Cooking a Turkey

A SIMIODE Modeling Scenario

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- A dilemma facing many cooks during the holiday season is determining the time needed to safely cook a turkey.
- Guidance that can be found in sources such as cookbooks, newspaper articles, and cooking websites is often based on the following USDA approximate cooking times for a whole turkey by weight provided in Table 1 (on the next slide).

Turkey Cooking Times

Table 1: USDA approximate cooking times for unstuffed turkeys.

4 to 8 pounds (breast)	1 1/2 to 3 1/4 hours
8 to 12 pounds	2 3/4 to 3 hours
12 to 14 pounds	3 to 3 1/4 hours
14 to 18 pounds	3 3/4 to 4 1/4 hours
18 to 20 pounds	4 1/4 to 4 1/2 hours
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- Oven temperature is 325 °F (163 °C) and a turkey is considered safely cooked when it reaches an internal temperature of at least 165 °F (73.9 °C).

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- Note that one kilogram $= 2.20462$ pounds.

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The time for heat to diffuse through the tissue so that the center of this spherical turkey reaches a certain temperature is proportional to the square of the radius of the turkey. Then we have to figure out the size of the idealized bird. By assuming that the spherical turkey has the same mass, M, as the real one, we can calculate the radius of the required spherical turkey.

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- The same type of formula has been suggested by Pief Panofsky, Stanford Linear Accelerator Center (SLAC) Director Emeritus.

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$$t = \frac{W^{2/3}}{1.5}$$

  where t is the cooking time in hours and W is the weight of the stuffed turkey, in pounds.

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$$t = \left[\frac{\left(\frac{3}{4\pi\rho}\right)^{2/3}}{\pi^2 \cdot \alpha} \ln \left\{ \frac{2(T_h - T_0)}{T_h - T_c} \right\} \right] \cdot W^{2/3}. \tag{1}$$

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- The parameters in their model are *turkey density* ρ, *oven temperature* T_h, *initial turkey temperature* T_0, desired *turkey center temperature* T_c, and *turkey thermal diffusivity* α, with $\alpha = \frac{k}{\rho c}$, where k and c are the *turkey thermal conductivity* and *turkey specific heat*, respectively.

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- Note that for the modified Panofsky formula, (1), weight is in kilograms and time is in seconds, while for the modified Panofsky coefficient, (2), weight is in pounds and time is in hours.

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- Note that for the modified Panofsky formula, (1), weight is in kilograms and time is in seconds, while for the modified Panofsky coefficient, (2), weight is in pounds and time is in hours.
- Unlike Panofsky's model, in which the coefficient's value of 1.5 depends on a fixed oven temperature and unknown initial turkey temperature, we see that the modified coefficient P is affected by several additional factors, including initial turkey temperature, T_0, and desired turkey center temperature, T_c.

Heating Curves for Turkeys of Various Weights

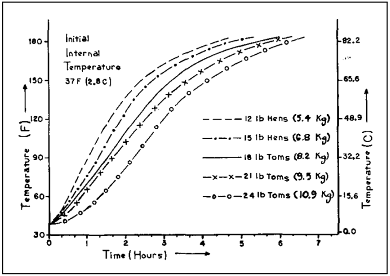

Figure 1: Rate of heating for unstuffed turkeys of different weights.

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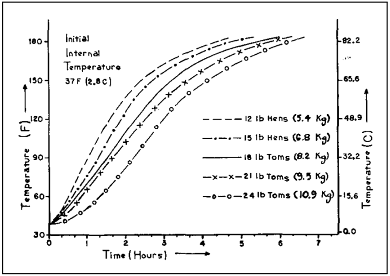
- From Figure 1, which reproduces Figure 2 in the paper "Heating Curves During Roasting of Turkeys" by Martinez, J. B., A. J. Maurer, and L. C. Arrington, one can measure the times to heat turkeys of weights 12 lb (5.4 kg), 15 lb (6.8 kg), 18 lb (8.2 kg), 21 lb (9.5 kg), and 24 lb (10.9 kg), to a safe temperature of 165 °F (73.9 °C (Celsius)).



