Tuesday	**5/11**	***Final exam week 5/6-5/13, LAB3 due Monday 5/10***	**LAB 3** Due ***Monday*** **5/10**

Guest Speakers – Scott Manley (YouTube Creator)



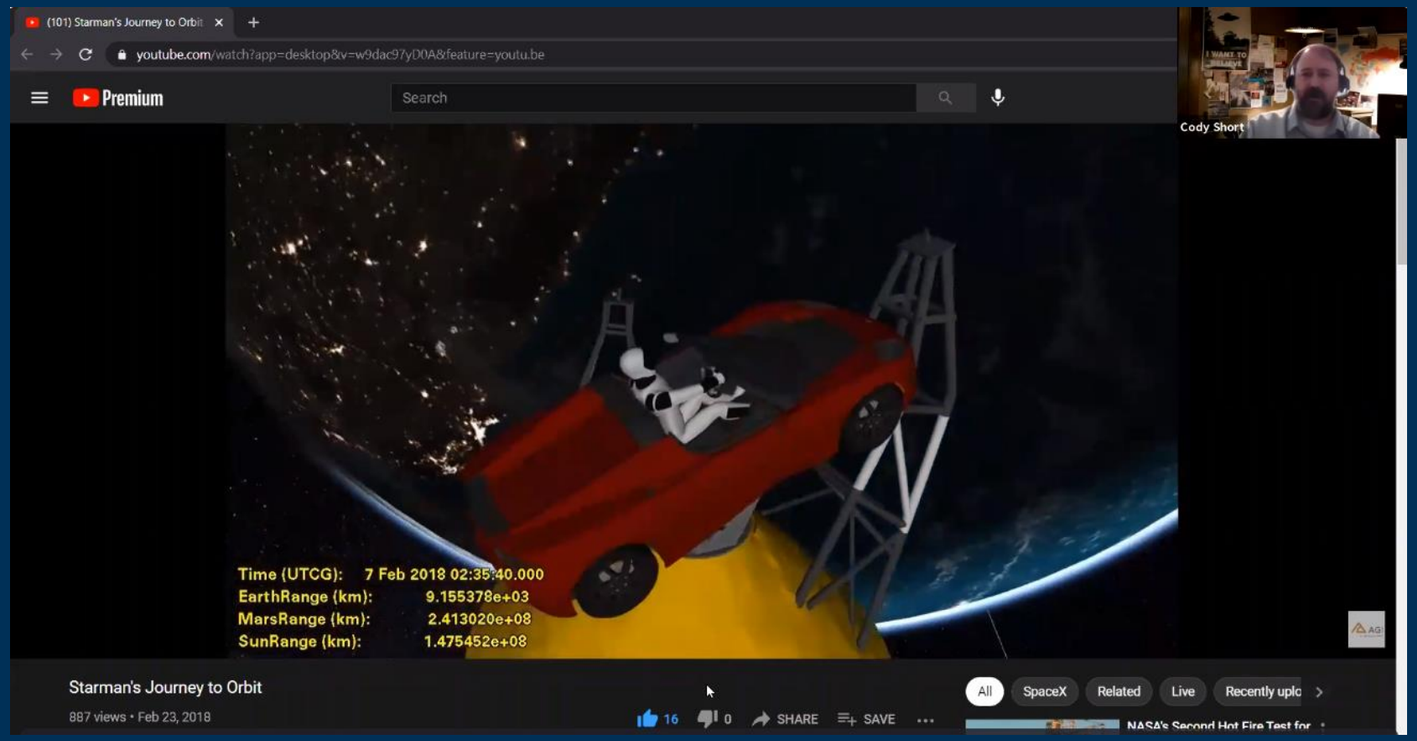
Fall 2020 – launch site latitude, azimuth and inclination
Spring 2021 - hyperbolic orbits, escape velocity, Netflix movie *Stowaway*

Guest Speakers – Game Developers from Private Division (KSP Publisher)



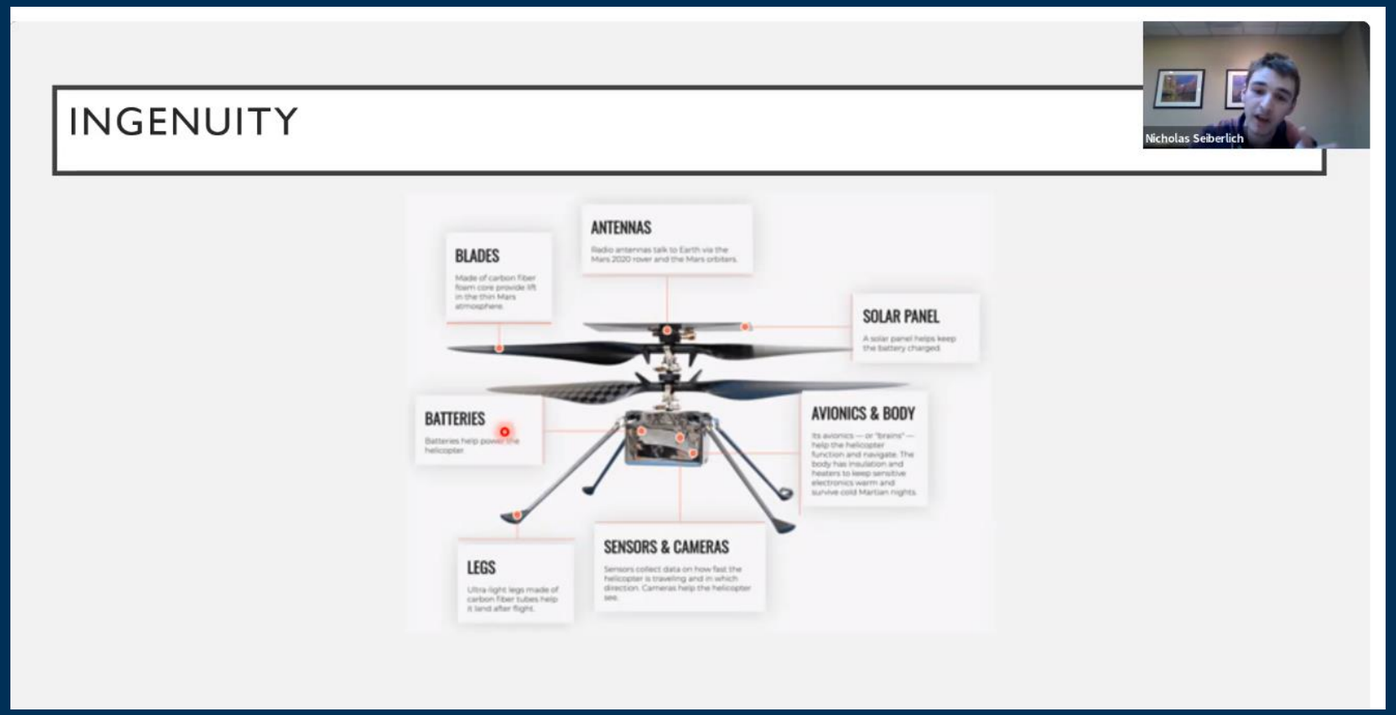
Presentations in Fall 2020 and Spring 2021 on mathematical skills required in game development and programming

Guest Speakers – Cody Short from Analytic Graphics, Inc.



Presentation on academic pathways to astrodynamics engineering and the work of an astrodynamics engineer

Guest Speakers – Nick Seiberlich, MCCC and Temple graduate



Nick, an MCCC and Temple graduate and now a manufacturing engineer, gave a presentation on the mathematics of control systems, and other mathematics applications in engineering

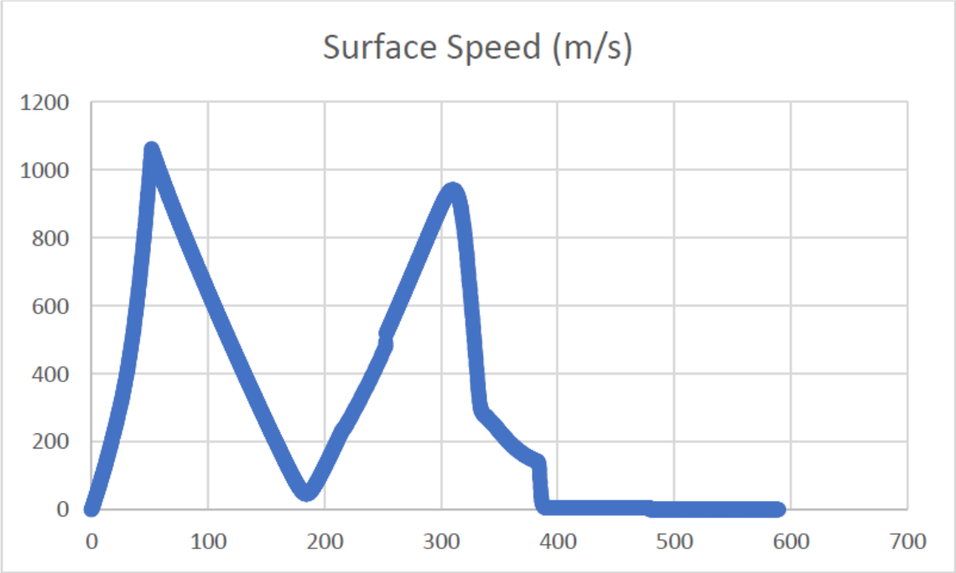
3D Graphing Calculator

KSP is a 3D ***graphing calculator***:

- ❑ Do the math and check answers in game
- ❑ Illustrate general principles
- ❑ Experiment by changing parameters

Virtual STEM Lab

3. Velocity for entire flight (launch to splash down).



Note: Absolute max speed at engine cut-off (at 52 seconds), speed then decreases until instant of max altitude (at 195 seconds), speed increases again as the rocket falls in space (to relative max speed at 310 seconds) after which speed is reduced again due to atmospheric drag and then parachutes deploying (at about 380 seconds) before splash down at about 8 minutes (or 480 seconds).

