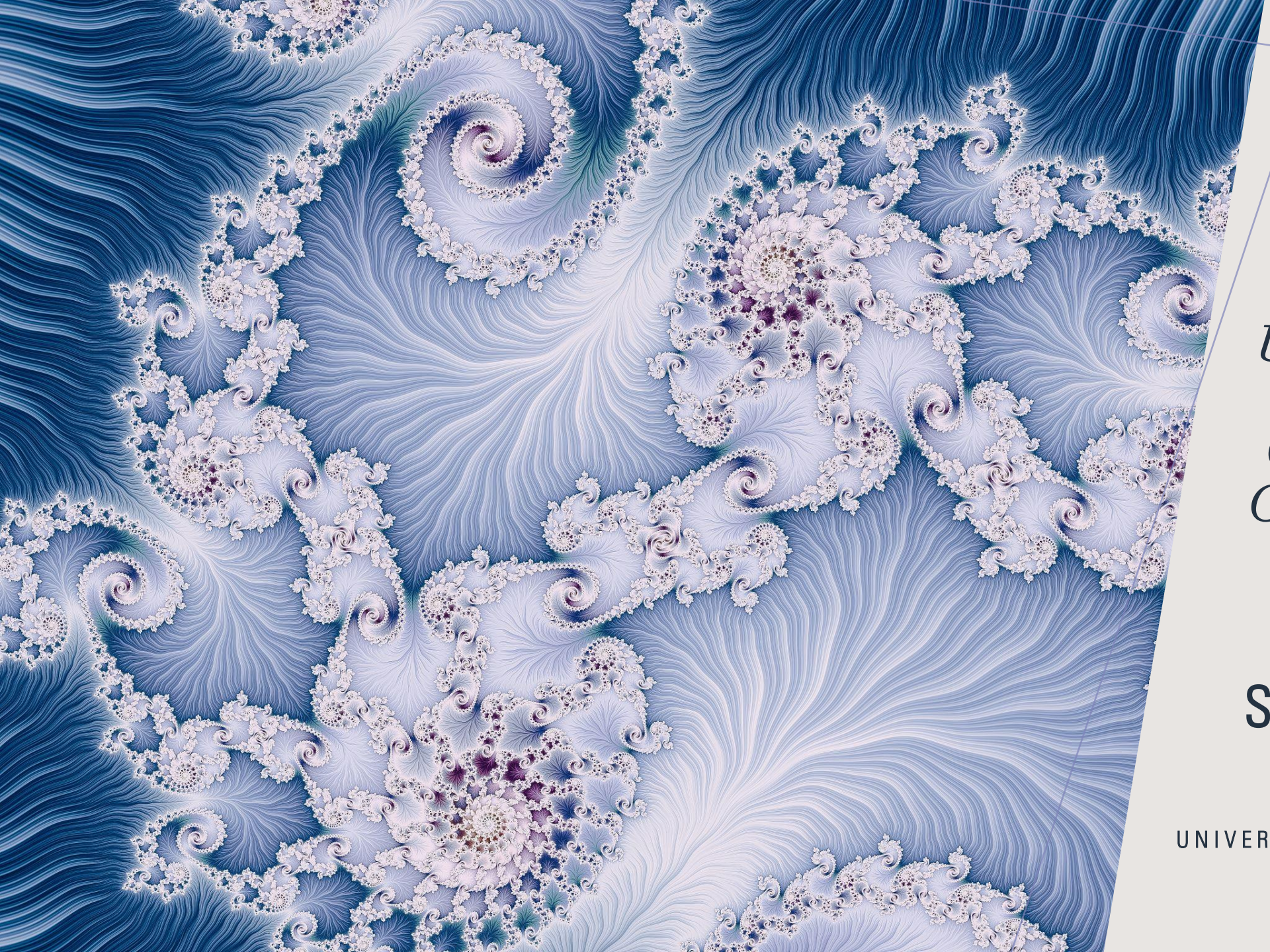

USING FRACTAL GEOMETRY TO QUANTIFY THE COMPLEXITY OF NATURE

SIMIODE 2023

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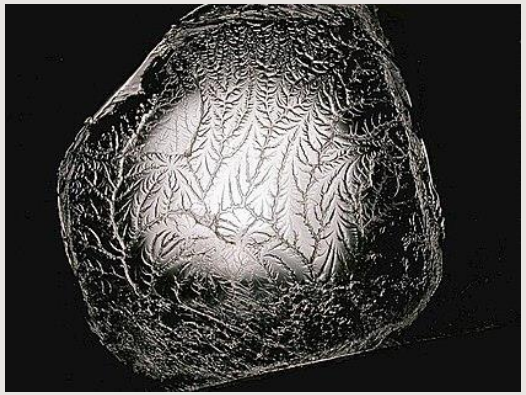
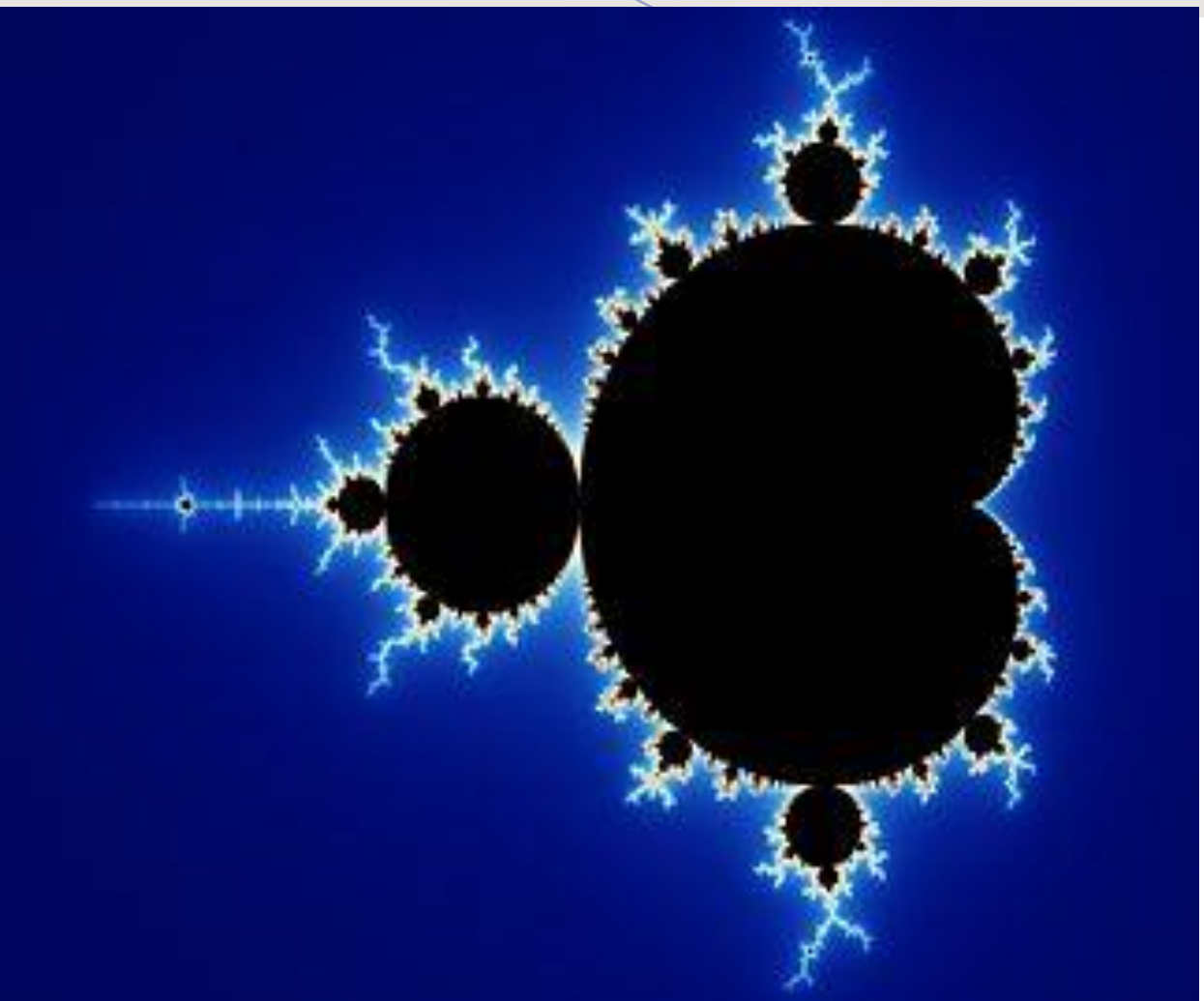


