Using Discrete Dynamical Systems and Differential Equations Projects in Calc II,
or
Digital Platforms modeling and usage.

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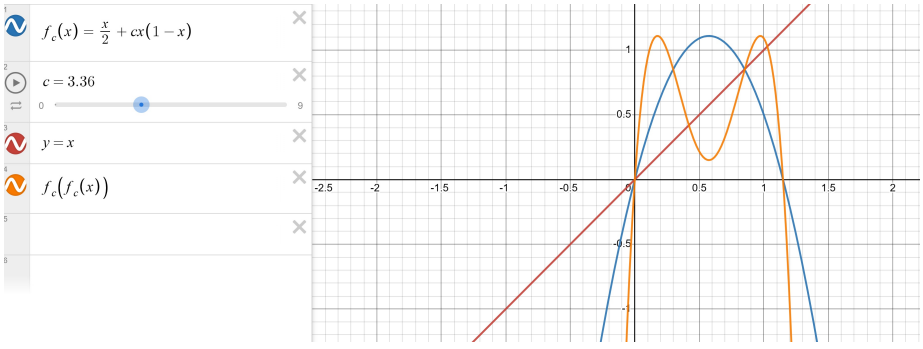

Plan

1. Discovering Dynamical Systems with technology tools
2. Project 1
3. Project development and assessments
4. TAs' role and benefits

Course Structure

- Discrete Dynamical Systems (DDS)

  *DDS Project
- Differential Equations (DE)

  *DE Project
- Integration ...

  *Regular "boring" Exam

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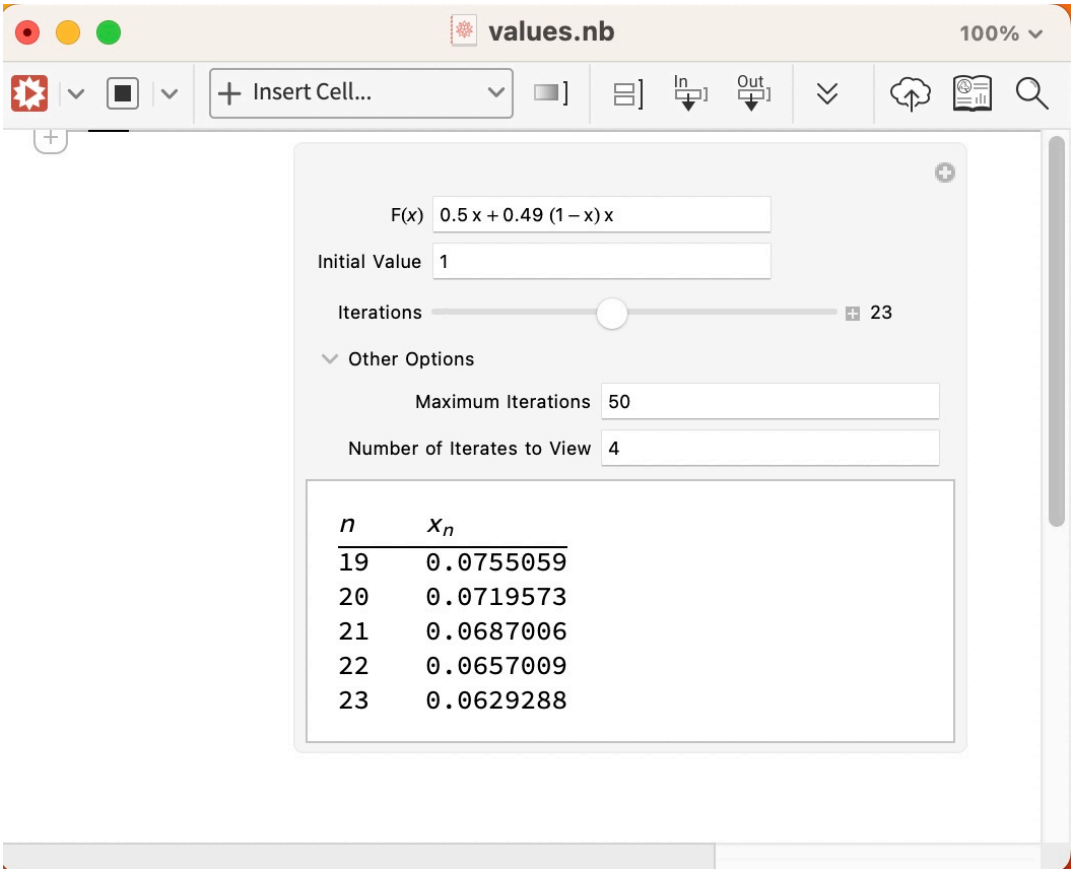
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Insert Cell...
In
Out
F(x) 0.5 x + 0.49 (1 – x) x
Initial Value 1
Iterations 23
Other Options
Maximum Iterations 50
Number of Iterates to View 4
n x_n
19 0.0755059
20 0.0719573
21 0.0687006
22 0.0657009
23 0.0629288

