Games are Mathematical Models

Teaching with Kerbal Space Program

Christopher Scott Vaughen
Assistant Math Professor
Montgomery County Community College
Blue Bell, PA
Outline

• Games as mathematical models

• *Kerbal Space Program* is a model solar system

• NASA KSP Math & Physics Lab *Grant*

• MAT199 NASA KSP Math & Physics Lab *Course* (Fall 2020-Spring 2021)

• Examples from the Kerbal Math & Physics Lab online *Workbook*
Games as Mathematical Models

Interesting parallels between doing mathematics and playing games:

• In both, we simulate a real-life activity or other phenomenon with a set of rules and assumptions.

• In math: What can we discover or achieve from a given set of axioms, while playing by the rules of math and logic?

• In a game: What can we discover or achieve while following the rules of the game?

Discovering the Armstrong Memorial on the Mun in KSP
Kerbal Space Program (KSP)
Computer game (PC, Mac, Xbox and PS4)

• Available at KerbalSpaceProgram.com and Steam.com
• Model solar system illustrates universality of math and physics
• Multidisciplinary environment
• Models textbook Newtonian physics with patched conic trajectories
• Multiple planets and moons, with and without atmosphere, create a virtual lab environment
• Players build rockets, satellites, landers, rovers, spaceplanes, jets, airplanes, and …
• Large active community of players, extensions and mods add options and realism
Goals:
• Create MAT 199 NASA KSP Math & Physics Lab course
• Promote math and physics literacy
• Promote STEM majors and careers
• Students complete algebra and pre-calculus chapters of the Kerbal Math & Physics Lab online workbook
• Study enrollment and demographic data in STEM at MCCC

Total enrolled in Fall 2020 and Spring 2021: 30 students
Total successfully completed: 23 students

Advertised to students at Intermediate Algebra level

STEM majors enrolled in course: 20
Non-STEM majors enrolled in course: 10

https://youtu.be/GC1jZ3qIvJo
MAT199 NASA KSP Math & Physics Lab Course

• Offered Fall 2020 and Spring 2021 at Montgomery County Community College in Blue Bell, PA

• Cost sharing with MCCC and NASA Pennsylvania Space Grant Consortium

• Students completed exercises and lab activities from the *Kerbal Math & Physics Lab* workbook

• 1 elective credit in math, meeting 2 hours/week for 15 weeks via Zoom

• Guest speakers:
  
  ❖ Lockheed Martin engineers and programmers
  ❖ Private Division game developers
  ❖ Scott Manley and Mike Aben (YouTube creators)
  ❖ Cody Short, orbital modeling engineer at Analytic Graphics, Inc
  ❖ Nicholas Seiberlich, former MCCC and Temple student, now professional engineer.
  ❖ Patrick Watt, Fall 2020 NASA KSP student, accepted into NASA Experience online program
The Kerbal Math & Physics Lab Online Workbook
by Christopher S. Vaughen

• Available online:  https://sites.google.com/view/kspmath

• Math and physics questions and solutions, lab activities

• Many questions don’t require access to the game and provide examples for traditional lectures, assignments, exams.

• Based on both KSP and real world examples

• Useful for traditional math/physics courses, lab activities, project based learning, math, physics and engineering clubs, and summer enrichment programs
## MAT 199 - NASA KSP Math & Physics Lab
### Schedule Spring 2021

