Increasing STEM Students Retention Rates Using Mathematical Modeling in Curriculum Design

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2023 SIMIODE EXPO

My SoTL inquiry

Focused on exploring how connecting differential equations concepts to real-world scenarios through mathematical modeling impacts student confidence in their mathematical abilities for STEM majors.

Why?

- Differential equations is an important component of mathematics curricula for STEM majors.

- Students at my institution often find Differential Equations class as a "capstone" course of the first two years of college mathematics that they complete as part of their STEM degrees.

- It is important to learn to think creatively and logically and see the connection of real-world context in mathematics curriculum.

- This is particularly important for opening doors to STEM majors and retaining STEM students.

- Engaging students in contextualized context through modeling can provide rich and motivating learning experiences for students.

How?

- I gave a pretest and posttest survey and an end-of-semester interview to check the impact of incorporating mathematical modeling in the class activities.

- Both tests had questions about understanding first-order differential questions and how mathematical modeling incorporation had impacted their learning.

- Assessment results (n=20) indicated that there is an improvement in students' confidence and in their mathematical abilities.

Modeling Activities – Draft Outline

Lesson	Activity Description	SIMIODE Resources
Week 1: Intro to Modeling - collecting data, modeling, **Pre-Test 1** **First order ODE**	Simulation with M&M's for population model with immigration and parameter estimation	1-1-S-MandMDeathAndImmigration
Week 2:		
Week 3: Newton's Law of Cooling in a constant and changing regime **First order ODE - separation of variables**	Data offered for constant and changing temperature regimes and modeling hang time for second option	1-31-S-CoolIt
Week 4: **Post-Test 1**		
Week 5: Logistic equation modeling spread of information, technology, disease **Pre-Test 2** **First order ODE – nonlinear**	Basic building blocks and assumptions to get to powerful logistic model and many applications	1-18-S-LogisticPopModeling

Modeling Activities – Draft Outline		
Week 7: Modeling spring mass damper system with driver or forcing function **Second order ODE – nonhomogeneous**	Resonance and beats along with frequency response analysis are introduced Frequency Response Analysis	3-1-S-SpringMassDataAnalysis
Week 8: **Post-Test 2**		
Week 9: Modeling and using data to confirm LSD levels in human subjects **Pre-Test 3** **System - Linear nonhomogeneous**	Creating and analyzing two compartment models and parameter estimation	1-10-S-LSDAndProblemSolving
Week 11/12: Modeling spread of flu in English Boarding School **System – Nonlinear**	Using data from English Board School an SIR epidemic model is built and validated – and other models	6-1-S-Epidemic
Week 14: Modeling an optimal insect colony **Post-Test 3** **System – Nonlinear**	An introduction to optimal control of a nonlinear system in evolutionary study of insects.	6-5-S-InsectColonySurvivalOpt

Newton's Law of cooling/warming

STATEMENT

Suppose you're going away on vacation over Christmas, for a total of four days (96 hours). When you leave the house the inside temperature is a comfortable 21°C; you want it at that same temperature when you return. Since it's winter, this means you'll have to run the furnace. To minimize the costs/fuel consumption, should you

1. Just leave the furnace on (thermostat set to 21°C the whole time you're gone), or should you
2. Set the thermostat so the furnace stays off for most of the four days, but comes on just in time to heat the house back to 21°C when you return at $t = 96$ hours?

Scenario 2 assumes there's no problem with letting the house get cold, e.g., burst pipes, tropical plants, small pets, etc. It also assumes you have a suitably programmable thermostat.

In summary, the central question of interest is

Is it more economical to keep the house warm, or to reheat it?

Exercise 1: Solve ((1)) with initial condition $y(0) = y_0$; assume A is constant. What is the long-term behavior of the solution? How does it depend on k?

$$\frac{dy}{dt} = -k(y(t) - A) \quad (1)$$

$$\int \frac{dy}{y(t) - A} = -\int k \, dt$$

$$e^{\ln|y(t) - A|} = e^{-kt + c}$$

$$y(t) - A = C_1 e^{-kt}$$

$$y(t) = C_1 e^{-kt} + A$$

Let $y(0) = y_0$

$$y(0) = C_1 e^{k(0)} + A$$

$$y_0 = C_1 (1) + A$$

$$y_0 - A = C_1$$

$$\boxed{y(t) = (y_0 - A)e^{-kt} + A}$$

depending on k the long-term behavior would be getting hotter or colder. If we had a negative k it would be colder and positive k would be hotter.