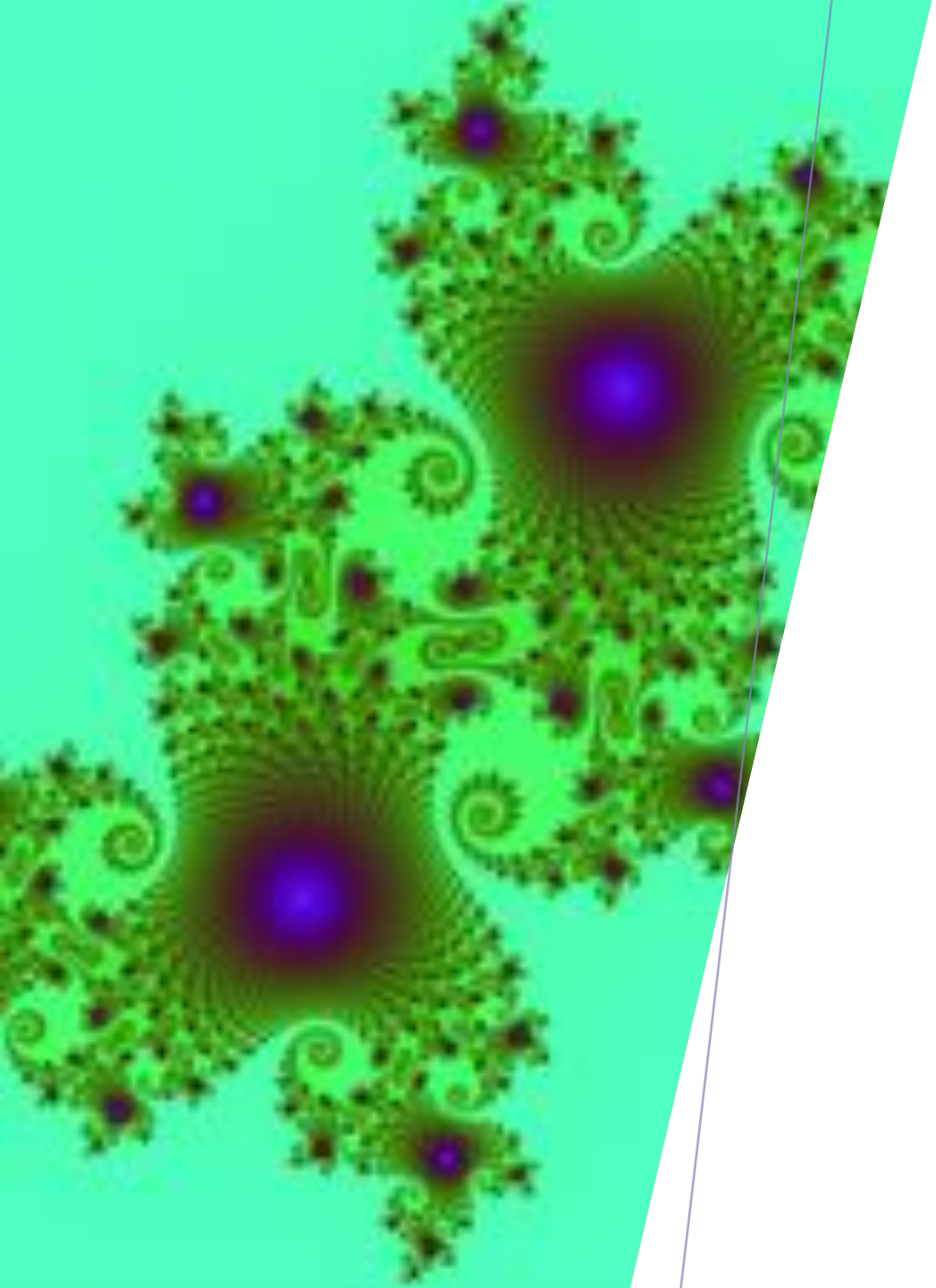
FRACTALS

- A fractal is a never-ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop.
- The term “fractal” was coined by Mandelbrot in the 1970s.
- Fractal geometry can be used to describe irregular and fragmented patterns in nature (Mandelbrot 1983; Aasum et al. 1991).
- In fact, fractal geometry presents a mathematical model for most of the complex phenomena such as coastline, mountain, cloud, and fracture patterns (Yin 1996; Li et al. 2009; Kim and Schechter 2009).
- One may think that fractals are only the result of mathematical calculations but the most interesting point is that many things in nature have fractals’ properties in them.