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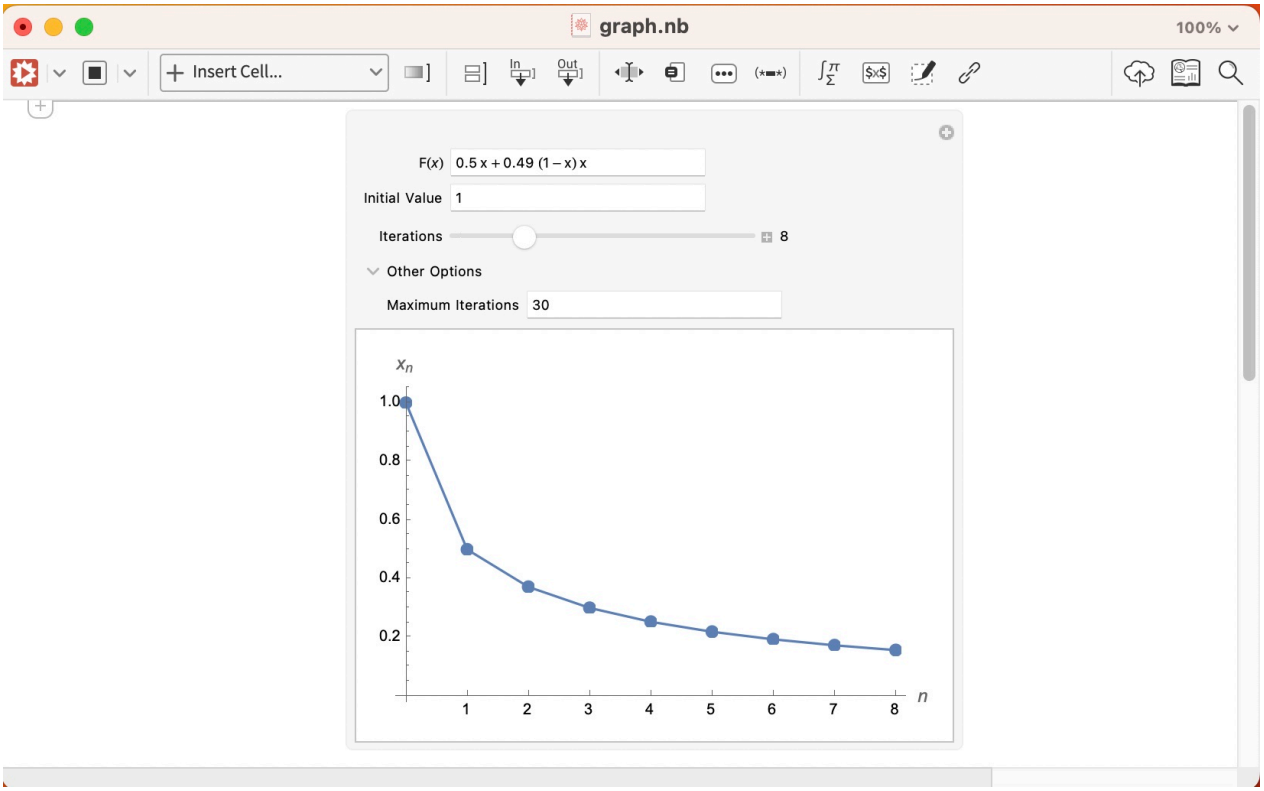
graph.nb
100%
Insert Cell...
F(x) 0.5 x + 0.49 (1 – x) x
Initial Value 1
Iterations 8
Other Options
Maximum Iterations 30
x_n
1.0
0.8
0.6
0.4
0.2
1
2
3
4
5
6
7
8
n