Exercise 2: Suppose we measure time t in hours and $y(t)$ denotes the temperature of the house in degrees Celsius. Take $t = 0$ as the time you leave the house, $y(0) = 21$, $k = 0.01$, and suppose $A = 0$. What are the units on k? Use the solution from Exercise 1 to determine how cold the house will be when you return at time $t = 96$ hours. Pretty cold, huh?

$y(0) = 21 \quad K = 0.01 \quad A = 0$

Let $t = 96$

$y(96) = (21 - 0)e^{-0.01(96)} + 0$

$y(96) = 8.041$

k is a proportional constant after 96 hours the house would be 8.041° C

Exercise 3: Suppose $r(t) = r_0$ in (2), where r_0 is a constant chosen so as to maintain a constant 21°C in the house—what constant r_0 is required? Hint: choose r_0 so that $y(t) = 21$ is an equilibrium solution to (2).

$$\frac{dy}{dt} = -K(y(t) - A) + r(t) \quad (2)$$

$r(t) = r_0$, $y(t) = 21$

$A = 0°C$, $K = 0.01$

$$\frac{dy}{dt} = -0.01(y) + r_0$$

$$\frac{dy}{dt} + 0.01y = r_0$$

IF $e^{\int 0.01 dt} = e^{0.01t}$

$$\int \frac{d}{dx}\left(ye^{0.01t}\right) = \int r_0 e^{0.01t} dt$$

$$ye^{0.01t} = \frac{r_0 e^{0.01t}}{0.01} + C$$

$$y = \frac{r_0}{0.01} + \frac{C}{e^{0.01t}}$$

$$y = \frac{r_0}{0.01} + \frac{C}{e^{0.01t}}$$

$$21 = \frac{r_0}{0.01} + \frac{C}{e^{0.01t}}$$

$$21 - \frac{C}{e^{0.01t}} = \frac{r_0}{0.01}$$

$$r_0 = .21 - C(0.01)e^{0.01t}$$

$$\lim_{t \to \infty} r_0$$

$$= \lim_{t \to \infty} .21 - C(0.01)e^{0.01t}$$

$$= C(0.01)$$

$$r_0 = C(0.01)$$

Exercise 4: Draw a phase portrait for (2) assuming that $r(t) = r_0$ is constant. What does an equilibrium represent?

$$\frac{dy}{dt} = -k(y - A) + r(t) \quad (2)$$

$A = 0$

$k = 0.01$

$r = r_0 + y(2r_0)$

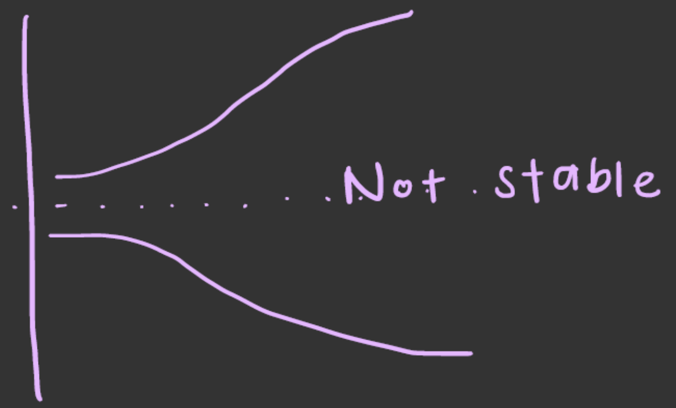
$$\frac{dy}{dt} = -0.01(y) + r_0$$

$$0 = -0.01(y) + r_0$$

$$y = \frac{r_0}{0.01}$$

$\frac{r_0}{0.01}$

$-y(-r_0)$



$-0.01(y) + 2r_0 = \frac{2r_0}{0.01}$

$-0.01(y) - r_0 = -\frac{r_0}{0.01}$

The phase portrait represents that the equilibrium solution is unstable. Which is a solution to the differential equation.