<table>
<thead>
<tr>
<th>Class</th>
<th>Day</th>
<th>Date</th>
<th>Topic</th>
<th>HW/LAB due date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thursday</td>
<td>1/21</td>
<td>Intro, gravity and acceleration</td>
<td>HW0 due 1/23</td>
</tr>
<tr>
<td>2</td>
<td>Tuesday</td>
<td>1/26</td>
<td>Gravity and acceleration</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Thursday</td>
<td>1/28</td>
<td>Gravity and acceleration</td>
<td>HW1 due 1/29</td>
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<tr>
<td>4</td>
<td>Tuesday</td>
<td>2/2</td>
<td>Thrust to weight ratio, acceleration and g-force</td>
<td>HW2 due 2/5</td>
</tr>
<tr>
<td>5</td>
<td>Tuesday</td>
<td>2/4</td>
<td>Thrust to weight ratio, acceleration and g-force</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Tuesday</td>
<td>2/9</td>
<td>Acceleration and g-force lab</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Thursday</td>
<td>2/11</td>
<td>Acceleration and g-force lab, orbital speed</td>
<td>LAB3 due 2/12</td>
</tr>
<tr>
<td>8</td>
<td>Tuesday</td>
<td>2/16</td>
<td>Orbital speed</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Thursday</td>
<td>2/18</td>
<td>Orbital speed, NASA mission objectives, Perseverance landing</td>
<td>HW3 due 2/19</td>
</tr>
<tr>
<td>10</td>
<td>Tuesday</td>
<td>2/23</td>
<td>Orbital period</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Thursday</td>
<td>2/25</td>
<td>Orbital period, SpaceX and private industry</td>
<td>HW4 due 2/26</td>
</tr>
<tr>
<td>12</td>
<td>Tuesday</td>
<td>3/2</td>
<td>Computing the mass of a planet or moon, kinetic and potential energy</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Thursday</td>
<td>3/4</td>
<td>Presentation by Patrick Wall, former NASA KSP student in Fall 2020,</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>describing his successful completion of the NASA ICAS online program</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Tuesday</td>
<td>3/8</td>
<td>Linear and Angular Speed</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Thursday</td>
<td>3/11</td>
<td>Great circle navigation</td>
<td>HW5 due 3/12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Spring Break - no class 3/15 – 3/19</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Tuesday</td>
<td>3/23</td>
<td>Great circle, rhumb line navigation</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Thursday</td>
<td>3/25</td>
<td>Presentation by Cody Short, Astrodynamics</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Software Engineer</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Tuesday</td>
<td>3/30</td>
<td>Presentation by Private Division/Squad – Mathematics in Game Programming</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Thursday</td>
<td>4/1</td>
<td>Yis-viva lab</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Tuesday</td>
<td>4/6</td>
<td>Presentation by Nick Sieberlich, mathematics of control systems,</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>rovers, impenuity helicopter</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Thursday</td>
<td>4/8</td>
<td>Delta-v for a Hohmann transfer</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Tuesday</td>
<td>4/13</td>
<td>Presentation by Scott Money – Mathematics of hyperbolic orbits, the</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Netflix movie &quot;Stowaway&quot;</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Thursday</td>
<td>4/15</td>
<td>Yis-viva lab, last day to drop</td>
<td>HW6 due 4/16</td>
</tr>
<tr>
<td>24</td>
<td>Tuesday</td>
<td>4/20</td>
<td>The Rocket Equation</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Tuesday</td>
<td>4/22</td>
<td>The Rocket Equation</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Tuesday</td>
<td>4/27</td>
<td>Kepler's Laws of Planetary Motion, LAB2 due 4/23</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Thursday</td>
<td>5/4</td>
<td>Launch Azimuth, Latitude and Orbital Inclination, last official class</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>meeting</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Tuesday</td>
<td>5/11</td>
<td>Final exam week 5/6-5/13, LAB3 due Monday 5/10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Due Monday 5/10</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Wednesday</td>
<td>5/5</td>
<td>Reading day</td>
<td>HW7 due 5/7</td>
</tr>
</tbody>
</table>

### Spring 2021 Schedule
Guest Speakers – Scott Manley (YouTube Creator)

Fall 2020 – launch site latitude, azimuth and inclination
Spring 2021 - hyperbolic orbits, escape velocity, Netflix movie *Stowaway*
Guest Speakers – Game Developers from Private Division (KSP Publisher)

Presentations in Fall 2020 and Spring 2021 on mathematical skills required in game development and programming
Guest Speakers – Cody Short from Analytic Graphics, Inc.