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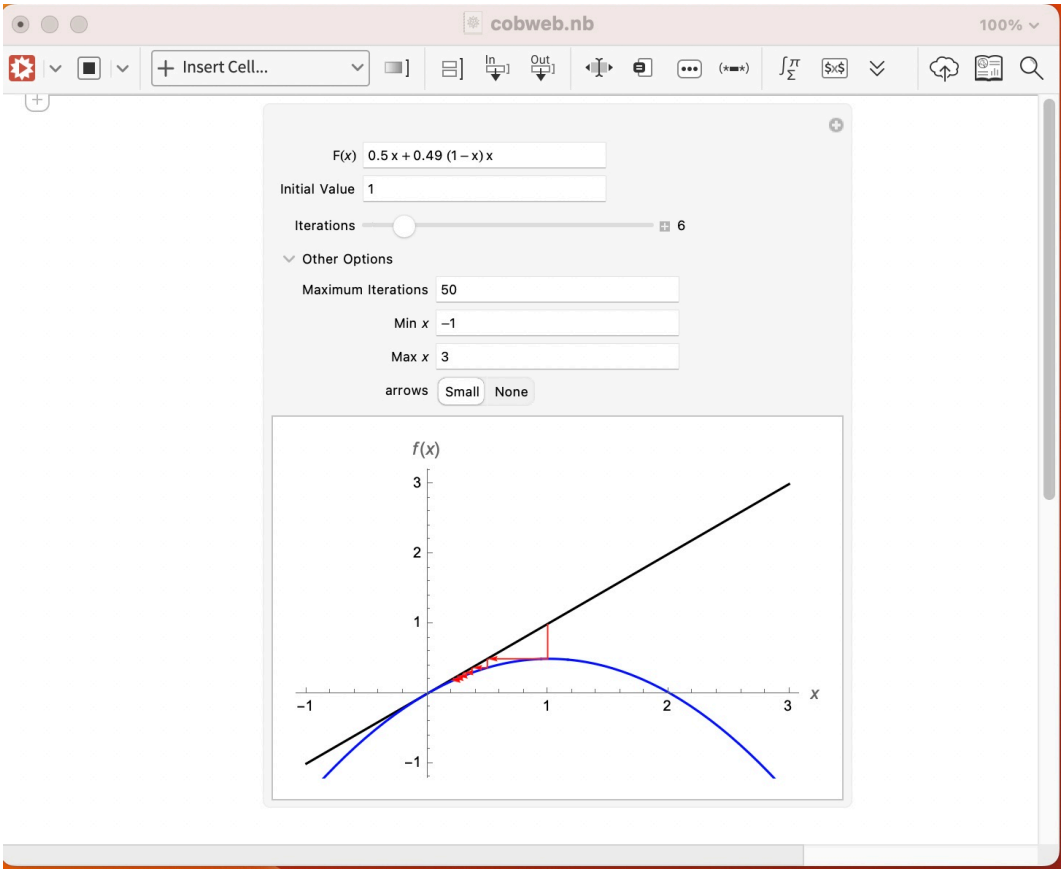
cobweb.nb
100%
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F(x) 0.5 x + 0.49 (1 – x) x
Initial Value 1
Iterations 6
Other Options
Maximum Iterations 50
Min x –1
Max x 3
arrows Small None
f(x)
3
2
1
–1
–1
1
2
3
x