KSP is a ***virtual STEM lab***

- ❑ Build models
- ❑ Conduct experiments
- ❑ Analyze exported game data

MAT 199 NASA KSP Course - Student Feedback

Student feedback in MAT 199 NASA KSP course on an end-of-semester questionnaire:

1. Why did you decide to take this course?

To try something new out of my comfort level.

2. What are some of the things you learned using the KSP program?

- The concept that math and physics are universal
- The ingenuity behind rocket science

6. Have you decided to take certain courses or pursue a major because of your experience with this course?

I HAVE decided to continue to persue a degree in STEM/engineering - this class has boosted much confidence in making this decision.

7. Would you recommend this course to other students? Why?

This has been the highlight of my semester. I would recommend this course to anyone with even a slight interest in math, rocketry, astronomy, NASA, navigation, game design, or all of the above. The material is wonderful, deep, and made much more easy to grasp with the help of KSP.

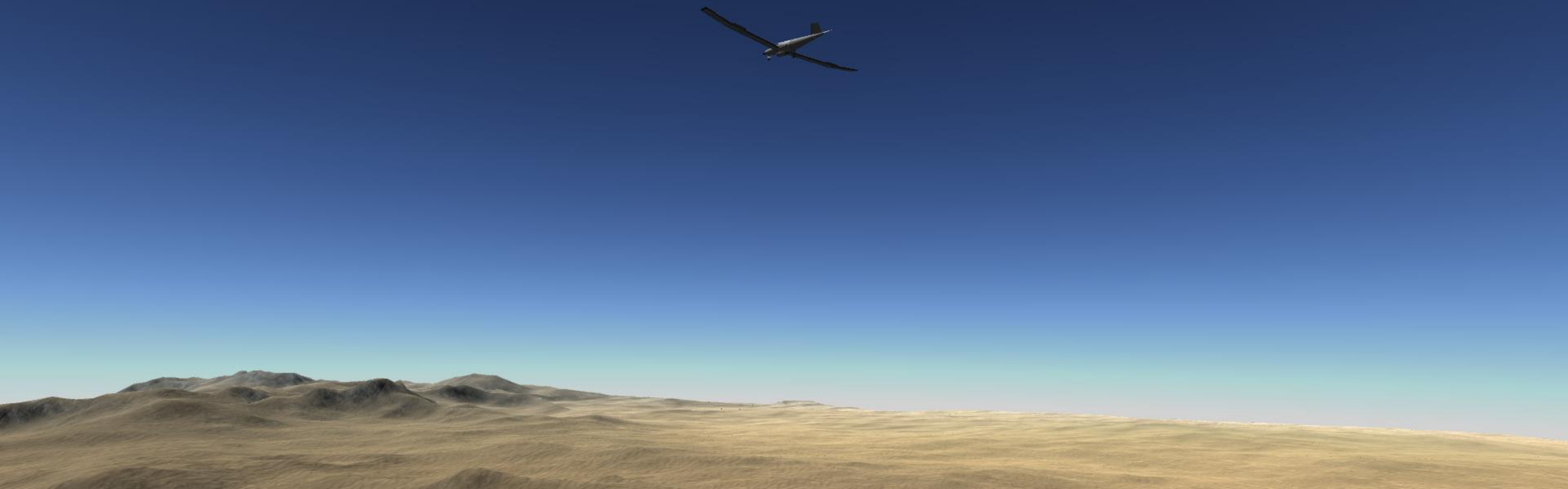
5. What are some examples of math or physics concepts you learned in the MAT 199 course?

Through MAT 199, I learned the Vis-Viva and the Rocket Equations.

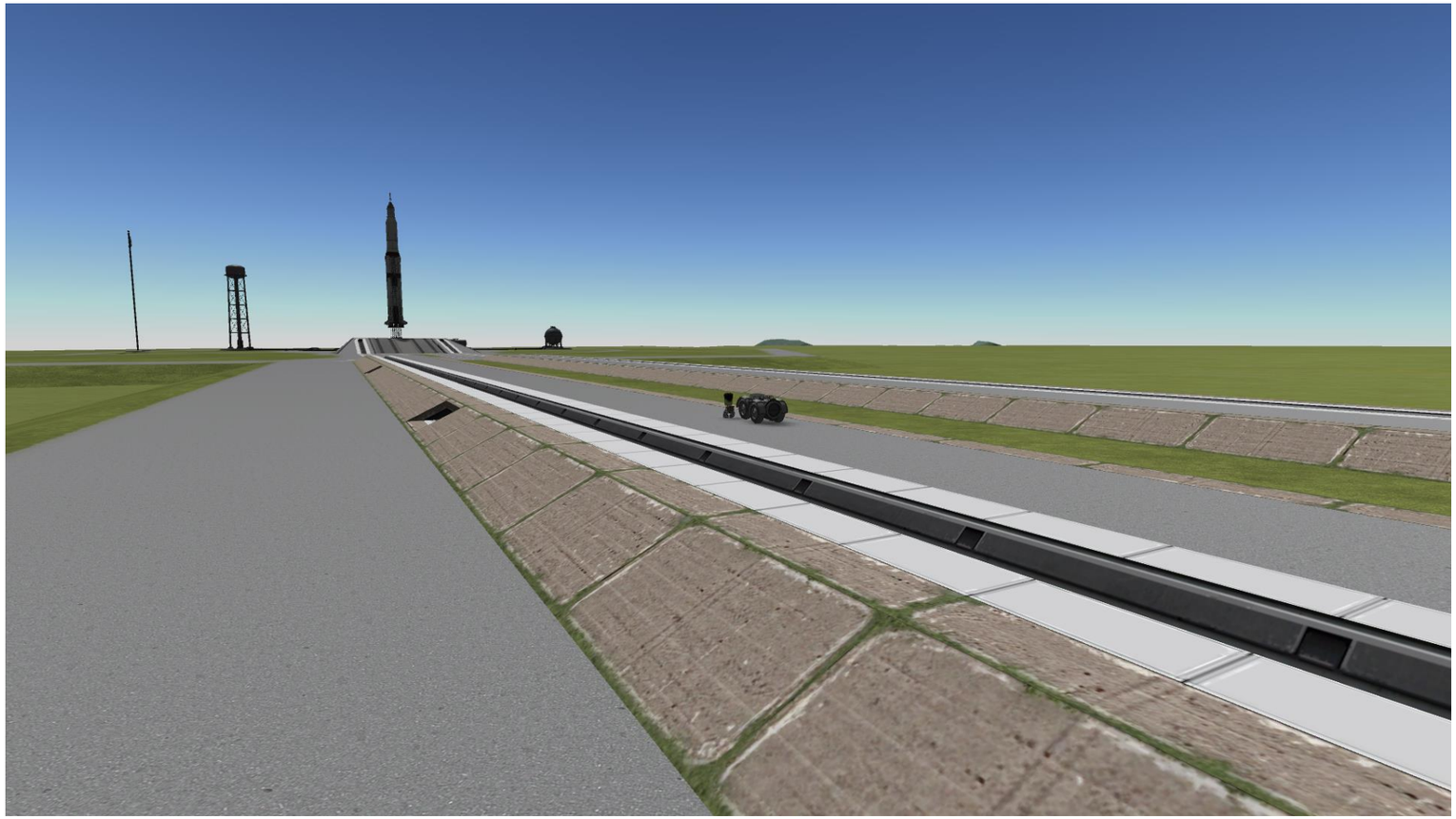
In traditional math classes, students can feel personally judged when they get something wrong

In a game, learning from failure is more acceptable and expected

And learning through interactive play can be very effective at developing intuition, an integrated understanding, and novel solutions

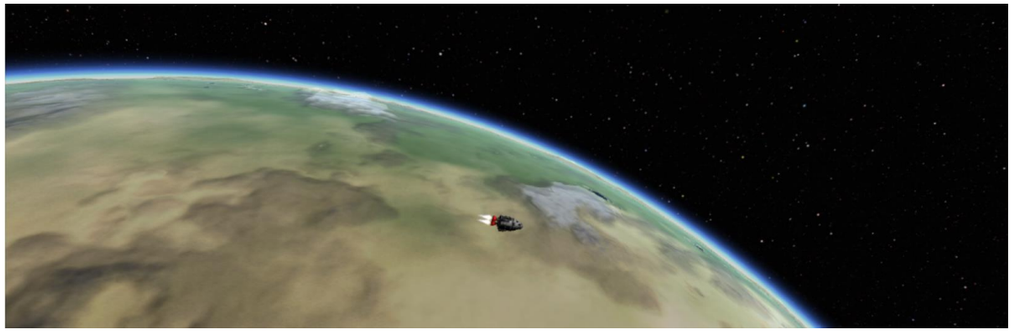


Modelling with KSP



Examples from the ***Kerbal Math & Physics Lab***
available at *sites.google.com/view/kspmath*

Algebra Examples



1. A rocket orbits at an altitude of 100 km above the surface of Kerbin which has a radius of 600 km and a mass of $M = 5.29 \times 10^{22}$ kg. Find the orbital radius of the rocket and then compute the orbital speed at 100 km.

$$r = 600\,000 + 100\,000 = 700\,000$$

$$GM = \mu = 3.53 \times 10^{12}$$

$$V = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.29 \times 10^{22})}{700\,000}} = \boxed{2245 \text{ m/s}}$$

You can check your answer above with the *Kerbal 1* rocket (available in Sandbox mode in KSP). Check your rocket's speed in the Navball display and in Map View.