The first fractals were discovered by a french Mathematician named Gaston Julia who discovered them decades before the advent of computer graphics.



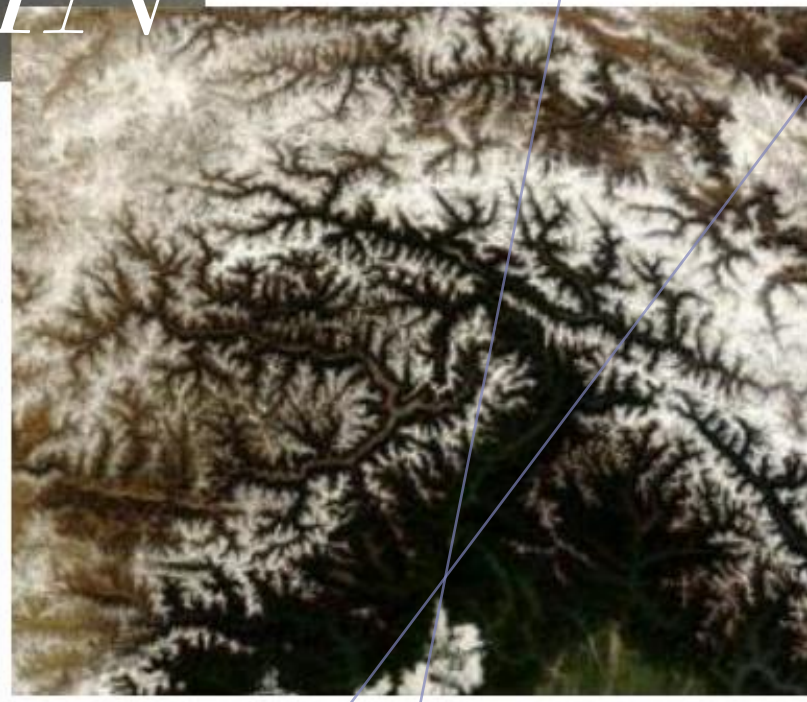
- Julia's work was rediscovered by Benoit Mandelbrot.
- The most famous of all fractals is the Mandelbrot set.

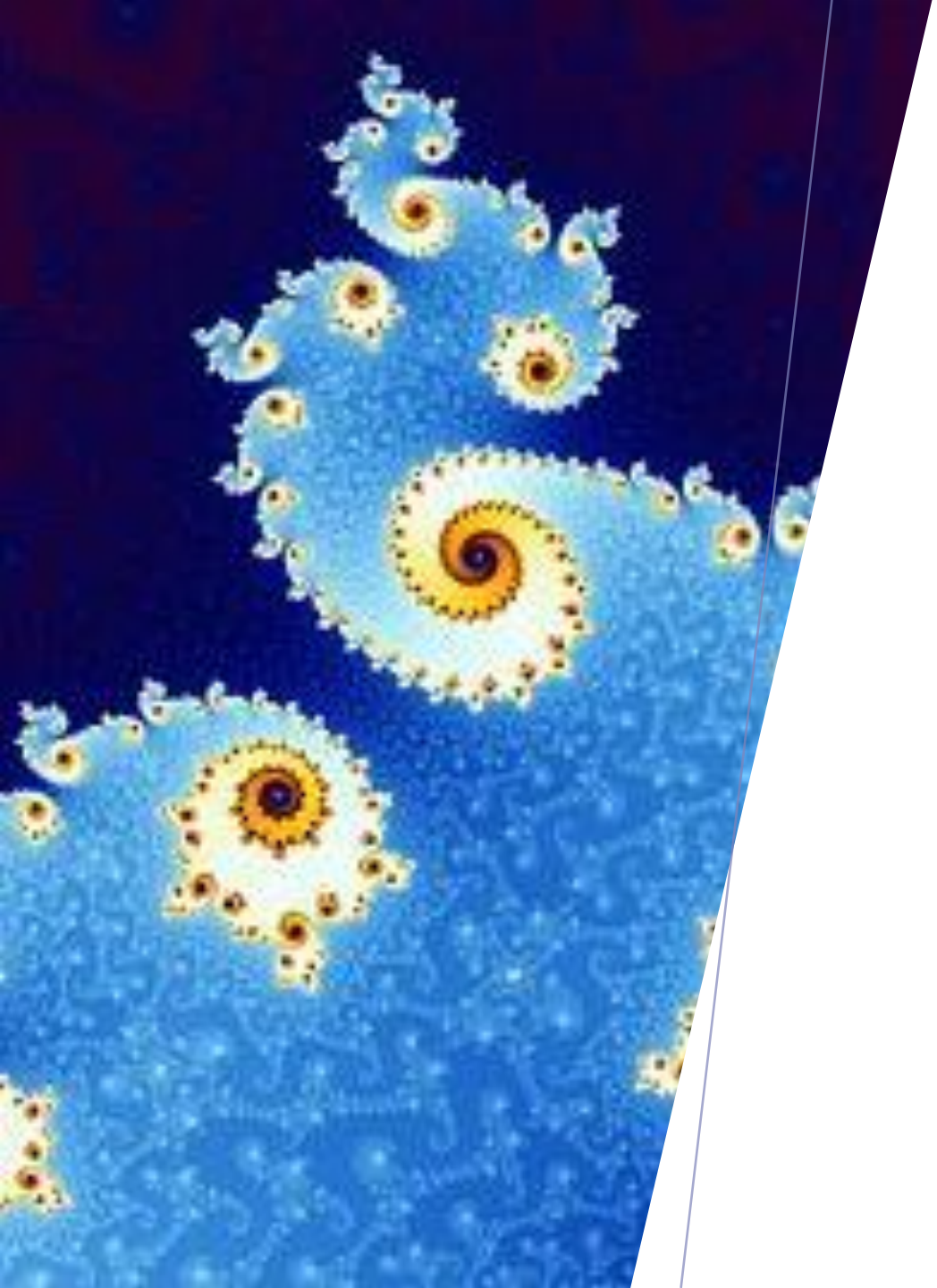


FRACTALS IN DYNAMICAL SYSTEMS

- A fractal is a mathematical object that is both self-similar and chaotic.
  - self-similar: As you magnify, you see the object over and over again in its parts.
  - chaotic: Fractals are infinitely complex.
- Amazingly, these beautiful objects of breath-taking complexity are generated by relatively simple mathematical processes.

