

# Slopes: A Free, Intuitive Mobile App to Enhance Learning in Differential Equations

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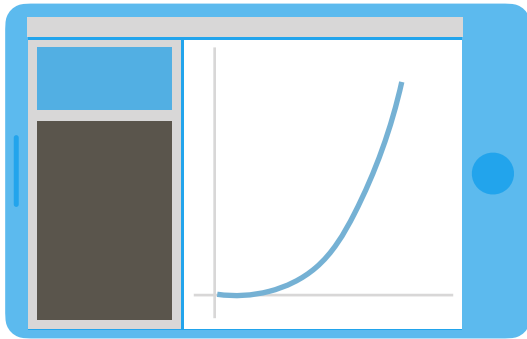
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Check out the apps at

<http://www.slopesapp.com>

<http://www.wavespdeapp.com>

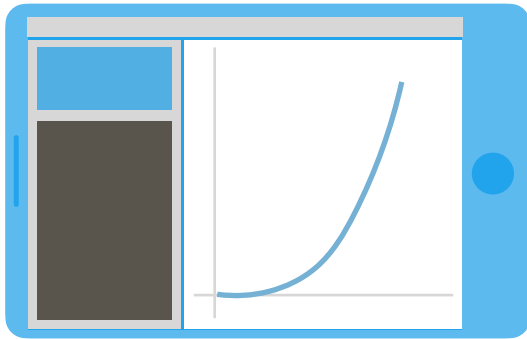




# Class Approach

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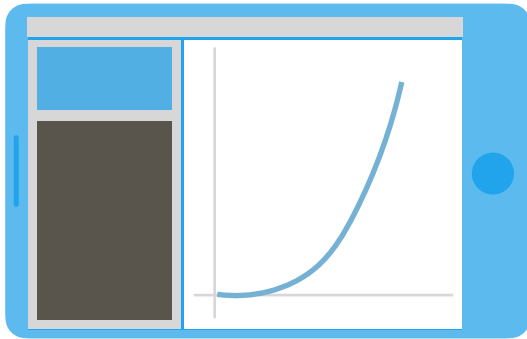
- \* Physical assumptions
- \* Mathematical expressions
- \* Think before you solve
  - \* Equilibrium solutions
  - \* Graphical analysis
    - \* Slopefields, Phase Planes, etc
- \* Analytical Solutions



## App Information

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- \* Languages: **Swift and Kotlin**
- \* Platforms: **iOS and Android**
- \* <http://slopesapp.com> (ODE)
  - \* iPad Release: **Nov 2016**
  - \* iPhone Release: **July 2017**
  - \* Android Release: **Jan 2021**
- \* <http://wavespdeapp.com> (PDE)
  - \* iOS Release Date: **June 2019**



# Why Mobile Devices?

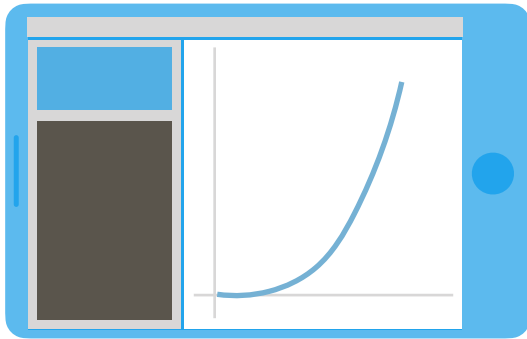
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- \* Portable
- \* Comparatively large screen
- \* Tactile interface

The Role of iPads in Constructing Collaborative Learning Spaces (Fisher, Lucas and Galstyan, 2013)

Using Slopes to Enhance Learning in Ordinary Differential Equations (K. Lucas and T. Lucas, 2022)





# Population Models

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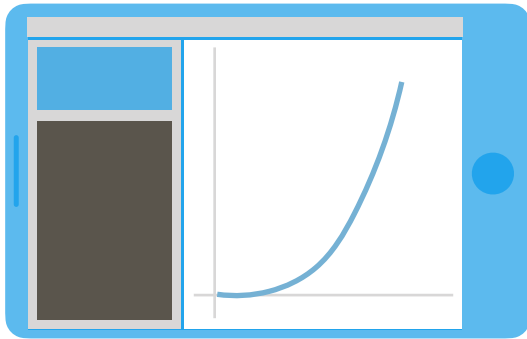
Invasive Crayfish - *Procambarus clarkii*







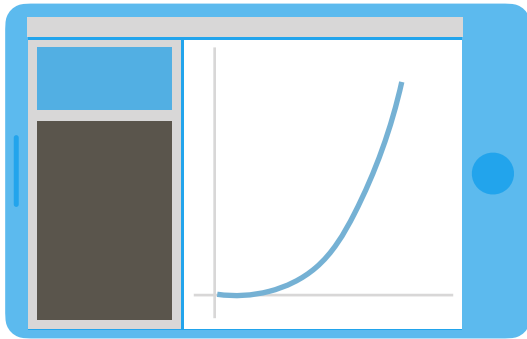




# Crayfish Models

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- \* Exponential Growth/Decay:  $y' = ay$
- \* Exponential Growth with Constant Removal:  $y' = ay - r$
- \* Logistic Growth:  $y' = ay(1-y/b)$
- \* Logistic Growth with Constant Removal:  $y' = ay(1-y/b) - r$
- \* Logistic Growth with Proportional Removal:  $y' = ay(1-y/b) - ry$



# Class Activities

1. **Logistic Growth:** Recall the following population model

$$\frac{dy}{dt} = ay \left(1 - \frac{y}{b}\right)$$

Use the slopefields activity to examine the model with  $a = 0.5$  and  $b = 10$ .

Discuss any observations can you make about the slopefield and solution curves.