Algebra Examples

15. The International Space Station (ISS) orbits the Earth at an altitude of about 250 miles above the surface. Earth's radius is about 3960 miles. Calculate your own weight, in pounds, if you were onboard the ISS. Derive a formula that you could use to find anyone's weight, in pounds, if they were onboard the ISS.

Answer:

15. Let x = weight in pounds on the surface of Earth. Then $F = x\left(\frac{3960}{4210}\right)^2 = 0.88x$ will be weight at an altitude of 250 miles, for example on board the ISS. This means weight on ISS is about 88% of weight on surface of the Earth.

$$\frac{y}{x} = \frac{\frac{\cancel{GMm}}{(r+h)^2}}{\frac{\cancel{GMm}}{r^2}}$$

$$\frac{y}{x} = \frac{r^2}{(r+h)^2}$$

$$y = \left(\frac{r}{r+h}\right)^2 x$$

Algebra Examples

Acceleration and g-Force Lab Data: 1 MT = 1,000 kg

Build and launch the *Thumper Test Rocket* as described above (or a similar rocket), and collect flight data to complete the table below. Record data for at least 10 different points in time between launch and engine cut-off.

Rocket Name: THUMPER Launch Date: 9/17 Launch Site: KSC

Initial Mass: 6.37 MT Engine burn time (in seconds): 52

Total Mass at engine cut-off: 2.62 MT

$TWR_{(i)} = \frac{125000\,N}{(6370\,kg)(9.8)} = 2.00$

Engine Thrust: ~ 137.5 kN

Initial TWR: 2.0 Final TWR (at engine cut-off): 6

$TWR_{(f)} = \frac{149000\,N}{(2620)(9.8)} = 5.99$

Theoretical delta-v: 1528 (use values provided in-game or the Δv formula in Ch. 2)

$\Delta v = V_e \ln\left(\frac{M_i}{M_f}\right) =$

Time (seconds)	Altitude ASL* (meters)	Speed (m/s)	Altitude (miles)	Speed (miles/hr)	Mach speed**
Ø	81	Ø	0.05	Ø	Ø
5	218	52	0.14	116.32	0.15
10	616	111	0.38	248.31	0.32
15	1291	170	0.80	380.29	0.50
20	2317	238	1.44	532.41	0.69
25	3697	311	2.30	695.71	0.91
30	5472	398	3.40	890.33	1.16
35	7632	502	4.74	1122.97	1.46
40	10495	639	6.52	1429.44	1.86
45	13492	796	8.38	1780.65	2.32
50	18443	996	11.46	2228.05	2.90
52	20024	1063	12.44	2377.83	3.10

4. Estimate the acceleration on the rocket (from launch to engine cut-off) using $a = \frac{\Delta v}{\Delta t}$ where Δv is the total change in velocity during the burn and Δt is the time interval of the burn.

$a = \frac{1063\ m/s}{52\ s} = 20.4\ m/s^2$ AVERAGE ACCELERATION TO MECO

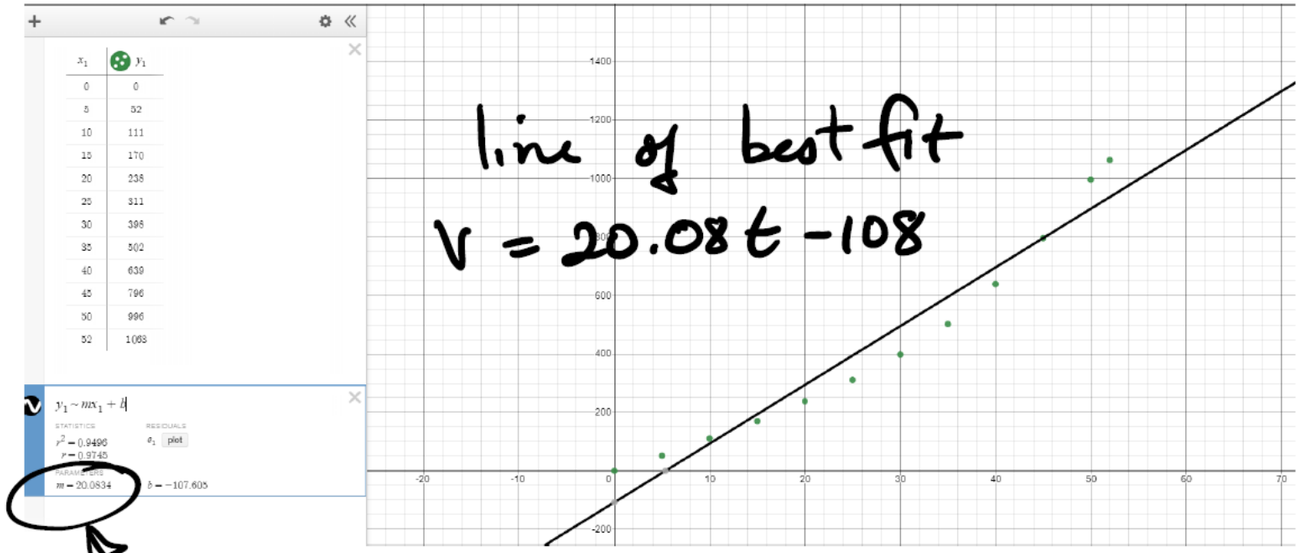
g-Force Lab:

Estimate acceleration from velocity

Calculate g-force

Algebra Examples

5. Using Desmos to get line of best fit to velocity data to find average acceleration:



$m = 20.08\ m/s^2$

average acc $= 20.08\ m/s^2$ approx

6. g-Force $= \frac{a}{g} + 1$

$= \frac{20.1}{9.8} + 1$

≈ 3 g's.

- Collect data in-game
- Compute average acceleration during launch
- Use acceleration to find average g-force
- Can be done with KSP or real-life data

Pre-Calc & Trig Examples

Calculate direction from point A to point B on any spherical planet or moon

7. Compute the initial bearing for the great circle path on the Moon from the Apollo 11 landing site to the Apollo 12 landing site. Use the Apollo 11 landing site coordinates at (0.68° North, 23.4° East) and the Apollo 12 landing site coordinates at (3.2° South, 23.4° West). Show work using the method presented here.

$\theta_A \quad \theta_A$

A: (.68°, 23.4°)

B: (-3.2°, -23.4°)

$\theta_B \quad \theta_B$

$\Delta\theta = -46.8$

$\beta' = \tan^{-1}\left(\frac{y}{x}\right)$

$\beta = 84.9° + 180 = 265°$

$x = \cos(.68)\sin(-3.2) - \sin(.68)\cos(-3.2)\cos(-46.8)$

$= .064$

$y = \sin(-46.8)\cos(-3.2)$

$= -.728$

Calculating initial heading from the Apollo 11 landing site to the Apollo 12 landing site

Planning a trip to see the UFO buried at the south pole of the Mun in KSP

Calculus based derivation of navigation formulas: https://youtu.be/wygNHhI3EgY

Pre-Calc & Trig Examples

Derive formula for delta-v required to initiate transfer orbit

2. Use the vis-viva equation, and the formula for speed in a circular orbit, to derive a general formula for the initial change in velocity (Δv_1) required for a Hohmann transfer from a circular orbit of radius r_1 to a circular orbit of radius r_2, where $r_2 > r_1$.

Vis-viva equation:

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

v = velocity

μ = gravitational parameter

r = distance between two bodies

a = semi-major axis

&

$$a = \frac{r_1 + r_2}{2}$$

r_1 = distance to periapsis

r_2 = distance to apoapsis

Speed in circular orbit at distance r_1

$$v = \sqrt{\frac{\mu}{r_1}}$$

Speed at periapsis:

$$v^2 = \mu\left(\frac{2}{r_1} - \frac{1}{\left(\frac{r_1 + r_2}{2}\right)}\right)$$

$$v^2 = \mu\left(\frac{2}{r_1} - \frac{2}{r_1 + r_2}\right)$$

$$v^2 = \mu\left(\frac{2(r_1 + r_2) - 2(r_1)}{r_1(r_1 + r_2)}\right)$$

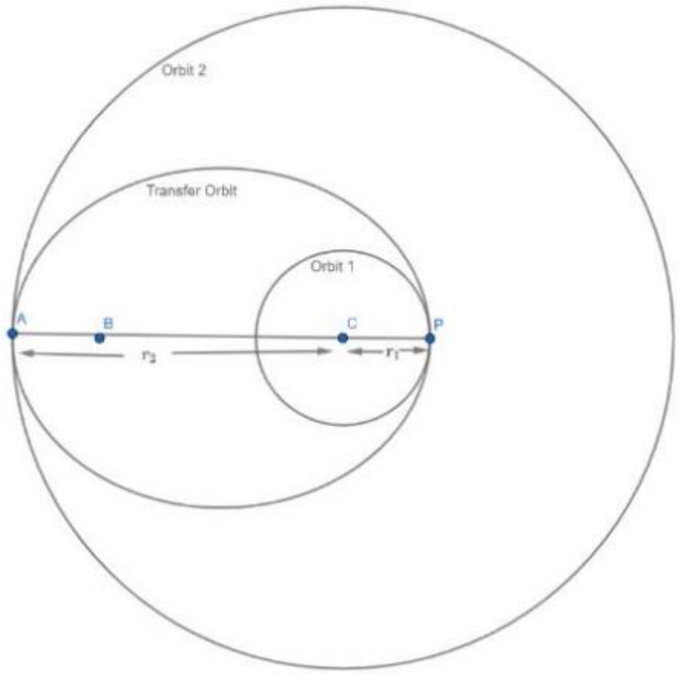
$$v^2 = \mu\left(\frac{2r_2}{r_1(r_1 + r_2)}\right)$$