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- This can be done by hand or using tools to more accurately measure data from these curves, such as the free Web Plot Digitizer, available online `https://automeris.io/WebPlotDigitizer`.

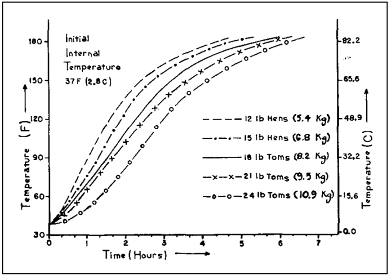

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Table 2: Model Parameters

Density ρ $(\mathrm{kg}/\mathrm{m}^3)$	1050
Thermal Conductivity k (W/(m K))	0.464
Specific Heat c (J/(kg K))	3530
Thermal Diffusivity $\alpha = k/(c\rho)$ $(\mathrm{m}^2/\mathrm{s})$	1.25185×10^{-7}
Oven Temperature T_h (°C)	163
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- Using these parameter values, we can consider what each of the models (Panofsky, Modified Panofsky, and Barham's cookbook algorithm (Cook's 15 and 20 min)) predict for turkey cooking times for each of these weights.
- We can also compare these model predictions to the actual heating times obtained from Figure 1 for each turkey, as well as USDA approximate cooking times found in Table 1.

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- Table 3 and Figure 2 (on the next two slides) compare these actual heating times to the estimated heating times for the three models (Panofsky, Modified Panofsky, and Cookbook (Cook's) algorithm estimates for 15 minutes per pound and 20 minutes per pound), respectively.

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- Since we aren't sure which turkey weights correspond to a "large" vs. a "small" turkey, we apply the Cookbook algorithm for both 15 minutes per pound and 20 minutes per pound to all given weights.

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Table 3: Estimated cooking times in hours for each model.

Weight W (kg)	Time to 73.9 °C (hr)	Panofsky (hr)	Modified Panofsky (hr)	Cook's 15 (hr)	Cook's 20 (hr)
5.4	3.13	3.47585	3.29914	3.22624	4.30165
6.8	3.52	4.05325	3.84719	3.99785	5.33047
8.2	3.97	4.59206	4.3586	4.76947	6.35929
9.5	4.46	5.0654	4.80788	5.48597	7.31463
10.9	5.03	5.55157	5.26933	6.25759	8.34345

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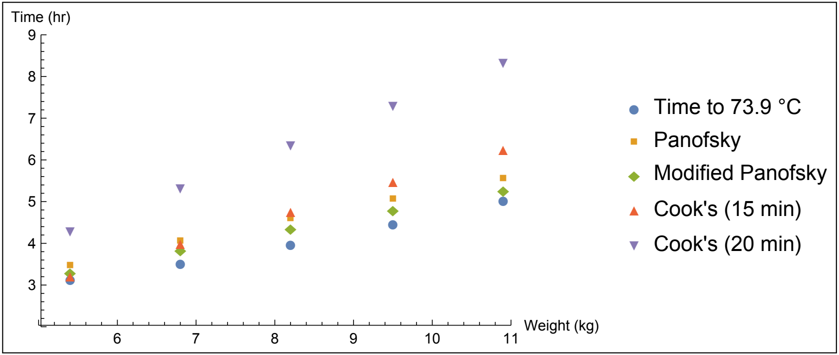

Figure 2: Comparison of turkey cooking models.

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$$\Delta u = u_{xx} + u_{yy} + u_{zz} \tag{4}$$

in rectangular coordinates.