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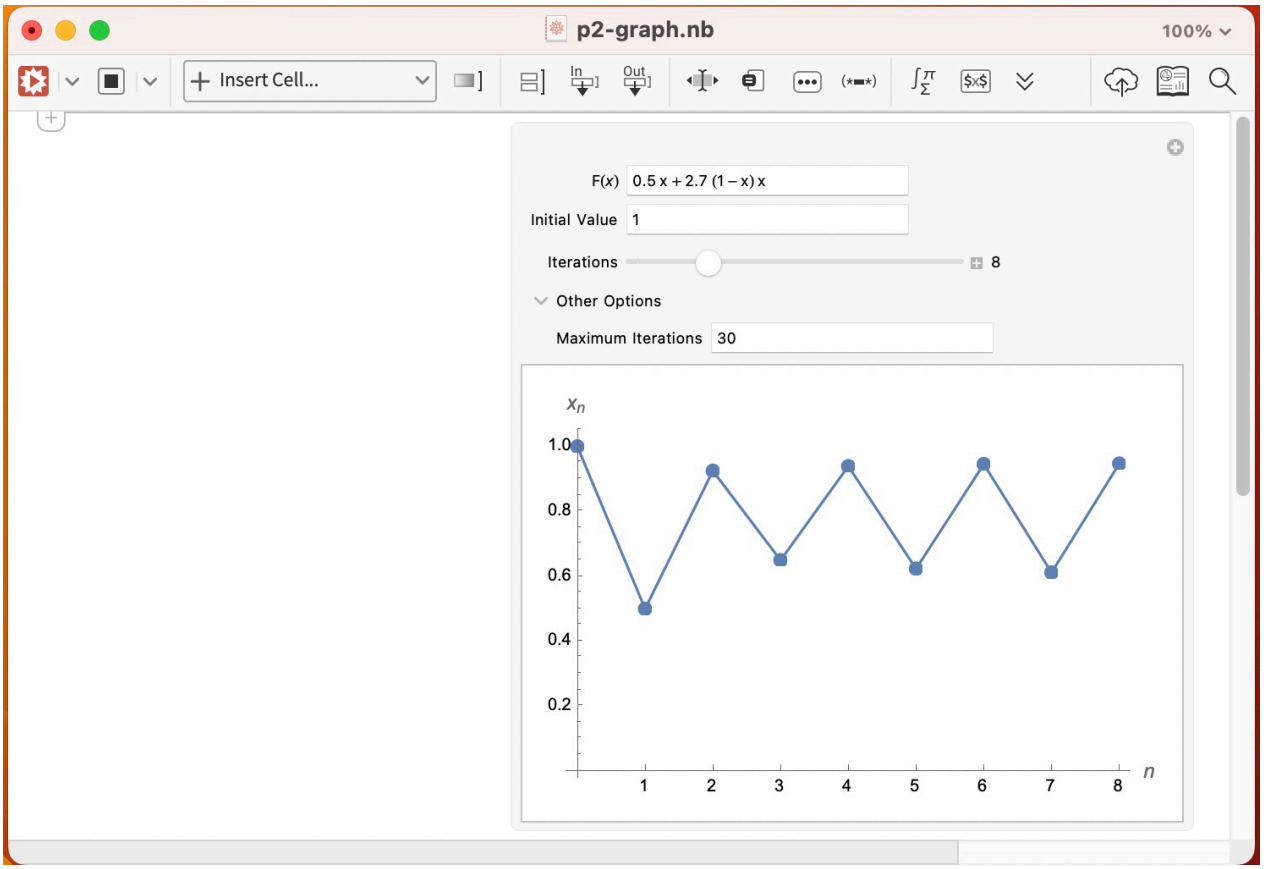
p2-graph.nb
100%
Insert Cell...
F(x) 0.5 x + 2.7 (1 – x) x
Initial Value 1
Iterations 8
Other Options
Maximum Iterations 30
x_n
1.0
0.8
0.6
0.4
0.2
1
2
3
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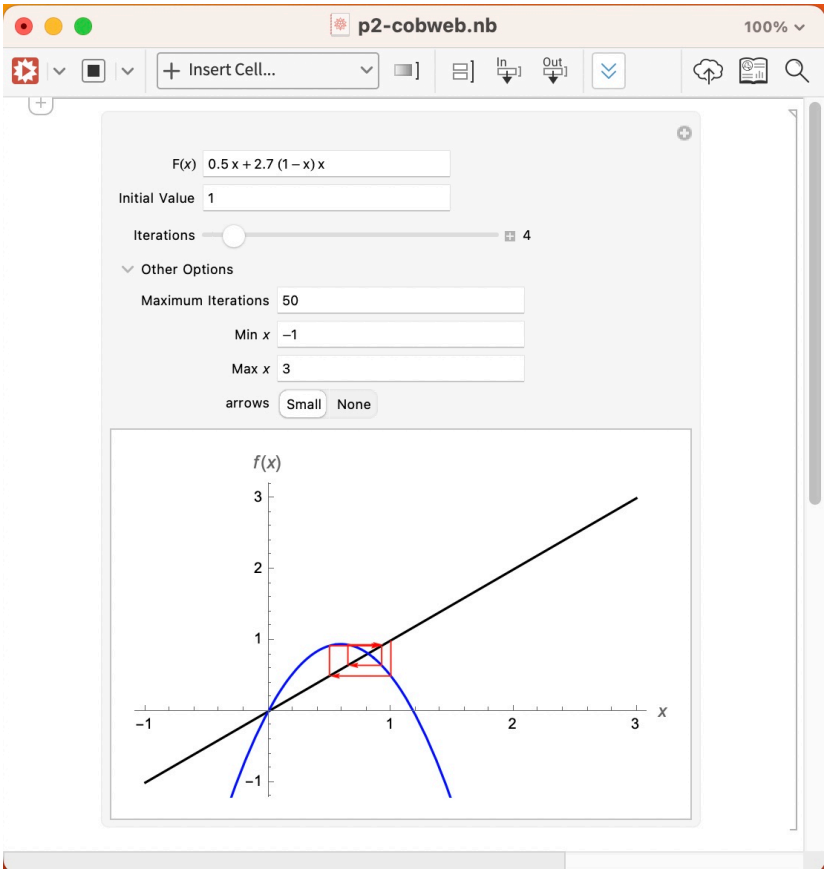
p2-cobweb.nb
100%
Insert Cell...
In
Out
F(x) 0.5 x + 2.7 (1 − x) x
Initial Value 1
Iterations 4
Other Options
Maximum Iterations 50
Min x −1
Max x 3
arrows Small None
f(x)
3
2
1
−1
−1
1
2
3
x