$$v = \sqrt{\mu\left(\frac{2r_2}{r_1(r_1 + r_2)}\right)}$$

$$\Delta v = \sqrt{\mu\left(\frac{2r_2}{r_1(r_1 + r_2)}\right)} - \sqrt{\frac{\mu}{r_1}}$$

or

$$\boxed{\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right)}$$

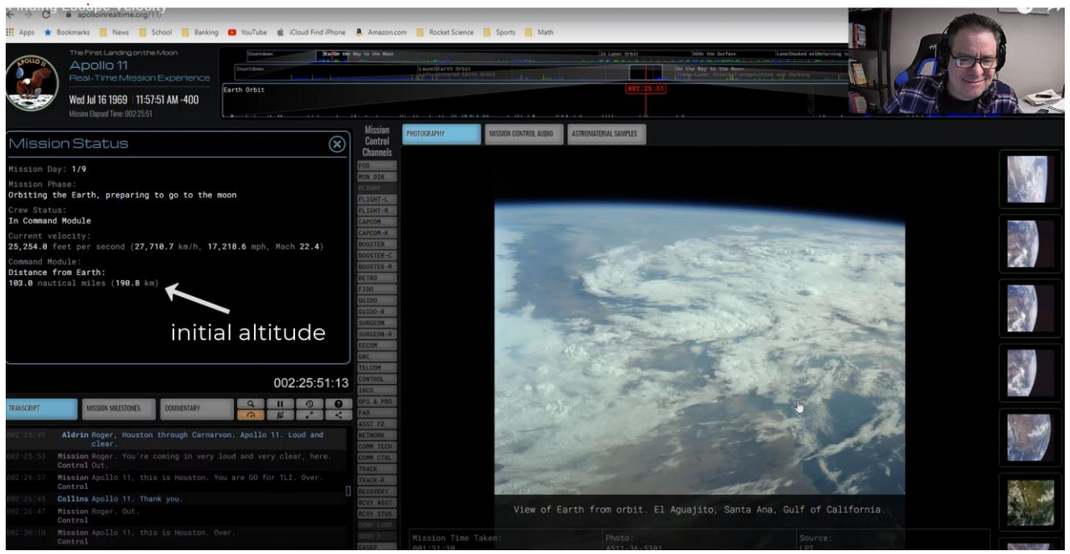


Pre-Calc & Trig Examples

Confirm delta-v calculations in KSP and in real-life

A Trip to the Moon, Part 3: Go For TLI

https://youtu.be/O3Yz-nNpa24



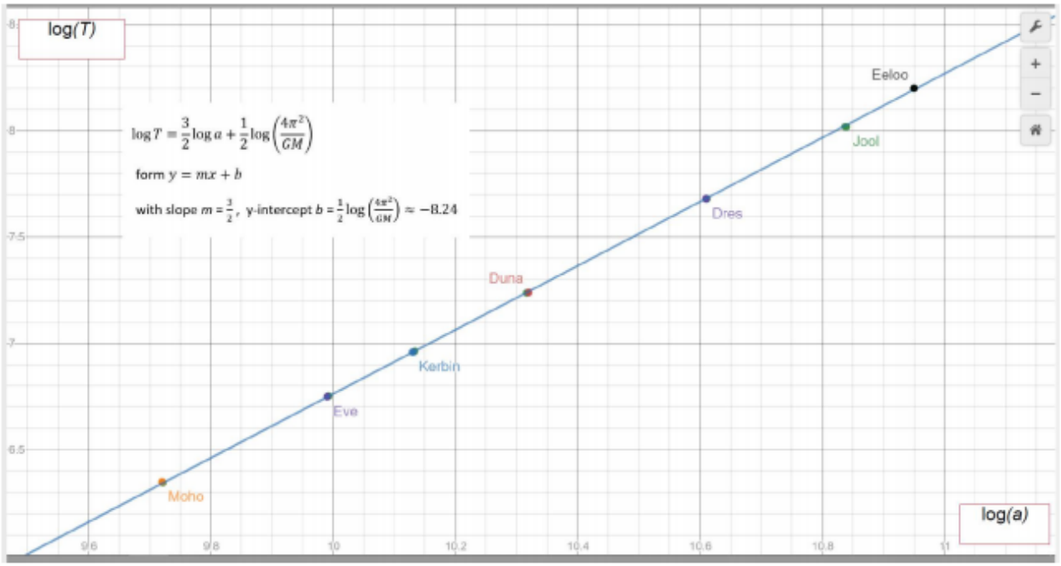
Apolloinrealtime.org

Pre-Calc & Trig Examples

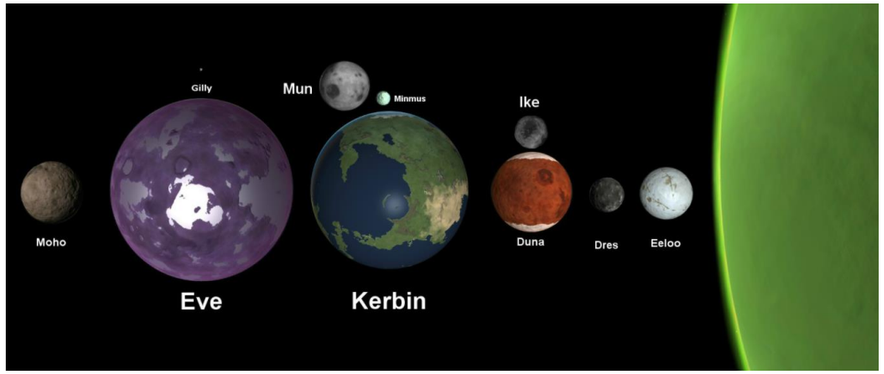
6. Completing the table to include Eeloo we have

Planet	log(*a*)	log(*T*)
Moho	9.721244782	6.3455
Eve	9.992672106	6.7526
Kerbin	10.13353381	6.9639
Duna	10.31651875	7.2384
Dres	10.6110788	7.6802
Jool	10.83742151	8.0197
Eeloo	10.9548155	8.1958

7. Updated graph includes the newly discovered planet Eeloo located at approximately (10.95, 8.20) in the log(*a*)-log(*T*) graph, confirming Kepler's third law of planetary motion.



Discovering a new planet, and confirming predicted orbit with Kepler's third law



- Collect orbit data (semi-major axis and period)
- Learn properties of log functions
- Learn Kepler's third law
- Apply to alternate solar system besides our own

Pre-Calc & Trig Examples

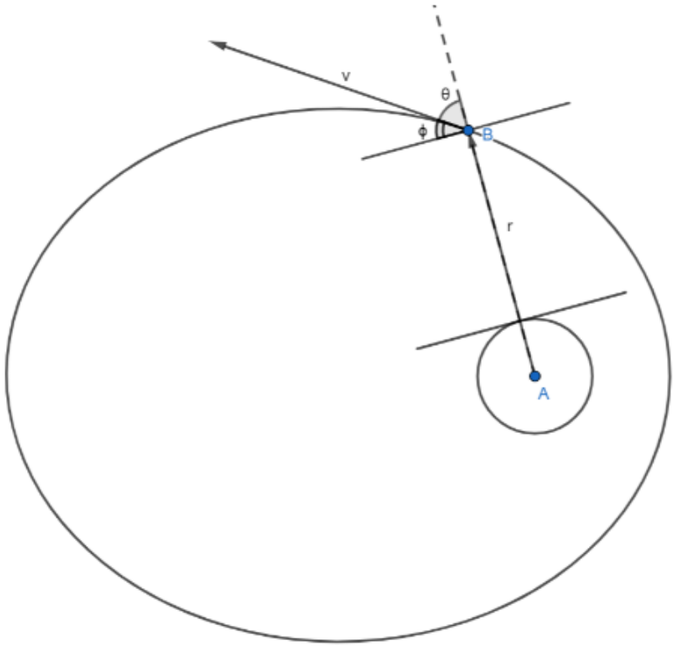
Vis-Viva Lab

Use Excel to complete the vis-viva lab report and answer the questions on the following page. Combine all pages into one single PDF document and upload to Blackboard using the filename format "yourname_LAB2.pdf"

trajectory	point 1 (in flight)	point 2 (in flight)	eccentricity
sub-orbital	altitude = 66183 m velocity = 879.5 m/s pitch = 58.1°, θ = 31.9° ε_1 = -4.9×10^6 h_1 = 3.1×10^8	altitude = 57662 m velocity = 958 m/s pitch = -59.9°, θ = 149.9° ε_2 = -4.9×10^6 h_2 = 3.2×10^8	e = 0.96
orbital (elliptical)	altitude = 89313 m velocity = 25563 m/s pitch = 3.6°, θ = 86.4° ε_1 = -1.9×10^6 h_1 = 1.8×10^9	altitude = 45,1320 m velocity = 1734 m/s pitch = -15.2°, θ = 105.2° ε_2 = -1.9×10^6 h_2 = 1.8×10^9	e = 0.28
orbital (nearly circular)	altitude = 82997 m velocity = 2274 m/s pitch = 0.2°, θ = 89.8° ε_1 = -2.6×10^6 h_1 = 1.6×10^9	altitude = 82025 velocity = 2277 m/s pitch = -0.2°, θ = 90.2° ε_2 = -2.6×10^6 h_2 = 1.6×10^9	e = .004
hyperbolic	altitude = 107694 m velocity = 3225 m/s pitch = 6°, θ = 84° ε_1 = 2.1×10^5 h_1 = 2.3×10^9	altitude = 12035000 m velocity = 990 m/s pitch = 80°, θ = 10° ε_2 = 2.1×10^5 h_2 = 2.3×10^9	e = 1.08