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so we can write equation (3) as

$$u_{rr}(r,t) + \frac{2}{r} u_r(r,t) = \frac{1}{\alpha} u_t(r,t). \tag{7}$$

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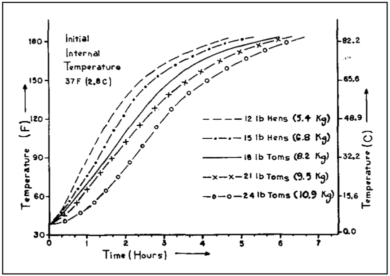
- Hint: Compare the size of coefficients of each term in the series – which is the dominant term?

Find and Test a Model to Predict Temperature of Turkeys of a Given Weight at *any* Time.



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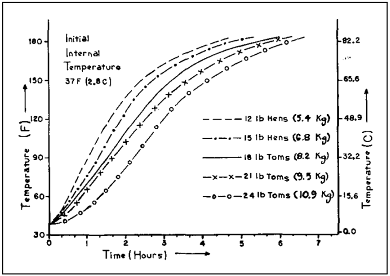

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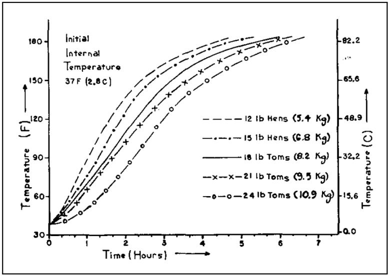

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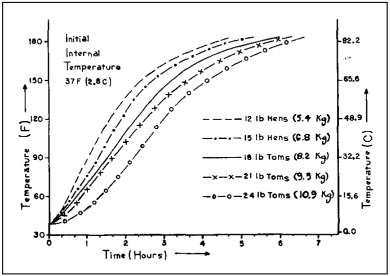

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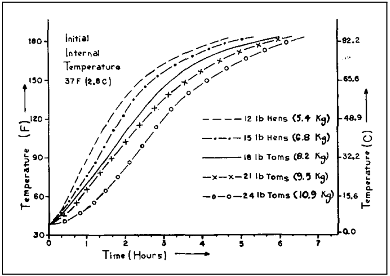
- Use this new model to predict the time needed to heat turkeys of weights 12 lb (5.4 kg), 15 lb (6.8 kg), 18 lb (8.2 kg), 21 lb (9.5 kg), and 24 lb (10.9 kg), to a safe temperature of 165 °F (73.9 °C).



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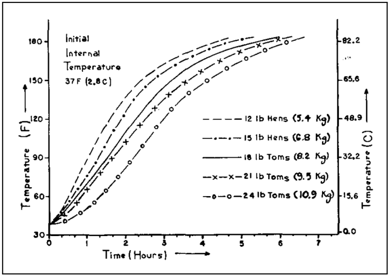
- Compare to the results for the models investigated above.



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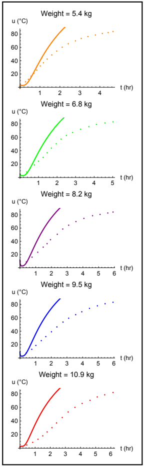

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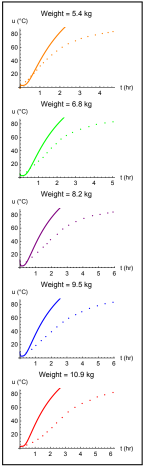
- For our model, we look at three cases: initial model, revised models, and further revised models.



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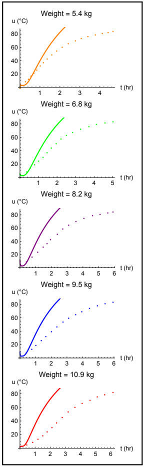

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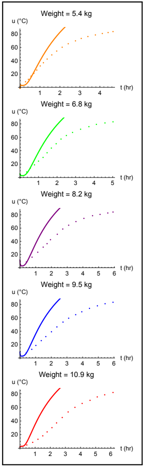