Exercise 5: Note that $r(t)$ is directly proportional to the power output of the furnace, and so $\int_0^{96} r(t)\,dt$ is directly proportional to the total energy expended by the furnace during the four days, which itself should be proportional to the cost of running the furnace for that time period. Compute this integral (hint: this is trivial). The result is a measure of the cost of keeping the house at 21°C.

$$\int_0^{96} r(t)\,dt = \int_0^{96} r(t)\,dt \Rightarrow \int_0^{96} 0.21\,dt \Rightarrow 0.21t\Big|_0^{96} = \boxed{20.16}$$

Exercise 6: Suppose the furnace is off from time $t = 0$ to some unspecified time $t = t_1 < 96$. Let's use $y_1(t)$ to denote the temperature of the house during this time period, so $y_1(t)$ obeys (1) with $y_1(0) = 21$. Use the results of Exercise 1 to write out $y_1(t)$ explicitly when $k = 0.01$ and $A = 0$.

$y(t) = (y_0 - A)e^{-kt} + A$

$A = 0$
$k = 0.01$
$y_1(0) = 21$

$0 \leq t \leq t_1$
$t_1 < 96$

$y_1(t) = y_0 e^{-0.01t}$

Let $y_1(0) = 21$

$21 = y_0 e^{-0.01(0)}$

$y_0 = 21$

$$\boxed{y_1(t) = 21e^{-0.01t}}$$

Exercise 7: At some time $t = t_1$ we need to turn the furnace on "full blast" to reheat the house. Assume that in this case the function $r(t) = 3$ degrees per hour (meaning the furnace is capable of heating the house at a max rate of 3 degrees per hour, at least when $y(t) = 0$). From time $t = t_1$ to time $t = 96$ the temperature of the house will be given by a function $y_2(t)$ that satisfies the DE (2) with the *final condition* $y_2(96) = 21$. Solve (2) with $y_2(96) = 21$ to find $y_2(t)$.

Could we simply have modeled $y_2(t) = 3t + b$, where b is chosen so that $y_2(96) = 21$? Explain.

$t_1 \le t \le 96$

$r(t) = 3$

$y_2(t) = 300 + Ce^{-0.01t}$

$y_2(96) = 21$

Let $y_2(96) = 21$

$21 = 300 + Ce^{-0.01(96)}$

$21 = 300 + C(0.38289)$

$-279 = 0.38289C$

$C = -728.669$

$y_2(t) = 300 - 728.669e^{-0.01t}$

if we used $y_2(t) = 3t + b$ our b would be -267. This simple equation can work for our needs because once we turn on the furnace it will be constantly getting 3°C hotter.

Exercise 8: Refer to Figure 1. The function $y_1(t)$ gives the temperature of the house for $0 \leq t \leq t_1$, while $y_2(t)$ gives the temperature for $t_1 \leq t \leq 96$. The value of t_1 when the furnace should be turned on is dictated the condition $y_1(t_1) = y_2(t_1)$ (think: why is this appropriate?) Use this observation to find t_1.