Recording data for the vis-viva lab

Confirming conservation of energy and angular momentum and corresponding orbital eccentricity



Pre-Calc & Trig Examples

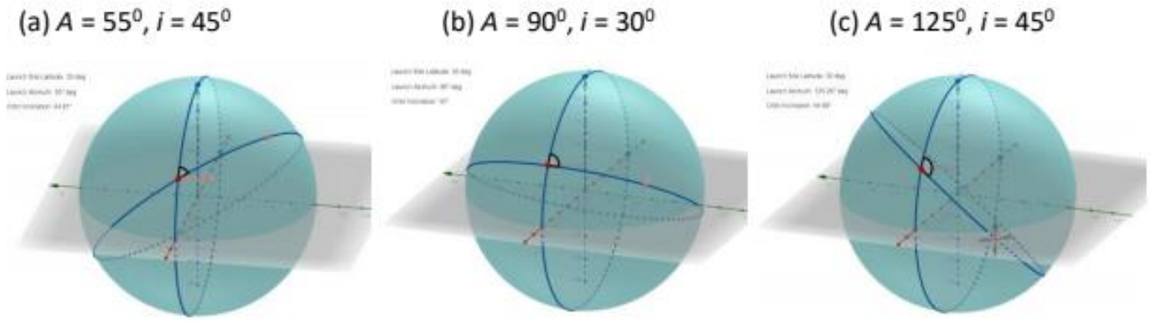


Figure 3. Launch azimuth and resulting orbital inclination from launch site at latitude 30^0 N.

Calculating launch azimuth (A) and orbit inclination (i) at different latitudes (β) on Earth and on Kerbin

Calculations which can be confirmed by observations of real life launches and with KSP

$$\cos(i) = \sin(A)\cos(\beta)$$

5. For the following questions, use the fact that the orbital inclination of the International Space Station (ISS) is approximately 51.6 degrees.

a) What launch azimuth is required from the Kennedy Space Center to reach the orbital inclination of the ISS? Show work, check with the Desmos and Geogebra interactive calculators.

$\phi = 28.6°$ KSC lat.
$\beta = ?$
$i = 51.6°$ I.S.S orbit inclination.

* find β using launch lat. + target orbit.

So, $\cos i = \sin\beta \cos\phi$

$\frac{\cos(51.6)}{\cos(28.6)} = \sin\beta \frac{\cos(28.6)}{\cos(28.6)}$

$\frac{\cos(51.6)}{\cos(28.6)} = \sin\beta$

$\beta = \sin^{-1}\left(\frac{\cos(51.6)}{\cos(28.6)}\right) \approx 45°$

6. Explain or show mathematically why it is impossible to enter directly into an equatorial orbit from Kennedy Space Center.

$\beta = \sin^{-1}\left(\frac{\cos i}{\cos\phi}\right) = \sin^{-1}\left(\frac{1}{.878}\right) = \sin^{-1}(1.14)$

$=$ error

if $i = 0°$ and $\beta = 28.6°$ no azimuth β possible

Consider the difference between

checking a calculation against a static answer in the back of a book
or against search results online

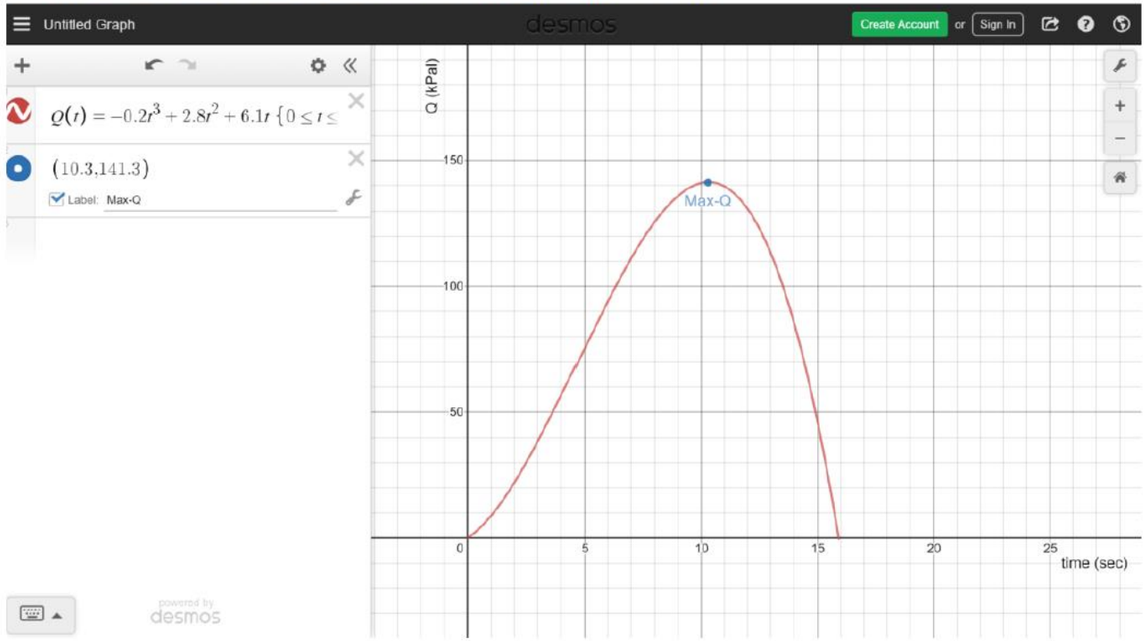
compared with

confirming a calculation in a simulation game
in an experience that is both dynamic and interactive

Calculus Examples

Maximum Aerodynamic Pressure

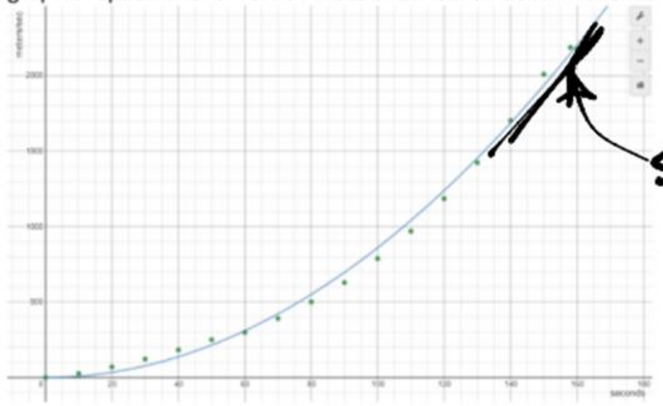
1. Rolle's theorem
2. Max-Q occurs at about 10.3 seconds, graph below.



Finding the instant of Max-Q
using cubic regression on data exported from game

Calculus Examples

3. $v(t) = 0.086t^2$
4. graph of speed over time from data and mathematical model using Desmos



Slope = acc.
= 27.2 m/s²

g-Force
$= \frac{a}{g} + 1$

5. $a(t) = 0.172t$
6. $a(10) = 1.72\ m/s^2$, $a(70)=12.04\ m/s^2$, $a(158)= 27.2\ m/s^2$
7. TWR (and g-Force) at 10 seconds is about 1.2, at 70 seconds TWR is about 2.2 and at 158 seconds it is about 3.8.

at 10 sec → $\frac{1.72}{9.8} + 1 = 1.2$

at 70 sec → $\frac{12.04}{9.8} + 1 = 2.2$ (Max Q)

at 158 sec → $\frac{27.2}{9.8} + 1 = 3.8$ (MECO)

Analyzing velocity data from a SpaceX Falcon-9 launch and estimating acceleration and g-forces.