- What do the arrows represent?

Arrows represent the slope

- Add some representative solution curves to plot. What is the relationship between the arrows and the solution curves?

Connect arrows, tracing the solution at particular starting point.

- Are there any equilibrium solutions? Use the plot to describe the stability of the equilibrium solutions.

$$y = 0, 10$$

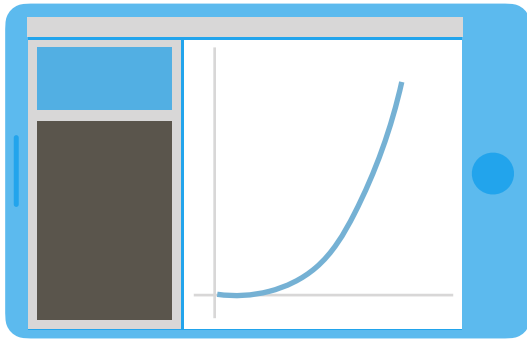
$0 \rightarrow$  unstable

$10 \rightarrow$  stable

- Interpret the solutions in the context of a population model.

$0 \rightarrow$  unstable  $10 \rightarrow$  stable

Start below 10 and  $\neq 0 \rightarrow 10$  Above 10  $\rightarrow 10$



# Class Activities

Consider the following modification of the previous model

$$\frac{dy}{dt} = ay \left(1 - \frac{y}{b}\right) \left(\frac{y}{c} - 1\right)$$

Use the slopefields activity to examine the model with  $a = 0.5$ ,  $b = 10$ ,  $c = 4$ .

Plot the slopefield and some representative solution curves. Discuss any observations.

- Describe the behavior of solutions for various initial values.

$$0 < y' < 4 \rightarrow 0 \quad y > 10 \rightarrow 10$$

$$4 < y' < 10 \rightarrow 10$$

- Are there any equilibrium solutions? Use the plot to describe the stability of the equilibrium solutions.

$$y' = 0, 4, 10$$

$$0 \rightarrow \text{stable} \quad 10 \rightarrow \text{stable}$$

$$4 \rightarrow \text{unstable}$$

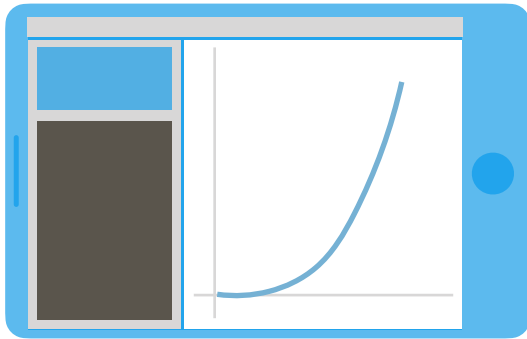
- Interpret the parameter  $c$  in the context of a population model.

$c$  is the threshold of which values above it will grow to carrying capacity and if below, will decay

to 0.

$$y' < c \rightarrow 0$$

$$y' > c \rightarrow b$$



# Class Activities

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Below is a discussion about the equilibrium solutions in the threshold model.

JAMES: So it would be anything greater than four.

ALEX: Ten, four, and zero.

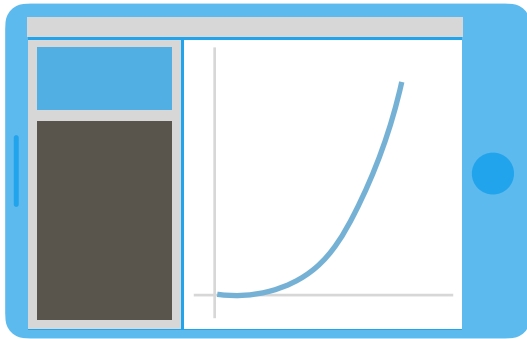
JAMES: And then four is unstable...It's like the opposite of a carrying capacity.

DOMINIC: It's a threshold, right?

JAMES: Oh yeah, it is. So if you're above the threshold, then you go to the carrying capacity.

DANIEL: If you're below, you're going to go to zero.

ALEX: It's like the minimum population you need to not die out.

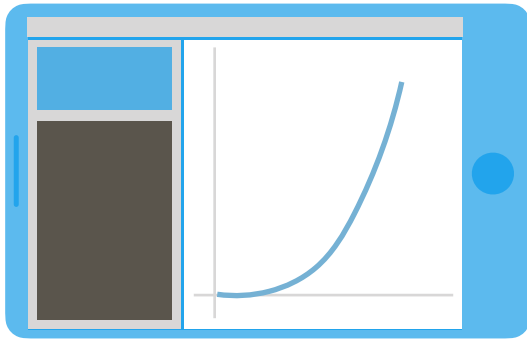


# Student Feedback

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“It's good just because visualizing helps a lot to be able to understand, especially when you get to higher levels of math and things get kind of hard to understand sometimes.”

“I really love how interactive it is ... You can move it around and manipulate it. I like being able to click to see, okay, what does the solution with this initial condition look like?”

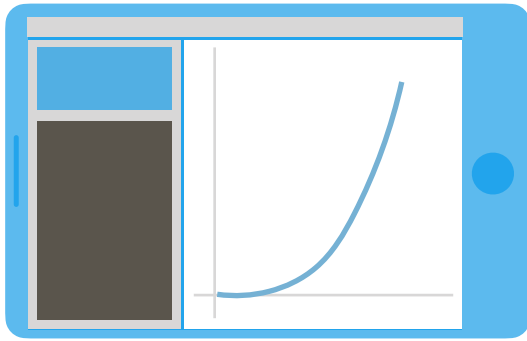


# Acknowledgements

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- \* Graphic Design: Dana Zurzolo
- \* Comp Sci: Stan Warford





# Questions?

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