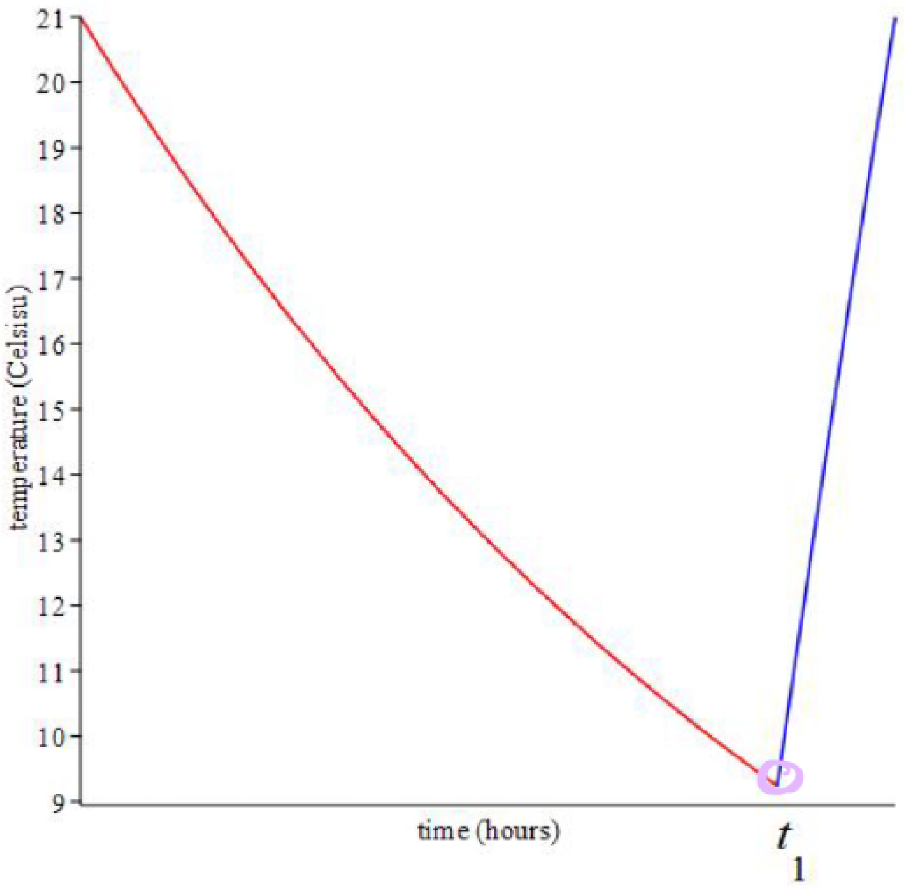


Figure 1. Function $y_1(t)$ (red), $y_2(t)$ (blue) for $0 \leq t \leq 96$.

$y_2(t_1) = 3t_1 - 267$

$y_1(t_1) = 21e^{-0.01t_1}$

$y_1(t_1) = y_2(t_1)$

$3t_1 - 267 = 21e^{-0.01t_1}$

$3(t_1 - 89)$

$e^{0.01t_1}(t_1 - 89) = 7$

$t_1 = 91.7953$

If the furnace does not shut off, will the temperature in the house continue to rise without bound after 96 hours?

with the equation we are using to model that temperature we don't have a cap on the temp. However, realistically the temperature would only go as not as the flames in the furnace or it would have switched off.

Exercise 9: As in Scenario 1, a measure of how much it costs to heat the house is given by $\int_0^{96} r(t)\,dt$. What is the integral in this case? (Note, $r(t) \equiv 0$ for $t < t_1$, so you really only need to worry about $t_1 \leq t \leq 96$).

$$\int_{t_1}^{96} r(t)\,dt \rightarrow \int_{91.7953}^{96} 3\,dt = 3t\Big|_{91.7953}^{96} = 288 - 275.3859$$

$$= \boxed{12.6141}$$

Exercise 10: Compare the total "energy" usage from Exercise 5 to that from Exercise 9. Which is more economical, reheating the house or just leaving it at 21°C?

energy usage exercise 5 (assuming $r(t) = 3$)

$= 20.16$

energy usage exercise 9 (assuming $r(t) = 3$)