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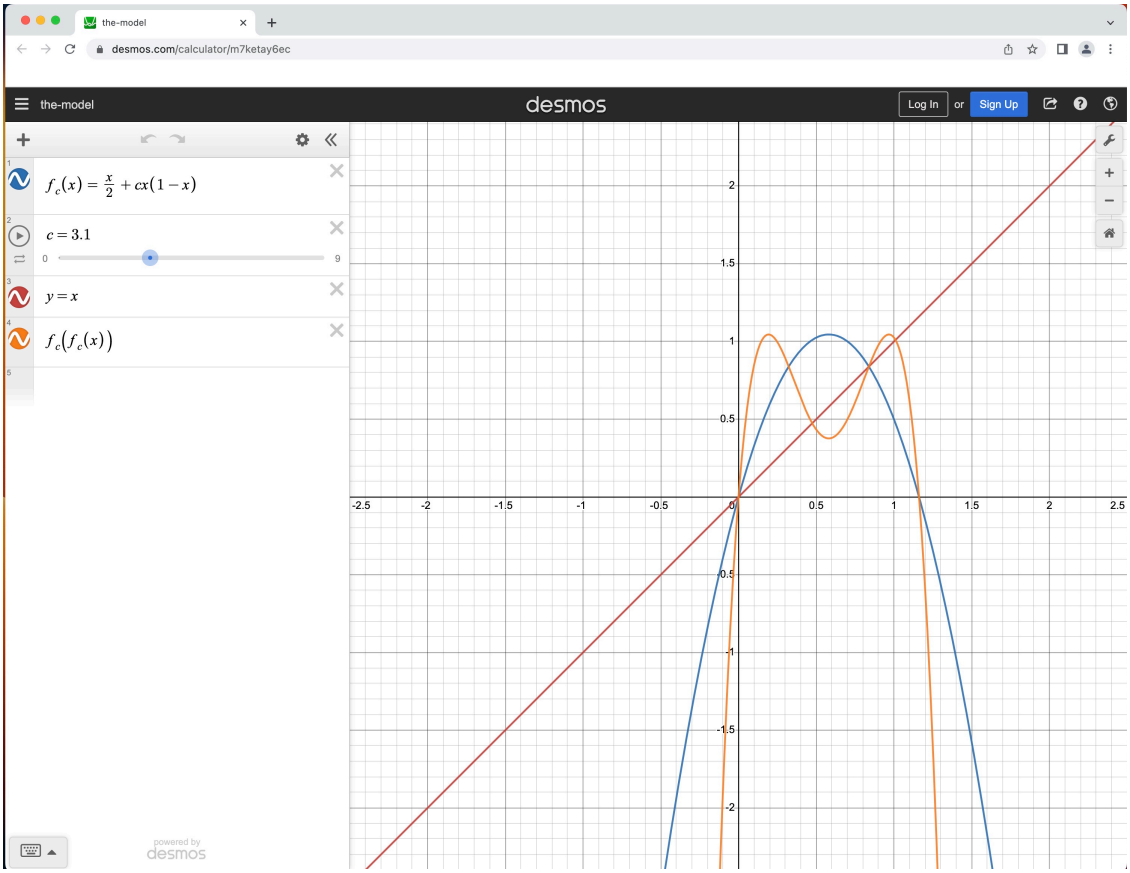
the-model
desmos.com/calculator/m7ketay6ec
the-model
desmos
Log In
or
Sign Up
$f_c(x) = \frac{x}{2} + cx(1-x)$
$c = 3.1$
0
9
$y = x$
$f_c(f_c(x))$
powered by
desmos