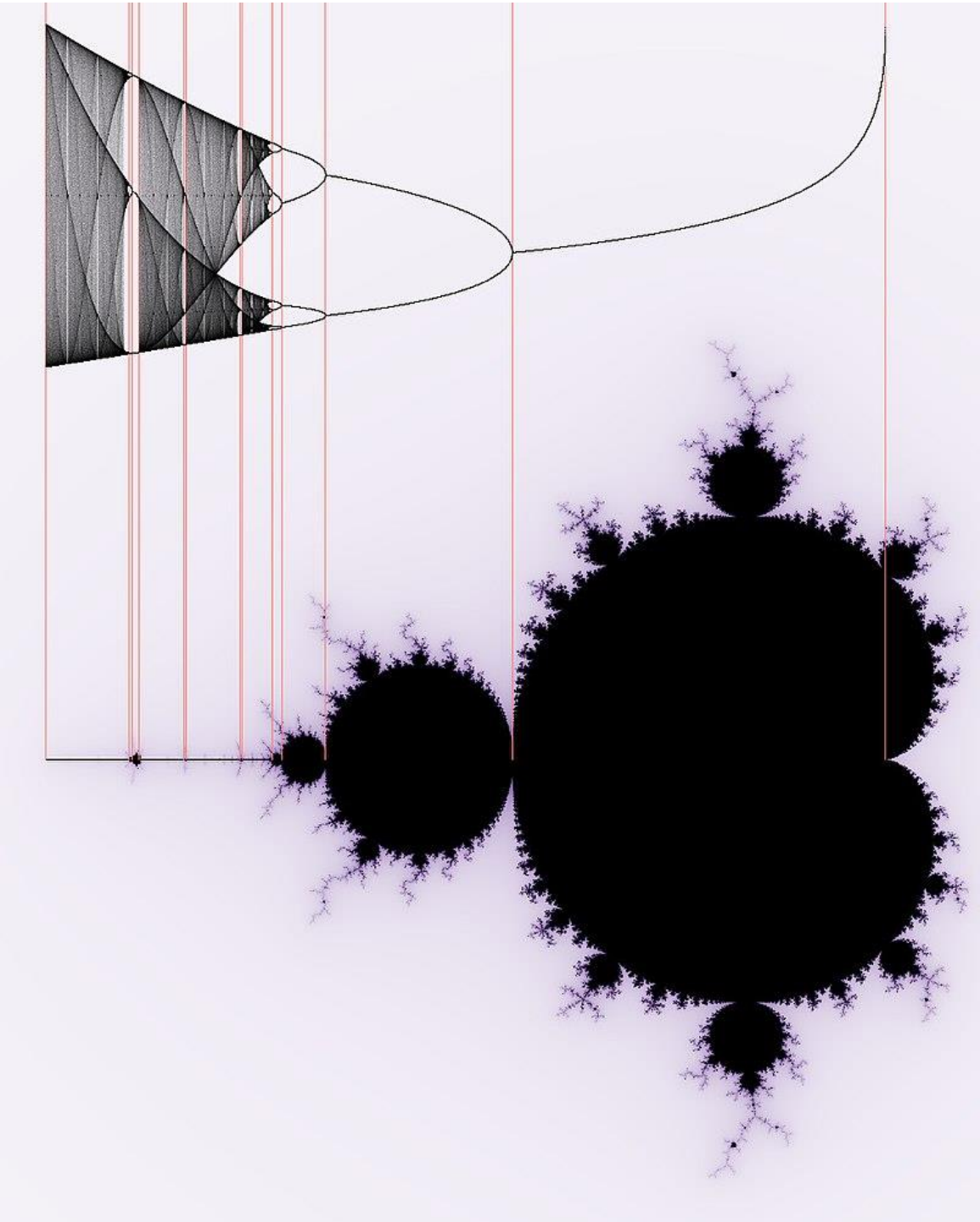
SELF SIMILARITY IN NATURE



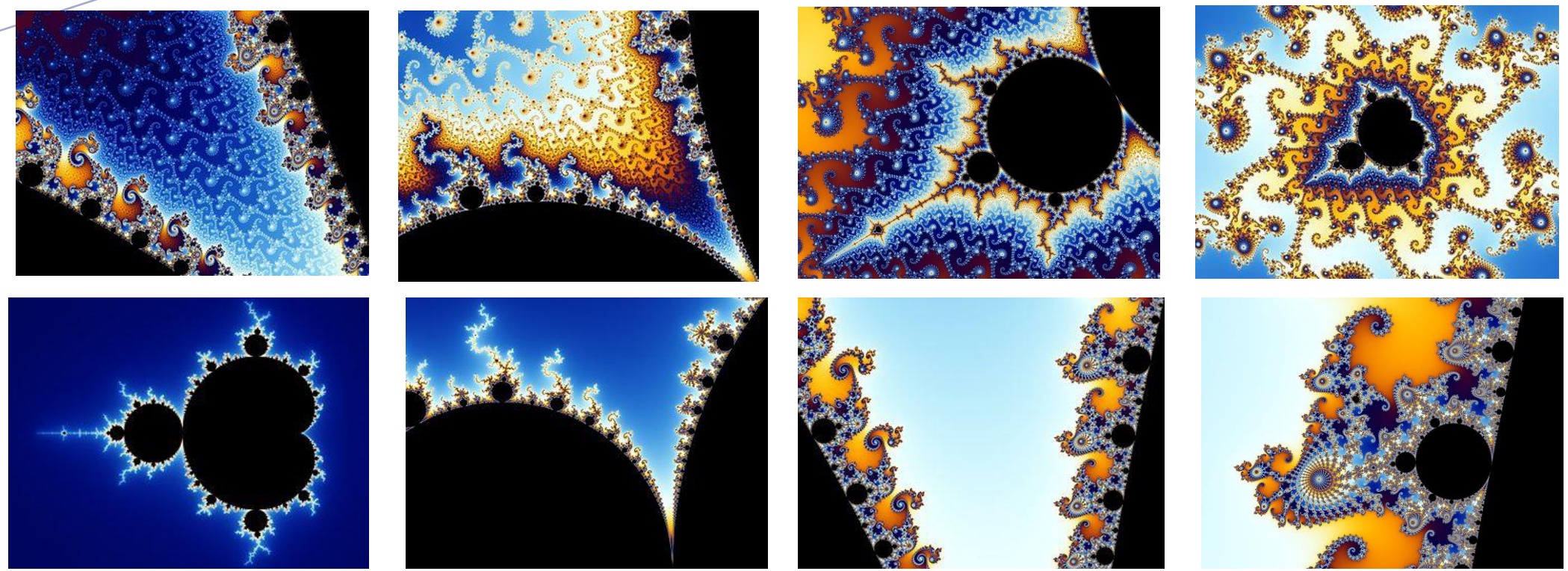


SELF-SIMILARITY DEFINED

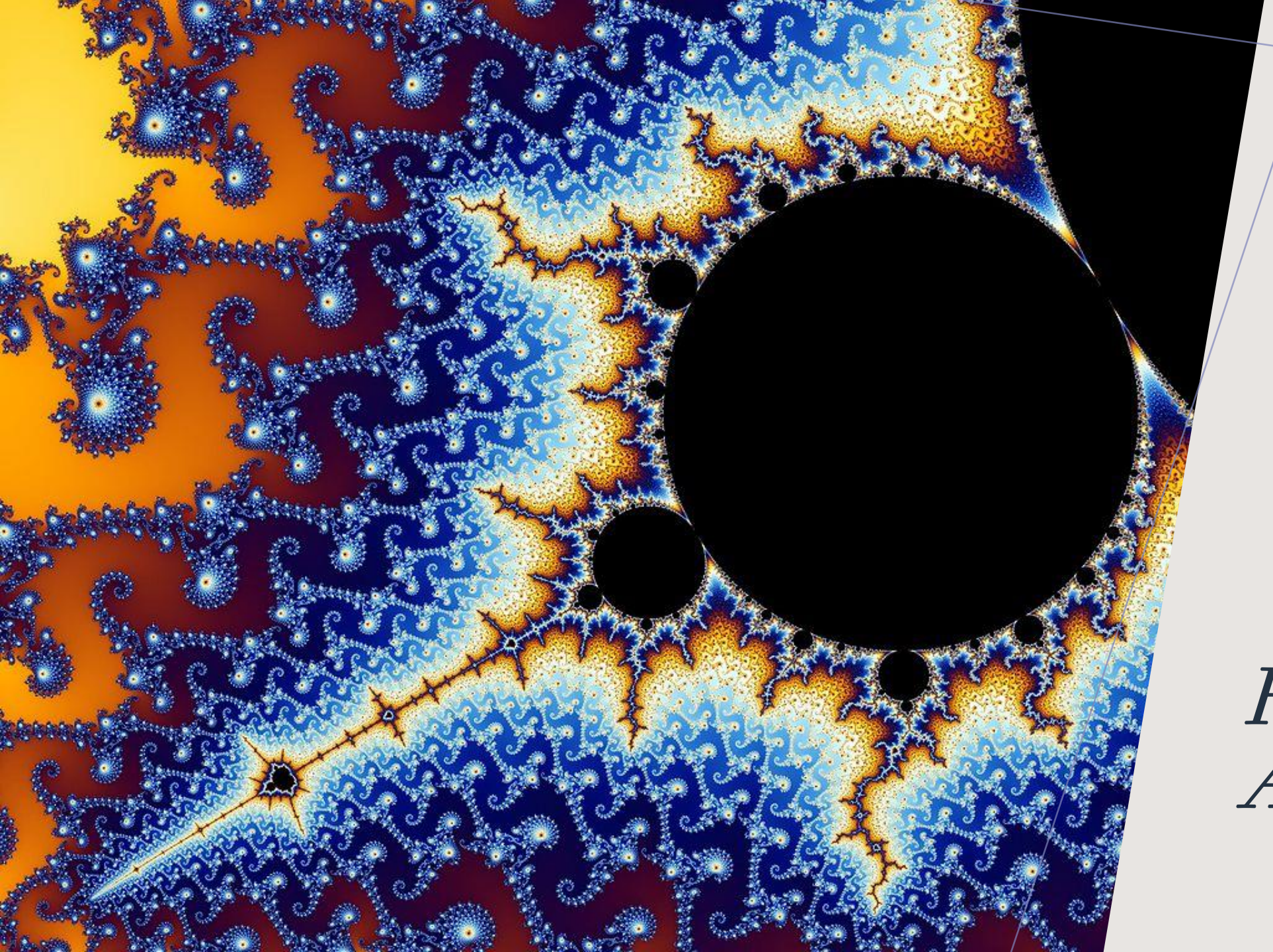
- Self-similarity is the unifying concept for the theories of fractals and chaos.
- A phenomenon that is self-similar looks the same or behaves the same when viewed at different degrees of magnification or different scales on a dimension. The dimension can be space (length, width) or time.
- Statistical self-similarity: coastline (paradox: length boundary is function of measuring unit), crack in wall.
- Exact (geometric) self-similarity: fern, spiral, binary tree
- Fractals in nature do no exhibit self-similarity over all time scales; self-similarity eventually breaks down.



SELF-SIMILARITY



- ...And we can continue to zoom in. As we magnify the object, we see the same thing over and over again.....This is Self Similarity