Presentation on academic pathways to astrodynamics engineering and the work of an astrodynamics engineer
Guest Speakers – Nick Seiberlich, MCCC and Temple graduate

Nick, an MCCC and Temple graduate and now a manufacturing engineer, gave a presentation on the mathematics of control systems, and other mathematics applications in engineering.
KSP is a 3D graphing calculator:

- Do the math and check answers in game
- Illustrate general principles
- Experiment by changing parameters
Virtual STEM Lab

3. Velocity for entire flight (launch to splash down).

KSP is a **virtual STEM lab**

- Build models
- Conduct experiments
- Analyze exported game data

*Note: Absolute max speed at engine cut-off (at 52 seconds), speed then decreases until instant of max altitude (at 195 seconds), speed increases again as the rocket falls in space (to relative max speed at 310 seconds) after which speed is reduced again due to atmospheric drag and then parachutes deploying (at about 380 seconds) before splash down at about 8 minutes (or 480 seconds).*
Student feedback in MAT 199 NASA KSP course on an end-of-semester questionnaire:

1. Why did you decide to take this course?
   To try something new out of my comfort level.

2. What are some of the things you learned using the KSP program?
   - The concept that math and physics are universal
   - The ingenuity behind rocket science

6. Have you decided to take certain courses or pursue a major because of your experience with this course?
   I have decided to continue to pursue a degree in space engineering - this class has boosted my confidence in making this decision.

7. Would you recommend this course to other students? Why?
   This has been the highlight of my semester. I would recommend this course to anyone with even a slight interest in math, rocketry, astronomy, NASA navigation, game design, or all of the above. The material is wonderful, deep, and made much more easy to grasp with the help of KSP.

5. What are some examples of math or physics concepts you learned in the MAT 199 course?
   Through MAT 199, I learned the V=V and the Rocket Equations.
In traditional math classes, students can feel personally judged when they get something wrong.

In a game, learning from failure is more acceptable and expected.

And learning through interactive play can be very effective at developing intuition, an integrated understanding, and novel solutions.
Modelling with KSP

Examples from the *Kerbal Math & Physics Lab* available at sites.google.com/view/kspmath
1. A rocket orbits at an altitude of 100 km above the surface of Kerbin which has a radius of 600 km and a mass of $M = 5.29 \times 10^{22}$ kg. Find the orbital radius of the rocket and then compute the orbital speed at 100 km.

$$\Gamma = 600,000 + 100,000 = 700,000$$

$$V = \sqrt{\frac{GM}{\Gamma}} = \sqrt{\left(\frac{6.67 \times 10^{-11} \times 5.29 \times 10^{22}}{700,000}\right)} = 2245 \text{ m/s}$$

GM = $\mu = 353,000$

You can check your answer above with the *Kerbal 1* rocket (available in Sandbox mode in KSP). Check your rocket’s speed in the Navball display and in Map View.
Algebra Examples

15. The International Space Station (ISS) orbits the Earth at an altitude of about 250 miles above the surface. Earth’s radius is about 3960 miles. Calculate your own weight, in pounds, if you were onboard the ISS. Derive a formula that you could use to find anyone’s weight, in pounds, if they were onboard the ISS.

Answer:

15. Let \( x \) = weight in pounds on the surface of Earth. Then \( \frac{y}{x} = (\frac{3960}{r+h})^2 = 0.88x \) will be weight at an altitude of 250 miles, for example on board the ISS. This means weight on ISS is about 88% of weight on surface of the Earth.
### g-Force Lab:

**Estimate acceleration from velocity**

**Calculate g-force**

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**Algebra Examples**

**Acceleration and g-Force Lab Data:**

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Altitude ASL* (meters)</th>
<th>Speed (m/s)</th>
<th>Altitude (miles)</th>
<th>Speed (miles/hr)</th>
<th>Mach</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>81</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>21.6</td>
<td>0.14</td>
<td>116.52</td>
<td>66</td>
<td>0.65</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>0.38</td>
<td>2.38</td>
<td>2.38</td>
<td>0.65</td>
</tr>
<tr>
<td>15</td>
<td>12.9</td>
<td>0.50</td>
<td>780.79</td>
<td>47.79</td>
<td>0.50</td>
</tr>
<tr>
<td>20</td>
<td>2.113</td>
<td>0.75</td>
<td>1,169.84</td>
<td>70.84</td>
<td>0.60</td>
</tr>
<tr>
<td>25</td>
<td>35.7</td>
<td>2.30</td>
<td>654.86</td>
<td>39.86</td>
<td>0.60</td>
</tr>
<tr>
<td>30</td>
<td>547.2</td>
<td>3.10</td>
<td>3,100.33</td>
<td>1,860.33</td>
<td>1.16</td>
</tr>
<tr>
<td>35</td>
<td>761.2</td>
<td>3.74</td>
<td>4,674.11</td>
<td>2,804.11</td>
<td>1.45</td>
</tr>
<tr>
<td>40</td>
<td>1,097.5</td>
<td>4.59</td>
<td>6,455.45</td>
<td>3,873.45</td>
<td>1.86</td>
</tr>
<tr>
<td>45</td>
<td>1,394.1</td>
<td>5.52</td>
<td>8,180.52</td>
<td>4,908.32</td>
<td>2.61</td>
</tr>
<tr>
<td>50</td>
<td>1,800.8</td>
<td>6.10</td>
<td>9,690.10</td>
<td>5,754.05</td>
<td>2.80</td>
</tr>
<tr>
<td>55</td>
<td>2,405.3</td>
<td>6.74</td>
<td>11,215.33</td>
<td>6,729.24</td>
<td>3.16</td>
</tr>
</tbody>
</table>

4. Estimate the acceleration on the rocket (from launch to engine cut-off) using $a = \frac{\Delta v}{\Delta t}$ where $\Delta v$ is the total change in velocity during the burn and $\Delta t$ is the time interval of the burn.

$$a = \frac{1,068 \text{ m/s}}{52 \text{ s}} = 20.4 \text{ m/s}^2 \text{ AVERAGE ACCELERATION TO MECO}$$
5. Using Desmos to get line of best fit to velocity data to find average acceleration:

\[ v = 20.08t - 108 \]

\[ a = \frac{20.08}{9.8} \approx 3 \text{ g's} \]

- Collect data in-game
- Compute average acceleration during launch
- Use acceleration to find average g-force
- Can be done with KSP or real-life data
Calculating initial heading from the Apollo 11 landing site to the Apollo 12 landing site

7. Compute the initial bearing for the great circle path on the Moon from the Apollo 11 landing site to the Apollo 12 landing site. Use the Apollo 11 landing site coordinates at (0.68° North, 23.4° East) and the Apollo 12 landing site coordinates at (3.2° South, 23.4° West). Show work using the method presented here.

\[
\begin{align*}
A & : (0.68°, 23.4°) \\
B & : (-3.2°, -23.4°) \\
\Delta \theta & = -16.8° \\
x & = \cos(0.68°) \sin(-3.2°) - \sin(0.68°) \cos(-3.2°) \cos(180°) \\
y & = \sin(0.68°) \cos(-3.2°) \\
z & = \sqrt{1 - x^2 - y^2} \\
B' & = \tan^{-1}\left(\frac{y}{x}\right) \\
B & = 85.9° \pm 180° = 245°
\end{align*}
\]

Planning a trip to see the UFO buried at the south pole of the Mun in KSP

Calculus based derivation of navigation formulas: 
https://youtu.be/wygNHhI3EgY
Pre-Calc & Trig Examples

Derive formula for delta-v required to initiate transfer orbit

2. Use the vis-viva equation, and the formula for speed in a circular orbit, to derive a general formula for the initial change in velocity (Δv) required for a Hohmann transfer from a circular orbit of radius $r_1$ to a circular orbit of radius $r_2$, where $r_2 > r_1$.

Vis-viva equation:

$$v^2 = \mu \left( \frac{1}{r} - \frac{1}{a} \right)$$

$v$: Velocity

$\mu$: Gravitational Constant

$r$: distance between two bodies

$a$: Semi-major axis

$$a = \frac{r_1 + r_2}{2}$$

$r_1$: distance to periapsis

$r_2$: distance to apoapsis

Speed at periapsis:

$$v^2 = \mu \left( \frac{1}{r} - \frac{1}{r} \right)$$

$$v^2 = \mu \left( \frac{a}{r_1} - \frac{a}{r_2} \right)$$

$$v^2 = \mu \left( \frac{a(r_2 - r_1)}{r_1(r_1 + r_2)} \right)$$

$$v^2 = \mu \left( \frac{a(r_2 - r_1)}{r_1(r_1 + r_2)} \right)$$

$$v = \sqrt{\frac{\mu}{r_1}}$$

$$\Delta V = \sqrt{\mu \left( \frac{a(r_2 - r_1)}{r_1(r_1 + r_2)} \right)} - \sqrt{\frac{\mu}{r_1}}$$
Pre-Calc & Trig Examples