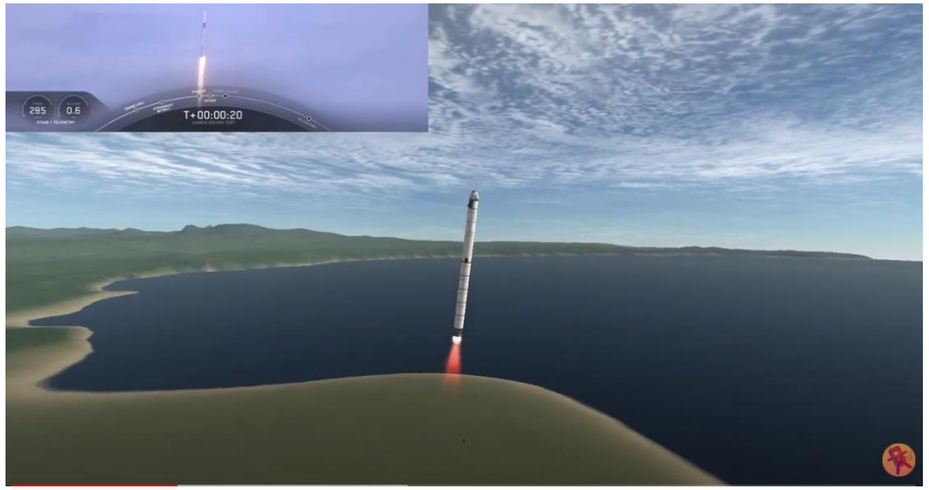


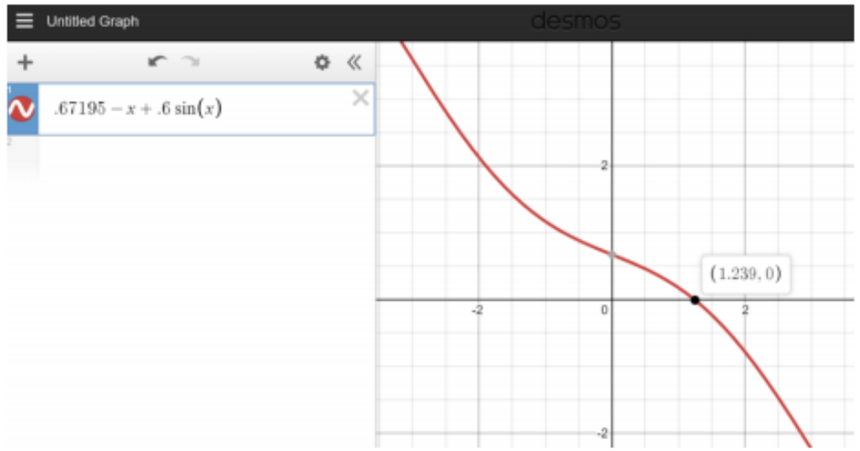
Image by *SpaceLab* on YouTube

Calculus Examples

2. Rounding to 3 decimal places, we have E = 1.239 after 4 iterations as shown below

n	Xn	f(X) = M - X + esinX	f'(X) = -1 + ecosX
0	0.67	0.373507955	-0.530434448
1	1.376105	-0.11549035	-0.883921652
2	1.245448	-0.004974343	-0.808216692
3	1.239293	-1.07605E-05	-0.804721246
4	1.23928	-5.07203E-11	-0.804713659
5	1.23928	approx. 0	-0.804713659

Graphing $f(x) = .67195 - x + 0.6\sin x$, we find an intercept at x = 1.239 as shown below



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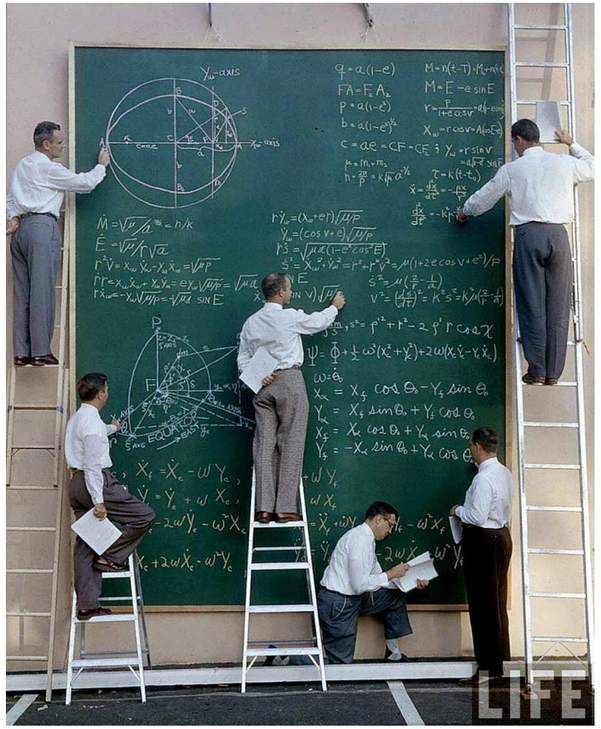
Kepler's Equation Interactive: https://www.geogebra.org/m/w3cby2qp

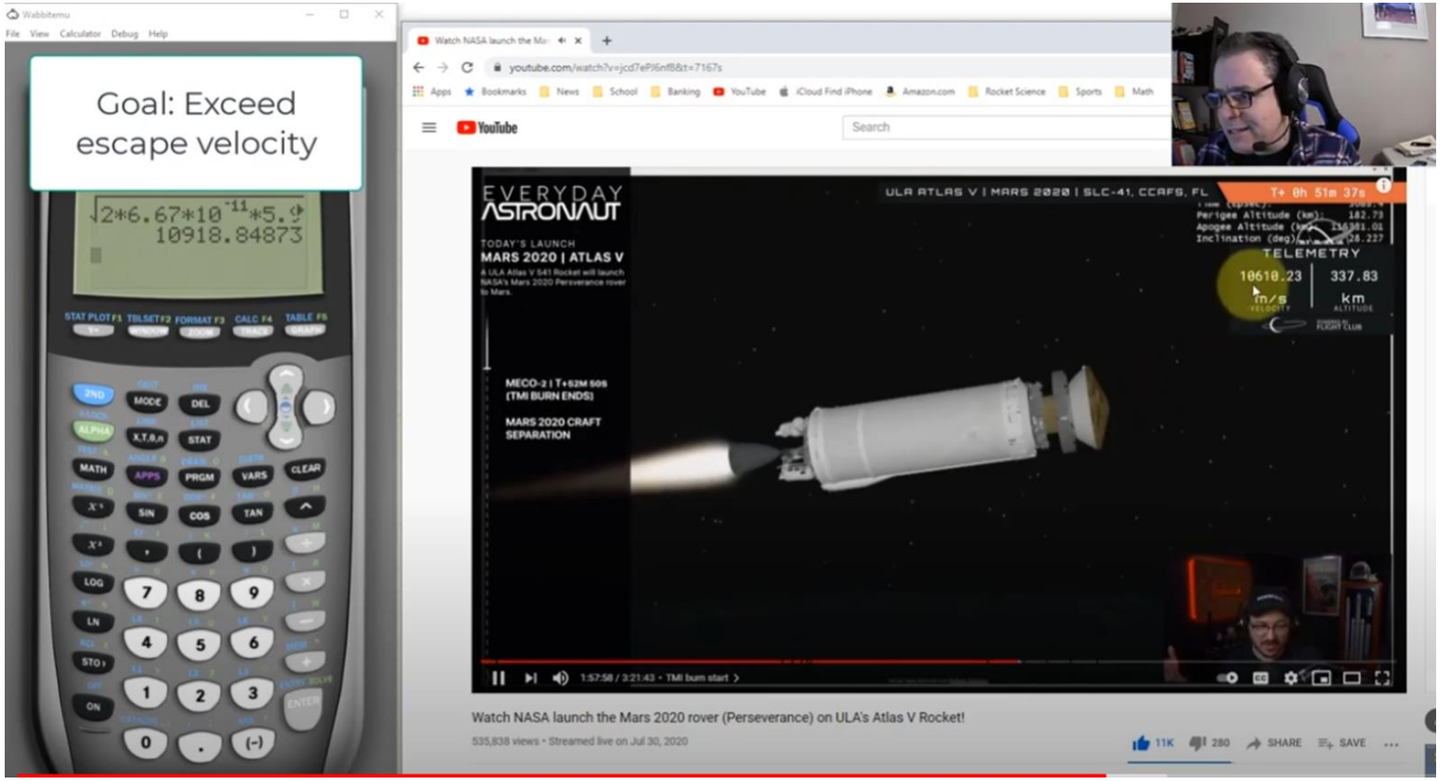
Use ***Newton's Method*** to solve ***Kepler's Equation***

Demonstrate Newton's method and a ***reason*** why a "numerical" method is required.

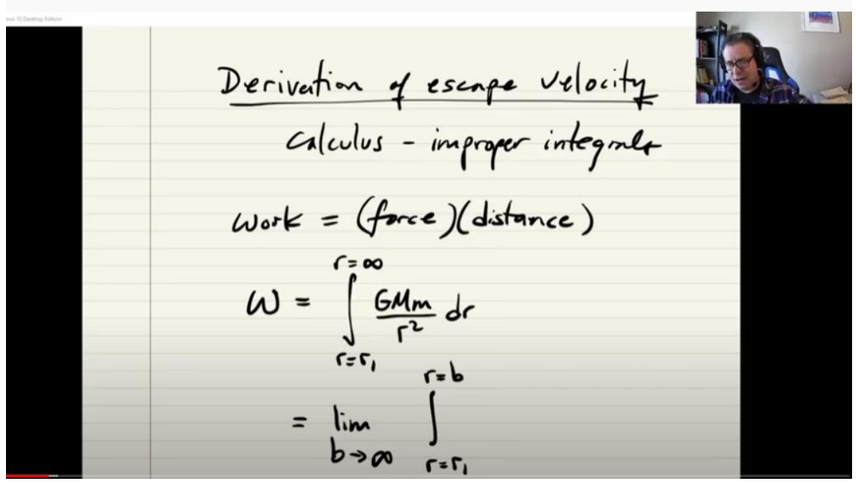


NASA before PowerPoint

Calculus Examples



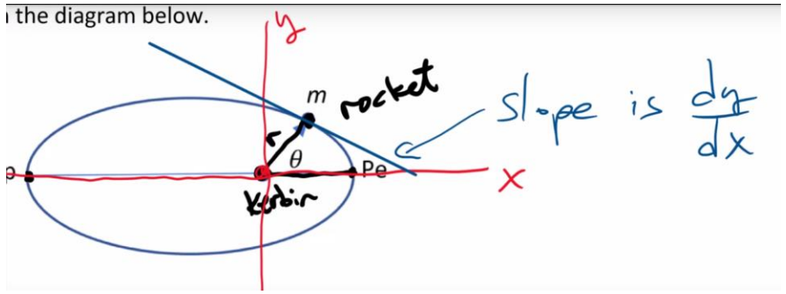
Finding Escape Velocity: https://youtu.be/9hox2a1GZug?t=1124



Calculus 2 classes:

- Derivation is application of improper integrals
- Connection with principle of work and energy
- Examples with Mars 2020 Perseverance mission, Apollo mission, and example in KSP

Calculus Examples



Finding slope of the tangent to the flight path, calculating flight path angle, confirm results in flight

We can use the relation $tan\varphi = \frac{1}{r}\frac{dr}{d\theta}$ to determine the flight path angle in orbit.