FRACTAL ANALYSIS

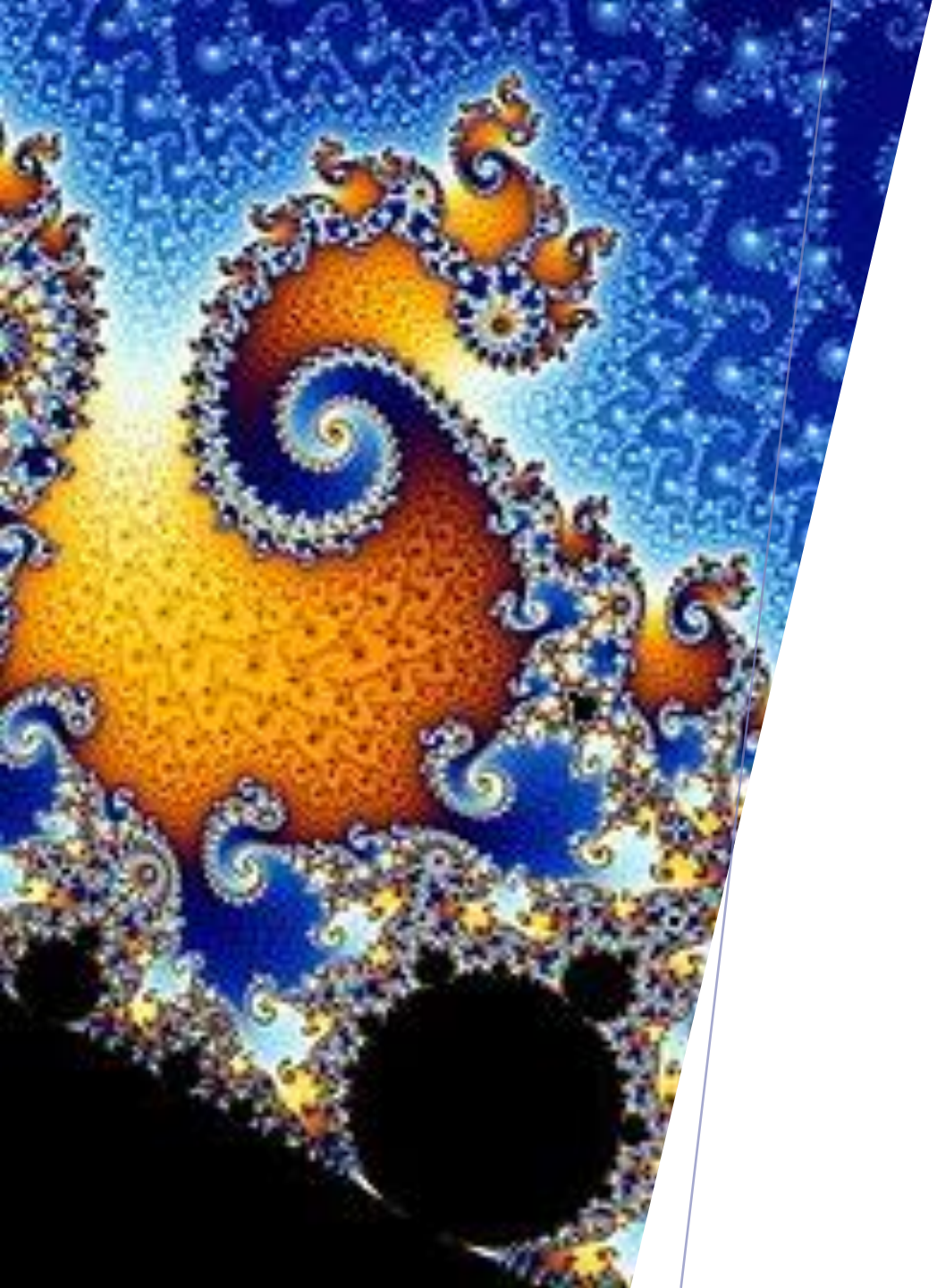
FRACTAL DIMENSION

Definition . (Box counting dimension) *Let A be a bounded set of $\mathbb{R}^k$. Let $N(A,\varepsilon)$ be the minimum number of ε-radius balls that are needed to cover A. If the limit*

$$D(A) = \lim_{\varepsilon \to 0} \{ \frac{\ln(N(A,\varepsilon))}{\ln(\frac{1}{\varepsilon})} \}$$

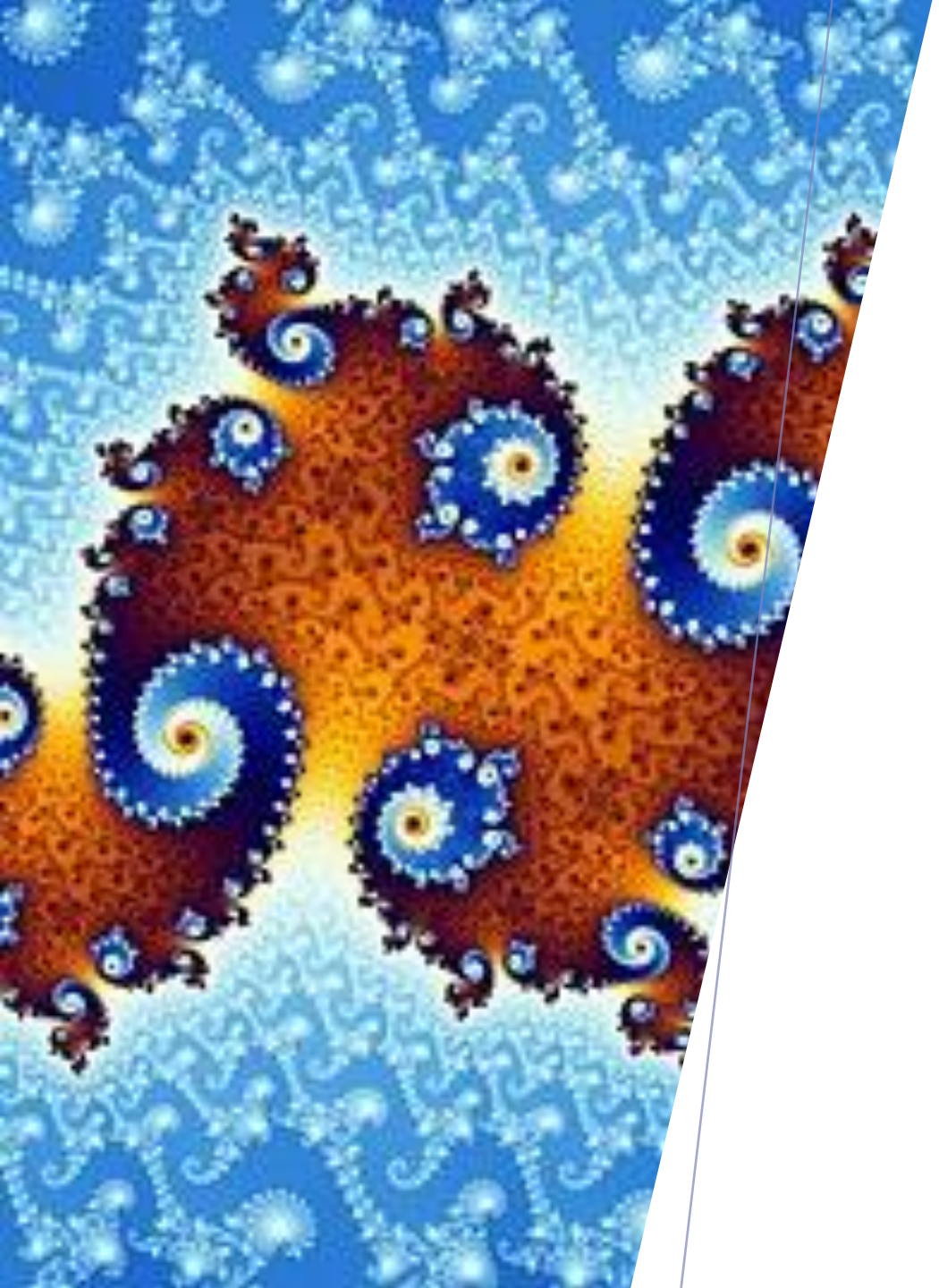
exists, then $D(A) = dim_B A$ is called the Box counting dimension of A.