Confirm delta-v calculations in KSP and in real-life

A Trip to the Moon, Part 3: Go For TLI
https://youtu.be/O3Yz-nNpa24

Apolloinrealtime.org
Discovering a new planet, and confirming predicted orbit with Kepler’s third law

6. Completing the table to include Eeloo we have

<table>
<thead>
<tr>
<th>Planet</th>
<th>log(a)</th>
<th>log(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moho</td>
<td>9.721268782</td>
<td>6.3455</td>
</tr>
<tr>
<td>Eve</td>
<td>9.992672106</td>
<td>6.7526</td>
</tr>
<tr>
<td>Kerbin</td>
<td>10.1355381</td>
<td>6.9639</td>
</tr>
<tr>
<td>Duna</td>
<td>10.31651875</td>
<td>7.2384</td>
</tr>
<tr>
<td>Dres</td>
<td>10.6110788</td>
<td>7.6802</td>
</tr>
<tr>
<td>Jool</td>
<td>10.83742151</td>
<td>8.0197</td>
</tr>
<tr>
<td>Eeloo</td>
<td>10.9548155</td>
<td>8.1958</td>
</tr>
</tbody>
</table>

7. Updated graph includes the newly discovered planet Eeloo located at approximately (10.95, 8.20) in the log(a)-log(T) graph, confirming Kepler’s third law of planetary motion.

- Collect orbit data (semi-major axis and period)
- Learn properties of log functions
- Learn Kepler’s third law
- Apply to alternate solar system besides our own
Pre-Calc & Trig Examples

Recording data for the vis-viva lab

Confirming conservation of energy and angular momentum and corresponding orbital eccentricity

<table>
<thead>
<tr>
<th>trajectory</th>
<th>point 1 (in flight)</th>
<th>point 2 (in flight)</th>
<th>eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>sub-orbital</td>
<td>altitude = 0.6672 m</td>
<td>altitude = 0.3962 m</td>
<td>e = 0.96</td>
</tr>
<tr>
<td></td>
<td>velocity = 930 m/s</td>
<td>velocity = 915 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pitch = 3.3°, θ = 26.9°</td>
<td>pitch = -3.5°, θ = 27°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e_1 = -4.5 \times 10^{-6}$</td>
<td>$e_2 = -9.9 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_1 = 3.1 \times 10^6$</td>
<td>$h_2 = 3.2 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>orbital (elliptical)</td>
<td>altitude = 2.337 m</td>
<td>altitude = 4.125 m</td>
<td>e = 0.28</td>
</tr>
<tr>
<td></td>
<td>velocity = 64.8 m/s</td>
<td>velocity = 1.237 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pitch = -1.5°, θ = 26.9°</td>
<td>pitch = -1.5°, θ = 26.9°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e_1 = -1.9 \times 10^{-5}$</td>
<td>$e_2 = -1.9 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_1 = 1.8 \times 10^8$</td>
<td>$h_2 = 1.8 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td>orbital (nearly circular)</td>
<td>altitude = 282.7 m</td>
<td>altitude = 830 m</td>
<td>e = 0.001</td>
</tr>
<tr>
<td></td>
<td>velocity = 2394 m/s</td>
<td>velocity = 2397 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pitch = -0.2°, θ = 28.8°</td>
<td>pitch = -0.2°, θ = 28.8°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e_1 = -2.6 \times 10^{-6}$</td>
<td>$e_2 = -2.6 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_1 = 1.6 \times 10^7$</td>
<td>$h_2 = 1.6 \times 10^7$</td>
<td></td>
</tr>
<tr>
<td>hyperbolic</td>
<td>altitude = 1074.5 m</td>
<td>altitude = 120.5 m</td>
<td>e = 1.08</td>
</tr>
<tr>
<td></td>
<td>velocity = 3225 m/s</td>
<td>velocity = 990 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pitch = 6°, θ = 84°</td>
<td>pitch = 80°, θ = 10°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e_1 = 2.1 \times 10^5$</td>
<td>$e_2 = 2.1 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_1 = 2.3 \times 10^7$</td>
<td>$h_2 = 2.3 \times 10^7$</td>
<td></td>
</tr>
</tbody>
</table>
Pre-Calc & Trig Examples