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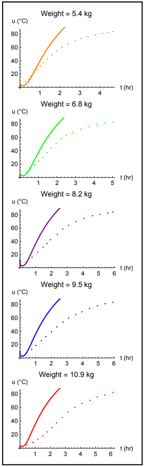

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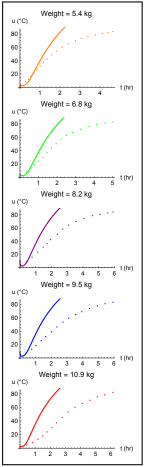

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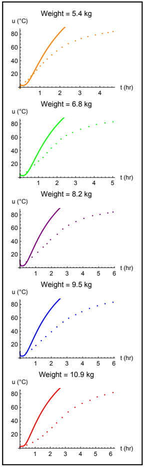
- Graphically, for each weight, there is a poor match between model (curves) and measured data (dots).



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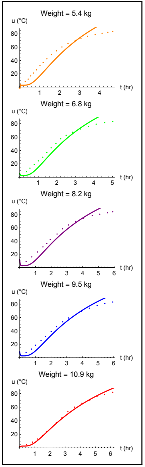

Figure 4: First model revision.

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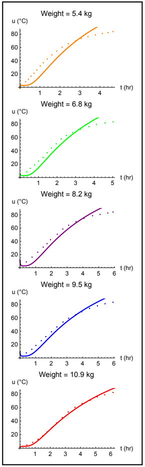
- For our first model revision, treating α as an unknown parameter, we notice that we can choose α to get a "good" graphical fit, with a different value of α for each weight.



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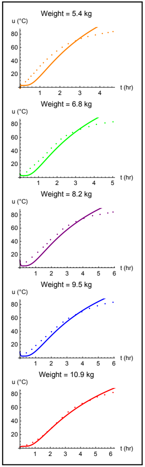

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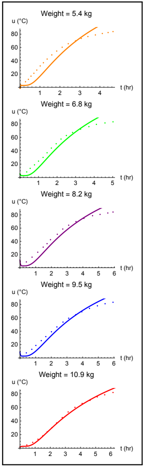
- Clearly we get a much better graphical fit, with square error ranging from approximately 2644 $(^{\circ}\mathrm{C})^2$ for 5.4 kg to 89 $(^{\circ}\mathrm{C})^2$ for 10.9 kg, with an RMSE of about 8.15 $^{\circ}$C.



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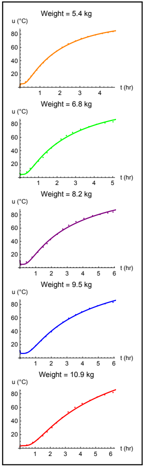

Figure 5: Second model revision.

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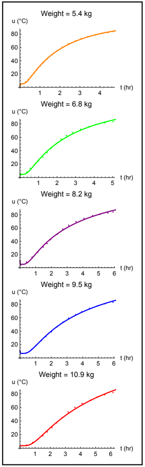
- For our second model revision, we treat T_h and T_0 as parameters that are unknown, along with parameter α and repeat the last case, choosing a set of parameters α, T_0, and T_h for each weight that minimize SSE for each weight.



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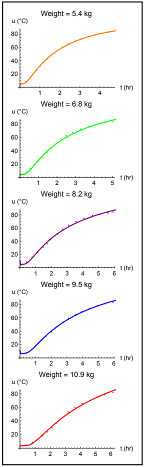

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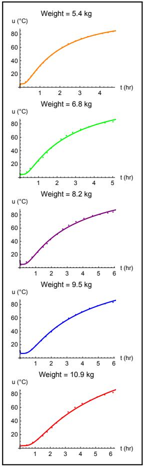
- Notice that SSE is significantly reduced with an RMSE of about 1.41 $^\circ$C.