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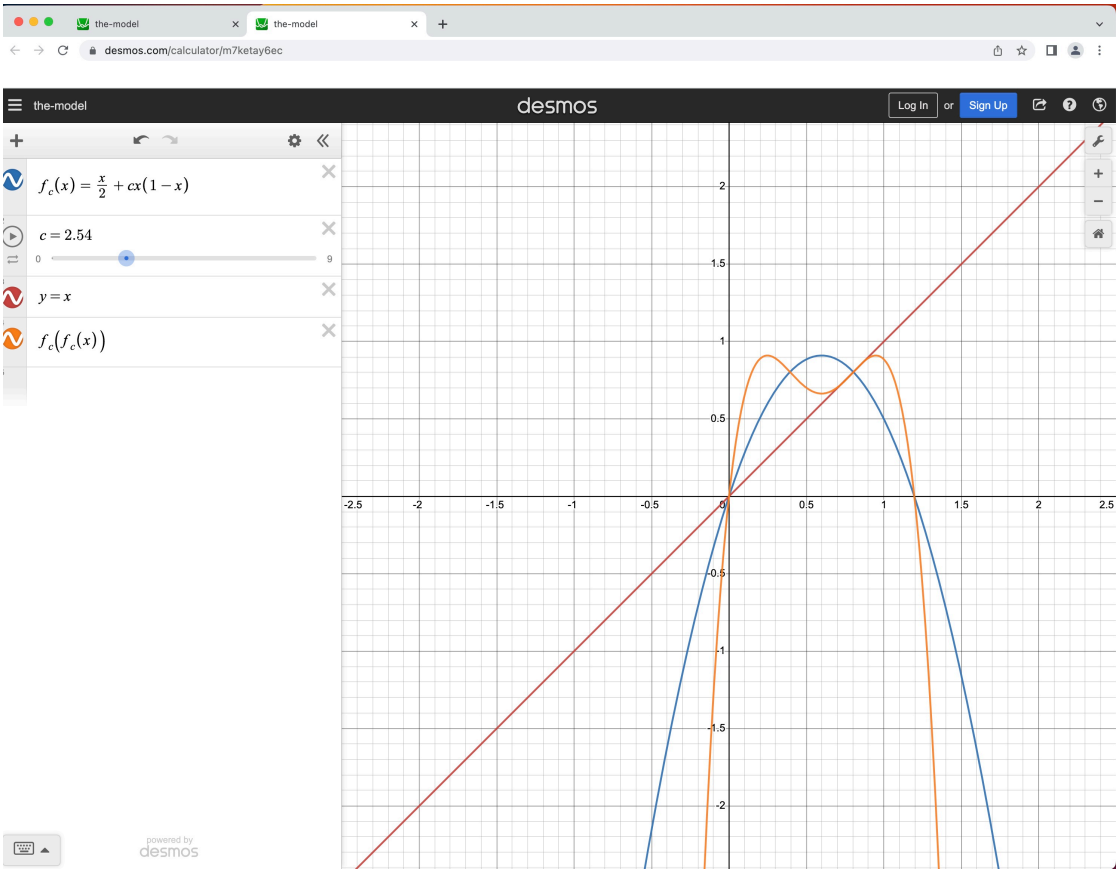
the-model
the-model
desmos.com/calculator/m7ketay6ec
the-model
desmos
Log In
or
Sign Up
$f_c(x) = \frac{x}{2} + cx(1-x)$
$c = 2.54$
0
9
$y = x$
$f_c(f_c(x))$
powered by
desmos