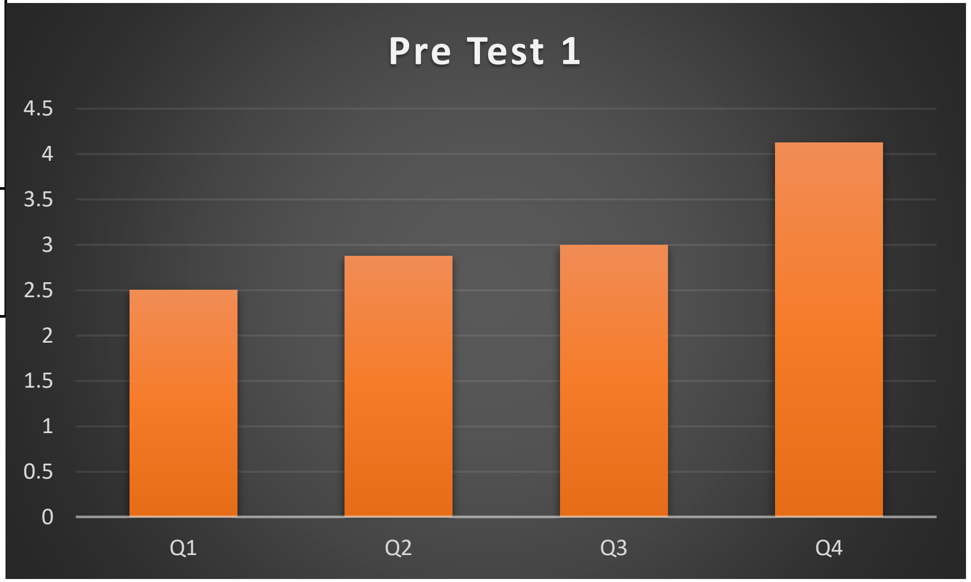
$= 12.6141$

reheating the house is significantly cheaper. If we were to just leave the house at 21°C it would cost \$20.16. And reheating would cost \$12.6141. So I would choose to reheat.

Pre-Test 1

Rate the following statements on a scale of 1-5, where 1 is strongly disagree and 5 is strongly agree:

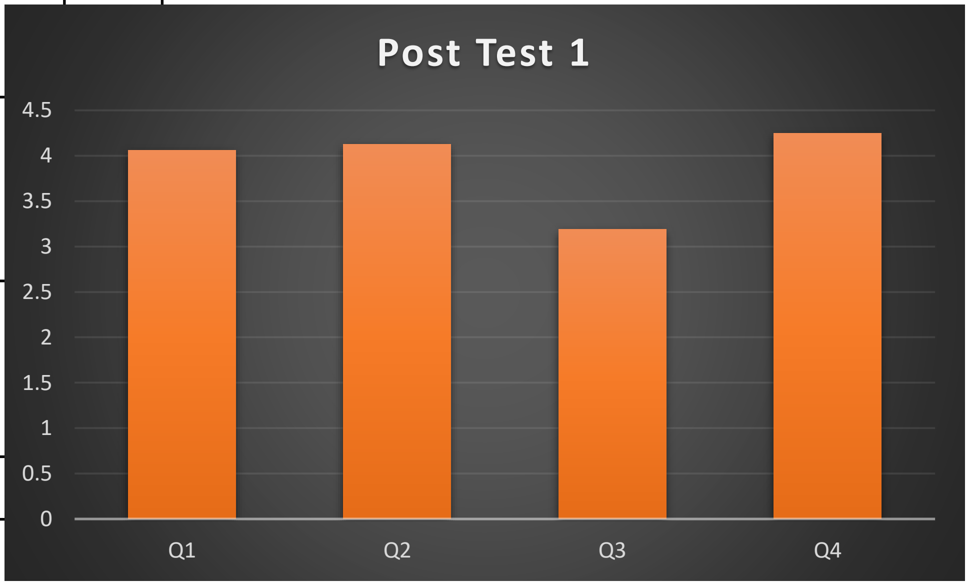
Statement	1	2	3	4	5
I am able to solve most problems involving first order linear equations.					
I am unsure which method to use when I attempt a problem involving first order linear equations.					
The math involved in solving problems on first order linear equations is very hard.					
I am unsure how the first order linear equations are utilized in real life scenarios					



Post-Test 1 - First order linear equations

Rate the following statements on a scale of 1-5, where 1 is strongly disagree and 5 is strongly agree:

Statements	1	2	3	4	5
Participating in modeling activities have increased my ability to solve most problems involving first order linear equations.					
Having participated in the class lectures and modeling activities, I am more confident in my ability which method to use when I attempt a problem involving first order linear equations.					
Having participated in the class lectures and modeling activities, The math involved in solving problems on first order linear equations does not seem as hard as before.					
Participating in modeling activities made me to understand how first order linear equations are utilized in real life situation					



Post-Test 1 - First order linear equations (Continued)

1. Write down one thing you learned from the modeling activities.

2. Write down one thing you are still unclear about first order linear equations.

3. Write down one thing you liked about the modeling activities.

4. Write down one thing you disliked about the modeling activities.

Post-Test 1 - First order linear equations (Continued)

1. Write down one thing you learned from the modeling activities.

I learned just how much these equations can apply to real world.

I am beginning to learn how to set up the equations.

2. Write down one thing you are still unclear about first order linear equations.

Sometimes I still don't know how to tell if it's first order.