A proof of this relation is outlined at *youtube.com/c/csvaughen*.

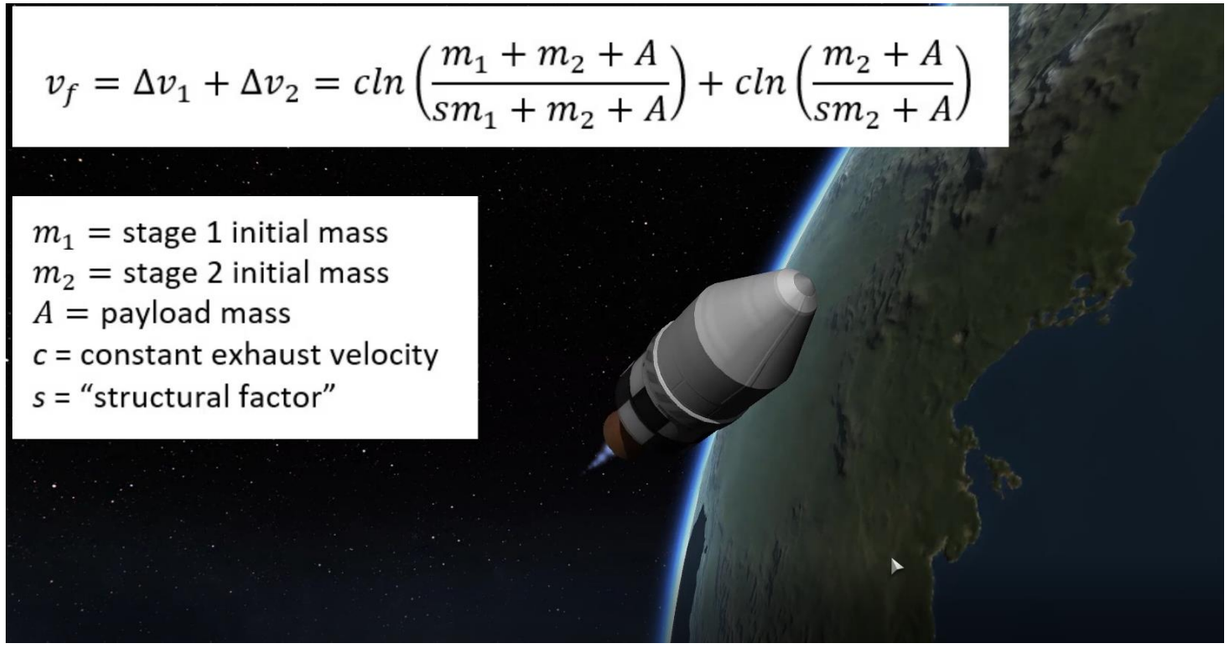
6. If $r = \frac{a(1-e^2)}{1+ecos\theta}$ describes an elliptical orbit, find an expression for $\frac{1}{r}\frac{dr}{d\theta}$.

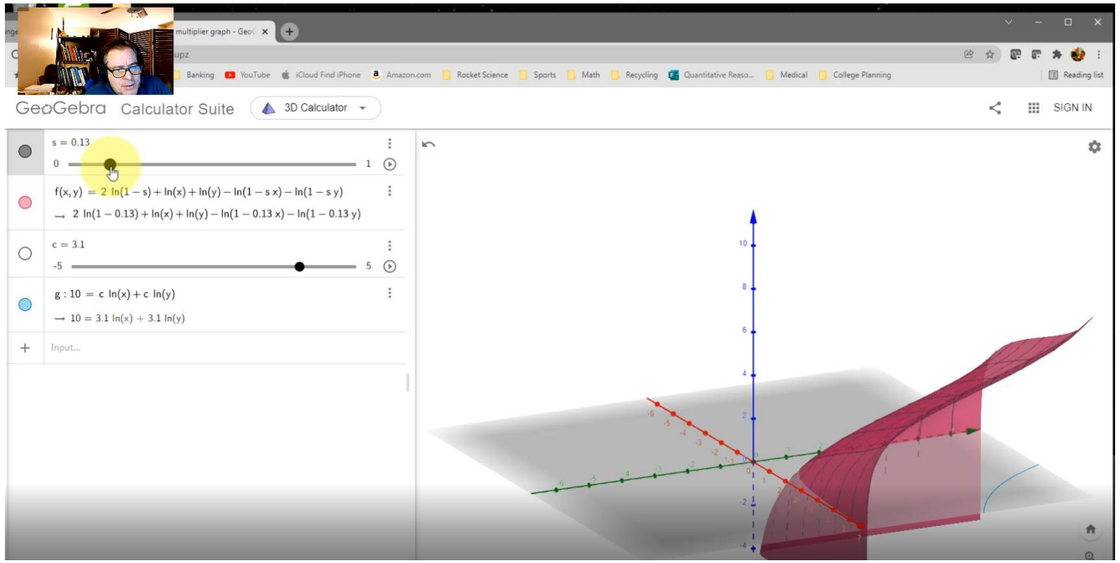
https://youtu.be/hdt7WnEVcII

Plot of flight path angle during orbit

Calculus Examples

Find optimal mass for each stage of a multi-stage rocket with Lagrange Multipliers





Video on YouTube https://youtu.be/eiGhqsNSMm0
and sites.google.com/view/kspmath

Derivation of formulas, comparisons with real-life rockets and examples in Kerbal Space Program
Examples, with solutions, beyond those done in video are at *sites.google.com/view/kspmath*
Extension to a similar project in the Stewart Calculus textbook

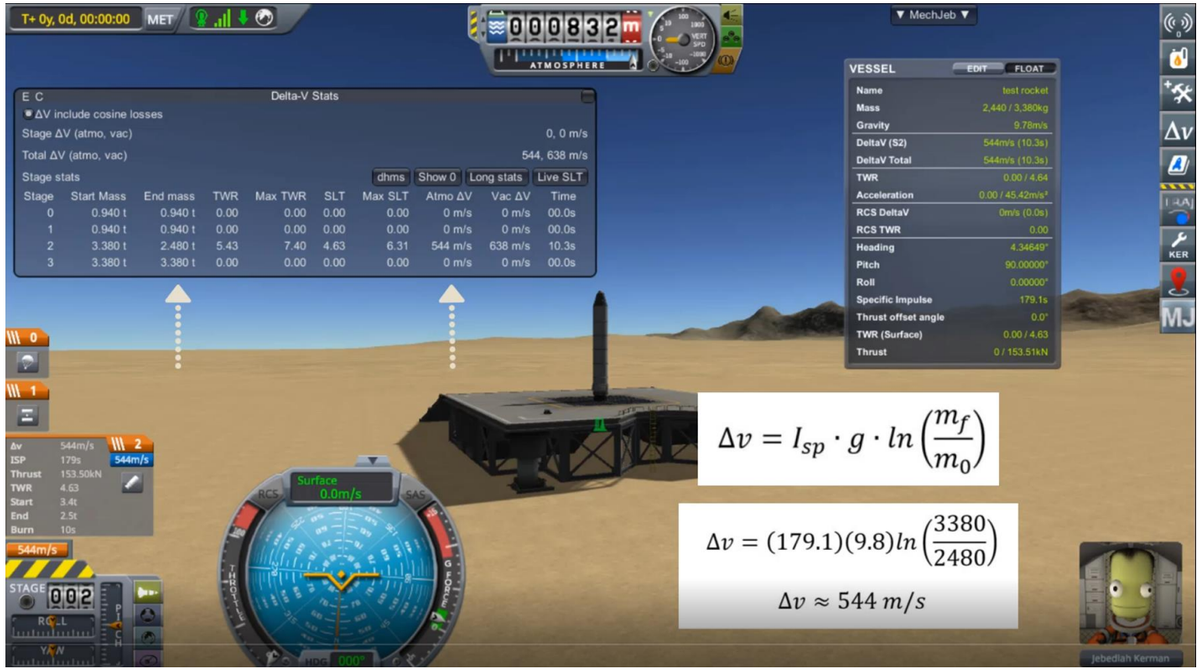
Differential Equations

Rocket Equation *with Gravity and Drag*

$$\frac{d}{dt}(mv) = F_{total}$$

$$\frac{dm}{dt}v + m\frac{dv}{dt} = R - mg - kv^2$$

With gravity and atmospheric drag, actual velocity should be less than theoretical value predicted by ideal rocket equation