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x: −2 y y: x

x_0: 1 y_0: 1

Iterations: 11

Other Options

Maximum Iterations: 50

Number of Iterates to View: 8

n	x_n	y_n
3	4	−2
4	4	4
5	−8	4
6	−8	−8
7	16	−8
8	16	16
9	−32	16
10	−32	−32
11	64	−32

https://www.wolframcloud.com/obj/80baa9e2-a838-40c

WOLFRAM NOTEBOOK

2D Phase Plane

Functions:

x=f(x,y)= x − y

y=g(x,y)= x + y

Seed:

x_0: 1 y_0: 0.5

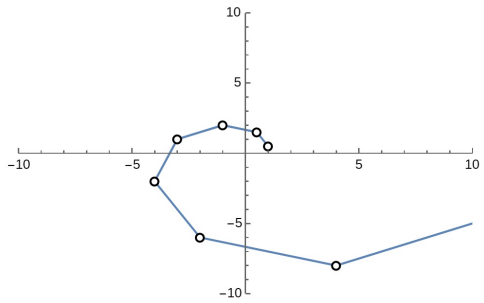
Iterations: 8

Plot Options:

Min x: −10 Max x: 10

Min y: −10 Max y: 10

Other Options



Go Back

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Shortened Letter from *"Dynamic Sales"* platform's owner:

Dear Math. students, I am Nicole Blum, the owner of the Internet platform *Dynamic Sales* that assists registered users of my platform (this number remains approximately constant over the last 5 years) in selling and buying new and used goods. Users can sell goods, if they pay \$50 fee for a month when they plan to sell. As a platform owner, I would like to increase my profit, and I invited Summer intern student to help me. Her notes say:

"Approximately $1/2$ *of sellers successfully sell their items within 1 month, and they do not renew their selling privileges for the next month. But many buyers that had interaction with sellers through the platform are so impressed (according to their reviews) with the functionality of the platform that they decide to get sellers' privileges for the next month. Let* I_0 *represent the factor of such successful interactions between sellers and buyers. Also, sellers usually do not purchase goods during the time of their selling activity.*

Let y_n *represents the proportion of people who pay \$50 at the beginning of the month* n*. Ms. Blum's profit depends on the factor of successful interactions between sellers and buyers,* I_0*, that is currently close to* 1*. I.e., we have the DDS:*

$$y_{n+1} = y_n - 1/2 \cdot y_n + I_0 \cdot y_n \cdot (1 - y_n), \text{ where } I_0 = 1."$$

Shortened Letter from *"Dynamic Sales"* platform's owner:

$$y_{n+1} = y_n - 1/2 \cdot y_n + I_0 \cdot y_n \cdot (1 - y_n)$$

Unfortunately, at the end if the internship, the student returned home without having had time to completely explain the ideas behind her model. Thus, I need your help to better understand and analyze the DDS, and to hear actionable recommendations for profitable business.