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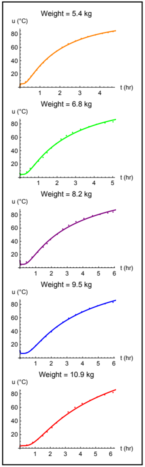
- We get even better results, which are shown in Figure 5.
- Notice that SSE is significantly reduced with an RMSE of about 1.41 °C.
- The only drawbacks to this model are that the parameters α, T_0, and T_h depend on the specific weights for which we have measured data and the model can only be used for these specific weights.



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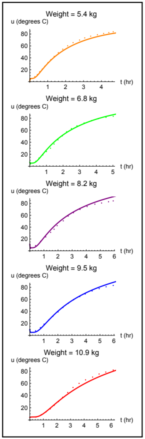

Figure 6: Third model revision.

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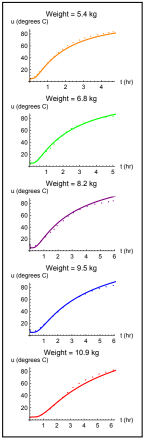
- Suppose we wish to apply the model to a turkey of *any weight*.



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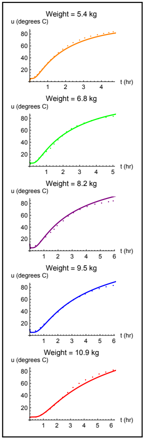

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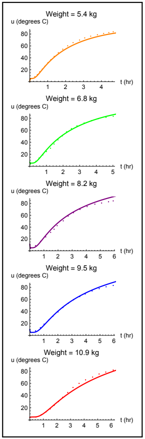
- Suppose we wish to apply the model to a turkey of *any weight*.
- Plotting each parameter vs. weight, we see that there is essentially a linear relationship between each parameter value and the weights.
- For our third model revision, fitting linear functions of weight W to the choices of α, T_0, and T_h from the last model (second revision), we get a model that is a function of weight only, that can be applied to *any turkey weight*!



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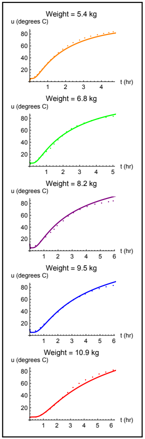
- Figure 6 shows that we get an excellent graphical fit with the trade-off from our last model of slightly higher SSE and an RMSE of about 2.75°C.



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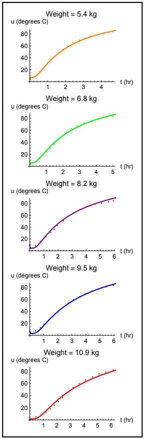
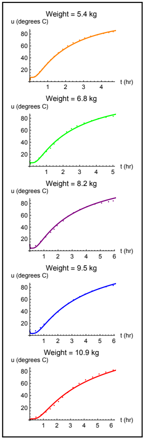


Figure 7: Fourth model revision.

Find and Test a Model to Predict Temperature of Turkeys of a Given Weight at *any* Time.

- Finally, as a fourth model revision, in order to reduce SSE, we construct a model that has six parameters, via new linear functions for α, T_0, and T_h, namely $\alpha(W) = m_\alpha W + b_\alpha$, $T_0(W) = m_1 W + b_1$, and $T_h(W) = m_2 W + b_2$.



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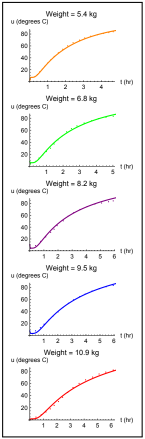

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- Figure 7 outlines these results.

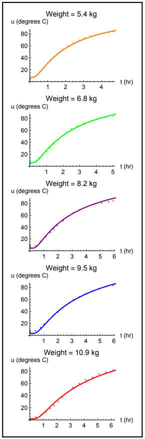

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Table 4: Estimated cooking times in hours for latest model.