Estimating theoretical delta-v with *ideal* rocket equation

Solve for *v* using Euler's method…

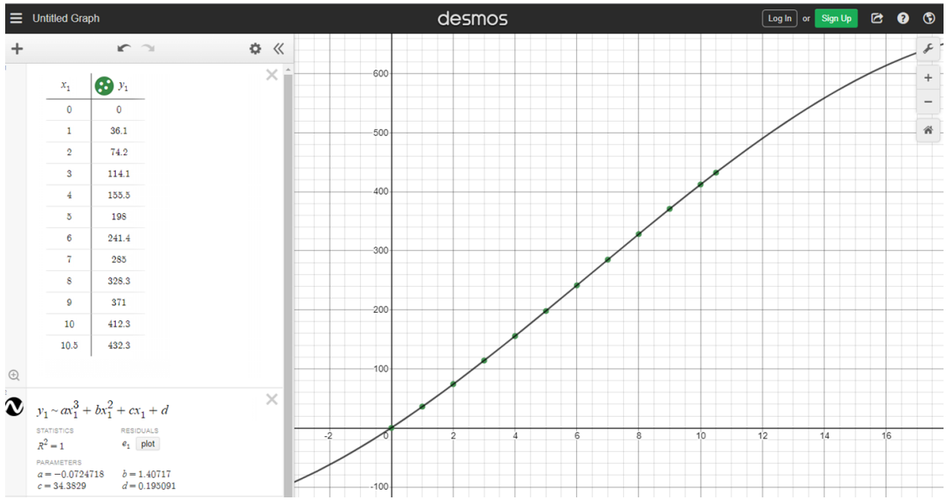
Differential Equations

Modelling with KSP – velocity in meters/sec

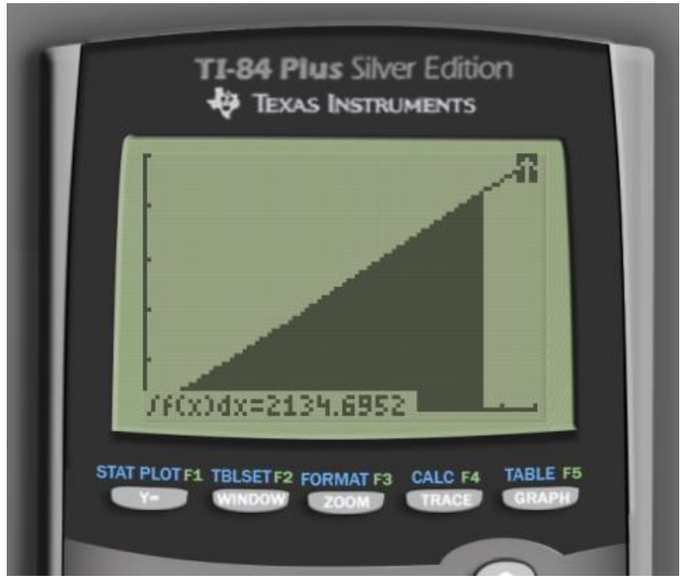
Time (seconds)	Excel	Mathematica	KSP
0	0	0	0
1	36.1	36.6	38.1
2	74.2	75.2	72.5
3	114.1	115.5	117.1
4	155.5	157.2	156.6
5	198	199.9	197.2
6	241.4	243.3	242.2
7	285	286.8	281.7
8	328.3	330	326
9	371	372.2	368
10	412.3	413	418.9
10.5	432.3	432.8	432.3

Best Fit Cubic to Velocity Data - Desmos

$$v(t) = -.0725t^3 + 1.4072t^2 + 34.3829t + .1951$$



Differential Equations



Integrating velocity on interval $t = 0$ to $t = 10.3$

Initial height : 832 meters (ASL)

2135 + 832 = 2967 meters

Estimated altitude at engine cut off: 2967 meters

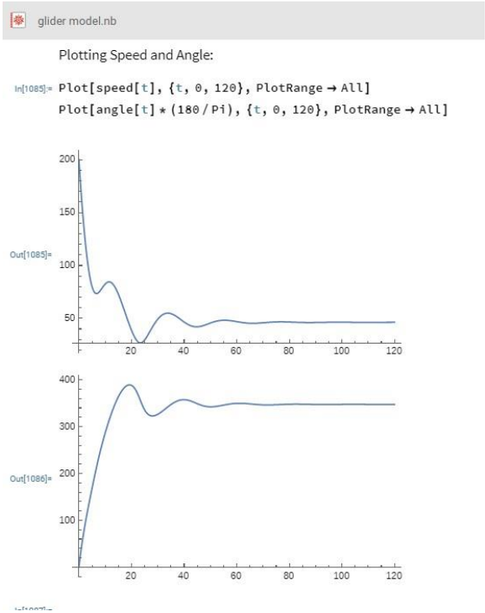
Confirming calculations in KSP

Differential Equations – *future addition – Thanks to Prof. Mera Couto!*

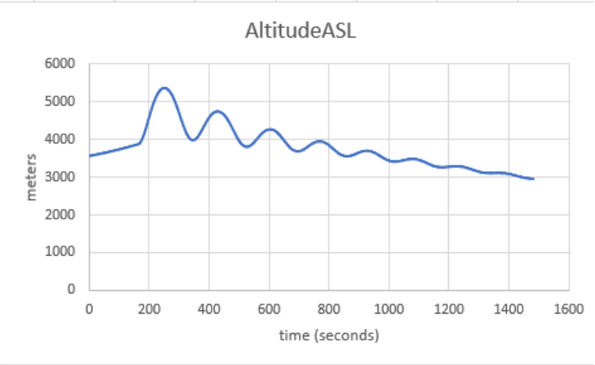
Modeling a glider's flight

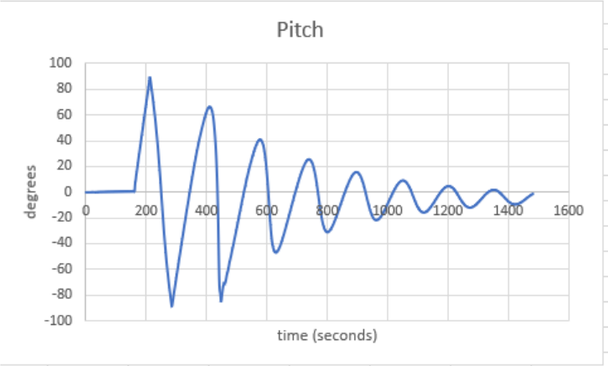
$$m\frac{\mathrm{d}V}{\mathrm{d}t} = -mg\sin\theta - \frac{1}{2}\rho V^2 C_D S$$

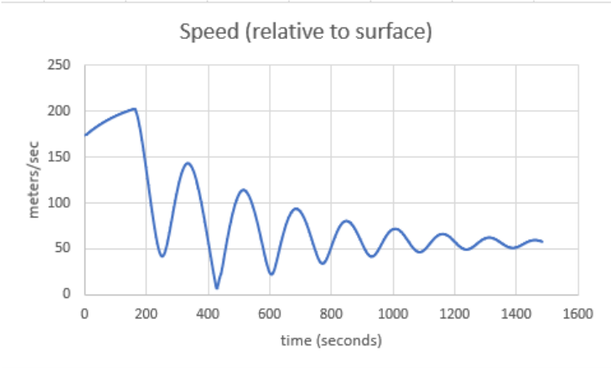
$$mV\frac{d\theta}{dt} = -mg\cos\theta + \frac{1}{2}\rho V^2 C_L S$$



Game data exported to Excel:







Thank you!!!

Questions?

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Blue Bell, PA

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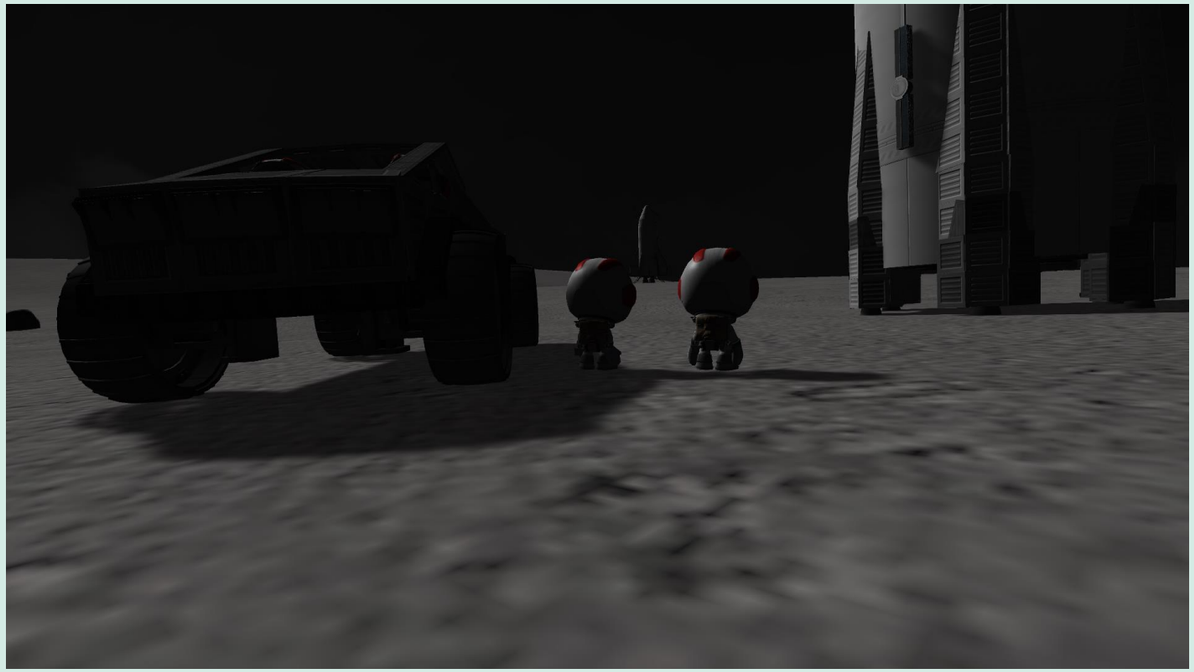
Kerbal Math & Physics Lab at

sites.google.com/view/kspmath

Christopher Scott Vaughen on YouTube

Also on YouTube:

- Scott Manley
- Mike Aben
- Tim Dodd
- Matt Lowne
- ShadowZone
- GameplayreviewUK
- SpaceLab



- KSP Math & Physics Lab Intro Video:
  https://www.youtube.com/watch?v=cnwJtBNGmgA

- Teaching with Kerbal Space Program – Fall 2020
  https://www.youtube.com/watch?v=WTVmKVEhb3s

- Teaching with Kerbal Space Program – Spring 2021
  https://youtu.be/GC1jZ3qIVdQ

- The Kerbal Effect – 10 year anniversary video celebrating KSP's ability to inspire
  https://youtu.be/GC1jZ3qIVdQ