Calculating launch azimuth \((A)\) and orbit inclination \((i)\) at different latitudes \((\beta)\) on Earth and on Kerbin

Calculations which can be confirmed by observations of real life launches and with KSP

\[
\cos(i) = \sin(A) \cos(\beta)
\]

5. For the following questions, use the fact that the orbital inclination of the International Space Station (ISS) is approximately 51.6 degrees.

a) What launch azimuth is required from the Kennedy Space Center to reach the orbital inclination of the ISS? Show work, check with the Desmos and Geogeebra Interactive calculators.

\[
\beta = 28.8^\circ, \text{KSC lat.}
\]

\[
\sin(\beta) = \sin(28.8^\circ) = 0.481
\]

If \(i = 0^\circ\) and \(\beta = 28.8^\circ\) no azimuth \(\beta\) possible.
Consider the difference between
cHECKING A CALCULATION AGAINST A STATIC ANSWER IN THE BACK OF A BOOK
or against search results online

compared with

cONFIRMING A CALCULATION IN A SIMULATION GAME
in an experience that is both dynamic and interactive
Calculus Examples

Maximum Aerodynamic Pressure

1. Rolle’s theorem
2. Max-Q occurs at about 10.3 seconds, graph below.

Finding the instant of Max-Q using cubic regression on data exported from game
Calculus Examples

Analyzing velocity data from a SpaceX Falcon-9 launch and estimating acceleration and g-forces.

Image by SpaceLab on YouTube
Calculus Examples

2. Rounding to 3 decimal places, we have $E = 1.239$ after 4 iterations as shown below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x_n$</th>
<th>$f(x_n)$</th>
<th>$f'(x_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.67</td>
<td>0.373507955</td>
<td>-0.530434448</td>
</tr>
<tr>
<td>1</td>
<td>1.376105</td>
<td>-0.11549035</td>
<td>-0.883921652</td>
</tr>
<tr>
<td>2</td>
<td>1.245448</td>
<td>-0.004974343</td>
<td>-0.808716682</td>
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<tr>
<td>3</td>
<td>1.239293</td>
<td>-1.07605E-05</td>
<td>-0.804721246</td>
</tr>
<tr>
<td>4</td>
<td>1.23928</td>
<td>-5.07208E-11</td>
<td>-0.804713659</td>
</tr>
<tr>
<td>5</td>
<td>1.23928</td>
<td>approx. 0</td>
<td>-0.804713659</td>
</tr>
</tbody>
</table>

Graphing $f(x) = 0.67195 - x + 0.6\sin x$, we find an intercept at $x = 1.239$ as shown below.

Kepler’s Equation Interactive: [https://www.geogebra.org/m/w3cby2qp](https://www.geogebra.org/m/w3cby2qp)

Use **Newton’s Method** to solve **Kepler’s Equation**

**Demonstrate** Newton’s method and a *reason* why a “numerical” method is required.

NASA before PowerPoint
Calculus Examples

Finding Escape Velocity:  https://youtu.be/9hox2a1GZug?t=1124

Calculus 2 classes:
- Derivation is application of improper integrals
- Connection with principle of work and energy
- Examples with Mars 2020 Perseverance mission, Apollo mission, and example in KSP
Calculus Examples

Finding slope of the tangent to the flight path, calculating flight path angle, confirm results in flight

We can use the relation $tan \varphi = \frac{1}{r} \frac{dr}{d\theta}$ to determine the flight path angle in orbit.

A proof of this relation is outlined at youtube.com/c/csvaughen.