To calculate this dimension for a fractal A, imagine this fractal lying on an evenly spaced grid, and count how many boxes are required to cover the set. The box-counting dimension is calculated by seeing how this number changes as we make the grid finer by applying a box-counting algorithm.



FRACTAL ANALYSIS

- **Fractals only tell us about an object in which a certain physical quantity is distributed evenly wherever it is found in the embedding space.**
- **There are situations in which physical quantities are unevenly distributed over the space.**
- **That is, the distribution may not be homogeneous and instead heterogeneous in the sense that the density of the content over the occupied regions only are different.**
- **We therefore have to invoke the idea of multifractality which tells us about the uneven or heterogeneous distribution of the content on a geometric support.**

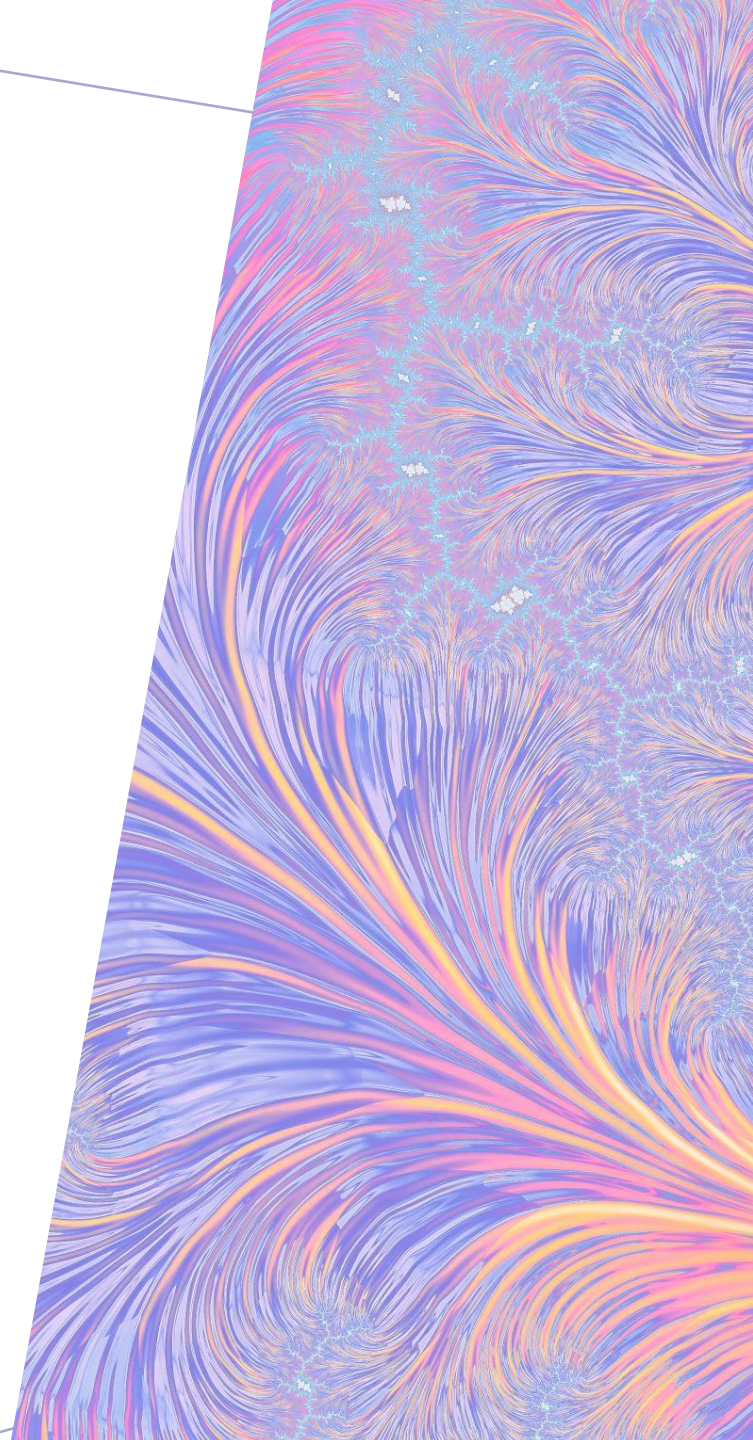