I am still uncertain how to form some of the equations when given a real world situation.

3. Write down one thing you liked about the modeling activities.

I liked the thought process when coming up with the equations and working in groups.

Doing them over and over is helping me learn to understand the ideas behind modeling.

4. Write down one thing you disliked about the modeling activities.

The ones we do at home take a lot more time than I expected and I fell behind.

When I got stuck in a problem I didn't know how to go about figuring out the problem.

Post-Test 1 - First order linear equations (Continued)

1. A cake removed from a hot oven cools over time until it reaches room temperature. In an attempt to describe this cooling mathematically, a student proposes the following equation where T is the temperature of the cake, t is time, and k is a positive constant.

$$\frac{dT}{dt} = -k$$

a) Do you think this student's equation models the cooling of the cake well? Please explain why/why not.

b) Are there any improvements you would make or situations where this model will not work? If so, explain why you would make each change or why no changes are necessary.

Post-Test 1 - First order linear equations (Continued)

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a) Do you think this student's equation models the cooling of the cake well? Please explain why/why not.

No. The model does not even involve T, which is the temperature of the Cake. Therefore the model does not actually describe the cooling of the cake.

No it doesn't, the model assumes that the cooling is constant and not effected by any other factors,

b) Are there any improvements you would make or situations where this model will not work? If so, explain why you would make each change or why no changes are necessary.

I would improve the equation by using Newton's law of cooling $\frac{dT}{dt} = k(T - T_m)$
I would also record the initial temperature of the cake $T(0)$ as well as the temperature of the room (or medium) T_m
Without these, we could not calculate the constant k, or solve the equation.

To improve on this model the student should realize that the temperature of the room and the current temperature of the cake influence how fast the cake is cooling at any given time. By making this change the student would see that the cake isn't cooling at a constant rate.

Further Exploration

To further explore the ways student experience with real-world scenarios through mathematical modeling impacted their perception of their mathematical abilities, eight students were interviewed after the course concluded.

Student Interview Questions

Q1: What are differential equations, and why are they useful?

Q2: Tell me everything you know about mathematical modeling?

Q3: How confident you are in solving differential equations? Does the modeling activities increase your confidence in solving differential equations? If so, how?

Q4: Have the modeling activities helped you understand how differential equations are applicable to STEM disciplines and real-world scenarios? If so, how?

Q5: Do you feel that participating in modeling activities better prepared you for differential equations and STEM disciplines you will encounter in future?

Further Exploration (continued)

- Seven of the survey participants indicated the mathematical modeling activities pushed their thinking beyond the textbook problems and connected real-world scenarios to the class material.
- For instance, students were asked to identify at least one real-world scenario and how it related to differential equations.
- Six participants immediately identified the spread of COVID-19 as a real-world scenario and how to model it through differential equations.

Overview of the impact

- Importantly, these students were able to explain the parameters in the differential equation and its physical significance.
- A majority of comments praised all the mathematical modeling activities.
- Many stated the hands-on activities helped them see differential equations in a real-world context and better gauge their learning process.
- Furthermore, they all indicated that mathematical modeling activities helped them see the value and positive perspectives towards STEM disciplines.

Important Takeaways and Future Work

- It has allowed me to think more deeply about the student learning process and identify issues within the delivery and course curriculum structure.

- Through this experience, I have found that when mathematical modeling of real-world scenarios is emphasized in the classroom, students can make important connections among the mathematics and other STEM fields, and this can positively impact their interest and confidence in doing mathematics.

- This study is vital because Differential Equations is a critical point where STEM students choose their interests and proceed with their mathematical adventures.

- In a future study, I would like to extend the study to more STEM majors. I also would like to explore the impacts of authentic mathematical modeling activities across the calculus curriculum.

Acknowledgement

AMATYC Project SLOPE (Scholarly Leaders Originating as Practicing Educators in Two-Year College Mathematics) is a collaboration between AMATYC, Anoka-Ramsey Community College, Florida International University, and Oregon State University. Support for Project SLOPE is provided by the National Science Foundation, NSF-IUSE:EHR #1726891.

Project SLOPE

Scholarly Leaders Originating as Practicing Educators in Two-Year College Mathematics

Thanks to the Project SLOPE Team!!!!!!!!