6. If $r = \frac{a(1-e^2)}{1+eccos\theta}$ describes an elliptical orbit, find an expression for $\frac{1}{r} \frac{dr}{d\theta}$.
Calculus Examples

Find optimal mass for each stage of a multi-stage rocket with Lagrange Multipliers

\[ v_f = \Delta v_1 + \Delta v_2 = c \ln \left( \frac{m_1 + m_2 + A}{s m_1 + m_2 + A} \right) + c \ln \left( \frac{m_2 + A}{s m_2 + A} \right) \]

- \( m_1 \) = stage 1 initial mass
- \( m_2 \) = stage 2 initial mass
- \( A \) = payload mass
- \( c \) = constant exhaust velocity
- \( s \) = “structural factor”

Derivation of formulas, comparisons with real-life rockets and examples in Kerbal Space Program
Examples, with solutions, beyond those done in video are at sites.google.com/view/kspmath
Extension to a similar project in the Stewart Calculus textbook

Video on YouTube https://youtu.be/ElGhqsNSMm0 and sites.google.com/view/kspmath
Differential Equations

Rocket Equation with Gravity and Drag

\[
\frac{d}{dt}(mv) = F_{total}
\]

\[
\frac{dm}{dt}v + m \frac{dv}{dt} = R - mg - kv^2
\]

With gravity and atmospheric drag, actual velocity should be less than theoretical value predicted by ideal rocket equation

Solve for \( v \) using Euler's method…

Estimating theoretical delta-v with ideal rocket equation

\[
\Delta v = l_{sp} \cdot g \cdot \ln \left( \frac{m_f}{m_0} \right) = (179.1)(9.8)\ln \left( \frac{3380}{2480} \right)
\]

\[
\Delta v \approx 544 \text{ m/s}
\]
### Differential Equations

#### Modelling with KSP – velocity in meters/sec

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Excel</th>
<th>Mathematica</th>
<th>KSP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>36.1</td>
<td>36.6</td>
<td>38.1</td>
</tr>
<tr>
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<tr>
<td>4</td>
<td>155.5</td>
<td>157.2</td>
<td>156.6</td>
</tr>
<tr>
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<td>199.9</td>
<td>197.2</td>
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<tr>
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<td>285</td>
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</tr>
<tr>
<td>8</td>
<td>328.3</td>
<td>330</td>
<td>326</td>
</tr>
<tr>
<td>9</td>
<td>371</td>
<td>372.2</td>
<td>368</td>
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<tr>
<td>10</td>
<td>412.3</td>
<td>413</td>
<td>418.9</td>
</tr>
<tr>
<td>10.5</td>
<td>432.3</td>
<td>432.8</td>
<td>432.3</td>
</tr>
</tbody>
</table>

#### Best Fit Cubic to Velocity Data - Desmos

\[
v(t) = -0.0725t^3 + 1.4072t^2 + 34.3829t + 0.1951
\]
Integrating velocity on interval $t = 0$ to $t = 10.3$

Initial height: 832 meters (ASL)

$2135 + 832 = 2967$ meters

Estimated altitude at engine cut off: 2967 meters

Confirming calculations in KSP
Differential Equations – future addition – Thanks to Prof. Mera Couto!

Modeling a glider’s flight

\[
\frac{m \, dV}{dt} = -mg \sin \theta - \frac{1}{2} \rho V^2 C_D S
\]
\[
\frac{mV \, d\theta}{dt} = -mg \cos \theta + \frac{1}{2} \rho V^2 C_L S
\]

Game data exported to Excel:
Thank you!!!

Questions?
Christopher Scott Vaughen
Assistant Math Professor
Montgomery County Community College
Blue Bell, PA

cvaughen@mc3.edu

Kerbal Math & Physics Lab at
/sites.google.com/view/kspmath

Christopher Scott Vaughen on YouTube

Also on YouTube:
• Scott Manley
• Mike Aben
• Tim Dodd
• Matt Lowne
• ShadowZone
• GameplayreviewUK
• SpaceLab

• KSP Math & Physics Lab Intro Video:
  https://www.youtube.com/watch?v=cnwJtBNGmgA

• Teaching with Kerbal Space Program – Fall 2020
  https://www.youtube.com/watch?v=WTVmKVEhb3s

• Teaching with Kerbal Space Program – Spring 2021
  https://youtu.be/GC1jZ3qIvDQ

• The Kerbal Effect – 10 year anniversary video celebrating KSP’s ability to inspire
  https://youtu.be/GC1jZ3qIvDQ