Weight W (kg)	Actual Time to 73.9 °C (hr)	Latest Model (hr)
5.4	3.13	3.24977
6.8	3.52	3.58626
8.2	3.97	3.99686
9.5	4.46	4.50452
10.9	5.03	5.29348

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- We find that for our latest model, the RMSE is 0.135 hr, or about 8.10 min, with an average overestimate of about six minutes.

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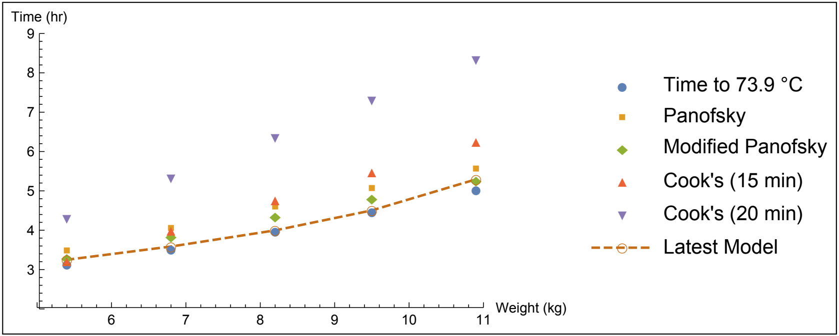

Figure 8: Comparison of turkey cooking models to latest model (dashed curve).

Conclusion

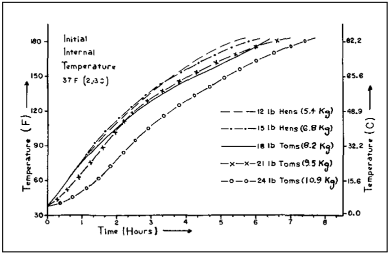

Figure 9: Rate of heating of stuffed turkeys of different weights [from Martinez, Maurer, and Arrington].

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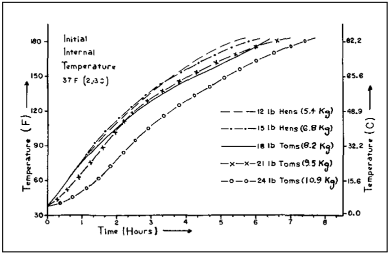

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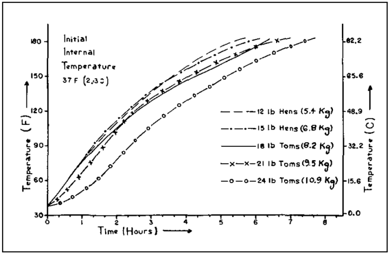

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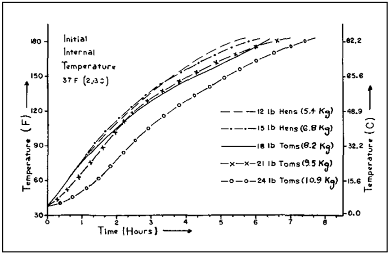

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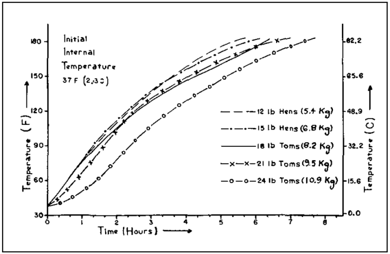

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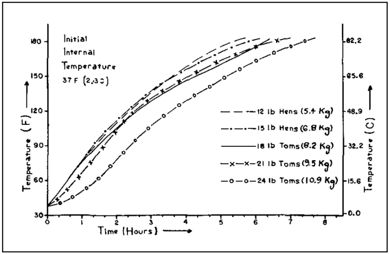

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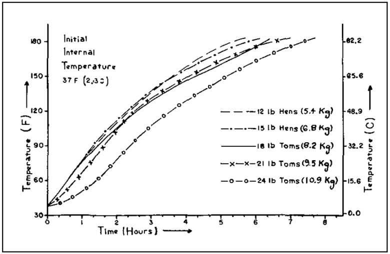
- This is left as an exercise!



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