Can you please explain what the different terms in the model represent and why? I understand that if I further advance functions of my platform, there will be more successful interactions. But this costs extra resources. So, my next request is to understand the long-term prognosis for my profit. My records show that at the beginning of the last month my platform had 20% of sellers and 80% of buyers. But my records are a bit inaccurate because some people joined/dropped the platform in the middle of the month, so would we get similar long-term behavior for other proportions of sellers & buyers? Also, I_0 is approximately 1. What if this number were 5.5 – what is the long-term effect on my business (assuming that I have 20%/80% sellers/buyers proportion)? I would like to thoroughly understand this, so can you please use multiple methods to explain your results?

And finally what I would really want to understand how the long-term prognosis depends on I_0. What value of I_0 should I strive for if on the long run, I want to triple the current proportion of sellers?

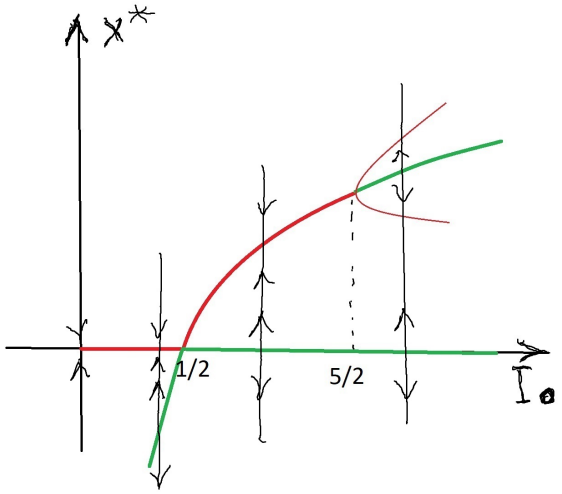
What we ask students to do

With all tools in their hands, we can ask students to:

- explain the terms of the equation
- explain if/when initial conditions affect long-term behavior
- play with several proposed examples of initial conditions and parameter I_0.
- perform analysis of this DDS using *several* methods, in particular:
  - find fixed points
  - analyze their stability
  - construct bifurcation diagram
  - provide actionable recommendations (profitable values of I_0 for a long term platform strategy AND the value of I_0 that should triple the current proportion of sellers on the long run).

What we expect to grade

At the end of the students' papers, we would like to see the following diagram and advice to have I_0 at some value between $1/2$ and $5/2$ for positive, stable (non-oscillatory) proportion of buyers and sellers. For tripling the current proportion of sellers on the long run, Ms. Blum should strive for $I_0 = 5/4$.



Project development and assessments

1a The project is developed in groups of 3 students

1b Each student performs peer review of 1 paper written by a different group (can learn some ideas and helps the other group).
- Students are given guidelines for reviewing and assessment:
- Assessment criteria (the same rubrics as for the final grading – presented at the beginning)
- Suggested comments

1c Each group receives 3 reviewers' feedback (each reviewer belongs to a different groups)

1d The project is submitted for grading.

2a All students write another, similar paper, reducing 2-dim DDS (with sellers and buyers variables) to the familiar 1-dim equation and check the results again.

2b Peer review process is repeated.

2c The final project is submitted for grading.

3a-c Construct a similar model for ODE... The same development and assessment.

TAs role

Students progress would not be good without TAs help.

- Flipped classroom; activities during each class meeting and group work with TAs' help
- TAs hours
- Side effect – TAs interest in Dynamics (SCUDEM competition)
  - "The project was well-executed and well-communicated. The presentation itself was extremely professional. I love the schematic diagram. The assumptions were very well explained and gave a clear in-depth understanding of the problem. I very much liked the simple and easy to follow explanation of the model and explanation without mathematical equations. Overall, using slides and script made the presentation feel professional." Meritorious Award

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References

Developed by Haverford College Department of Mathematics and Statistics, Calculus: Dynamics and Integration, Course Notes (copy of Sept. 2022)

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