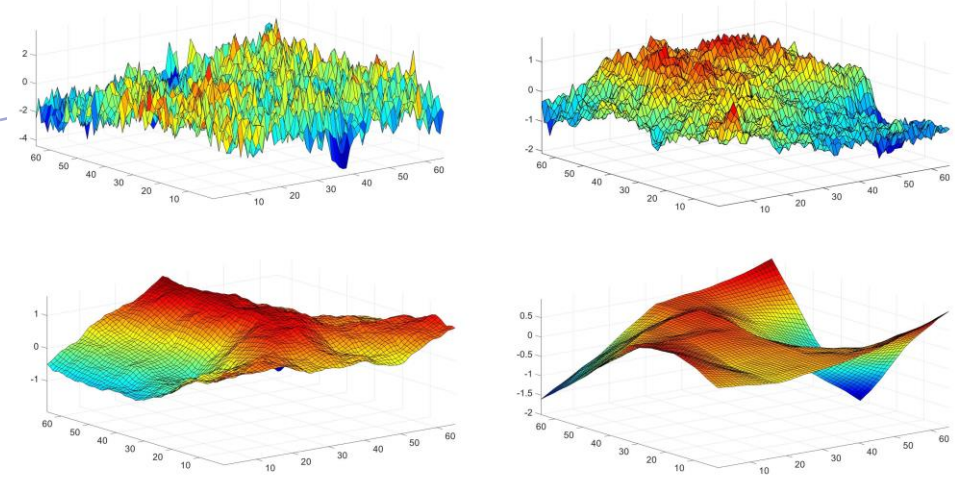
MULTIFRACTAL ANALYSIS

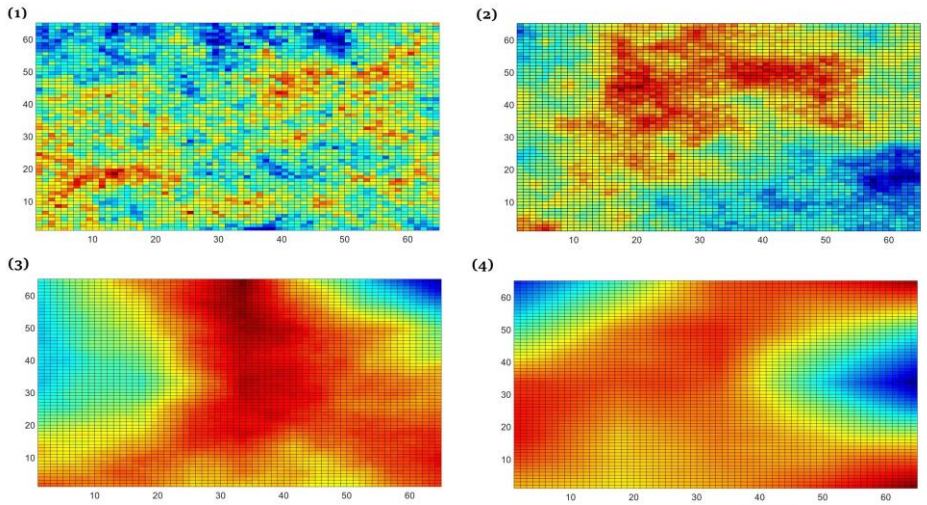
- **The multifractal spectrum effectively shows the distribution of scaling exponents for an object.**
- **Equivalently, the multifractal spectrum provides a measure of how much the local regularity of an object varies in time.**
- **An object that is monofractal exhibits essentially the same regularity everywhere in time and therefore has a multifractal spectrum with narrow support.**
- **Conversely, A multifractal object exhibits variations in regularity over time and has a multifractal spectrum with wider support.**

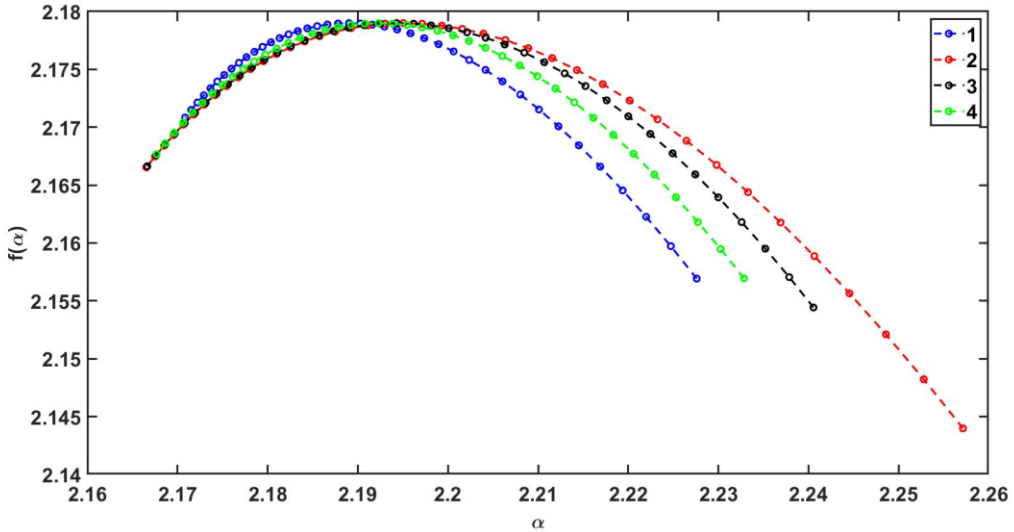

FRACTALS ARE
EVERYWHERE

FRACTAL GEOMETRY OF DIFFERENT SURFACES AND MATERIALS







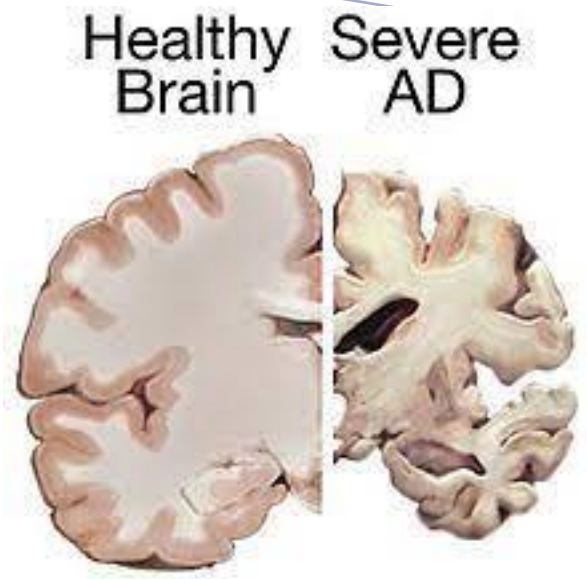


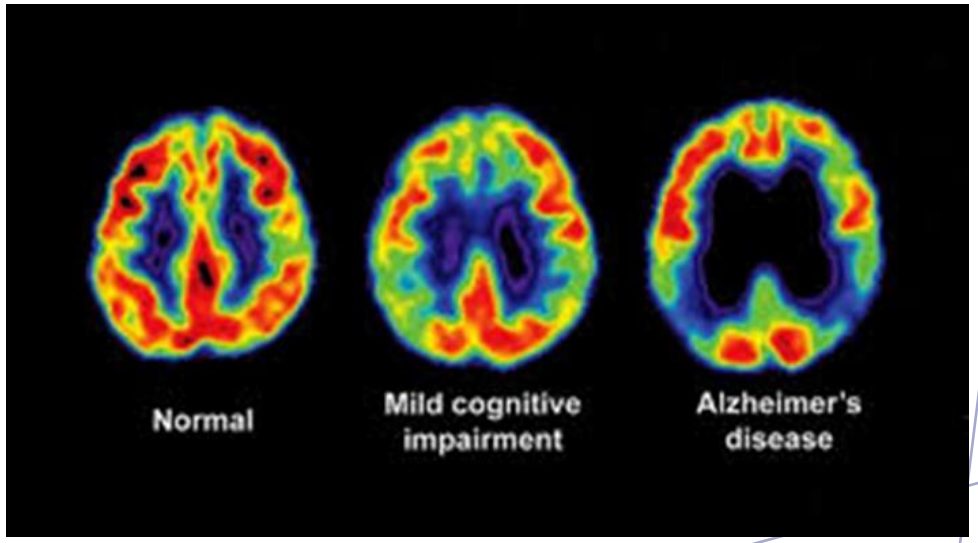
MULTIFRACTAL ANALYSIS

THE FRACTAL GEOMETRY OF BRAIN

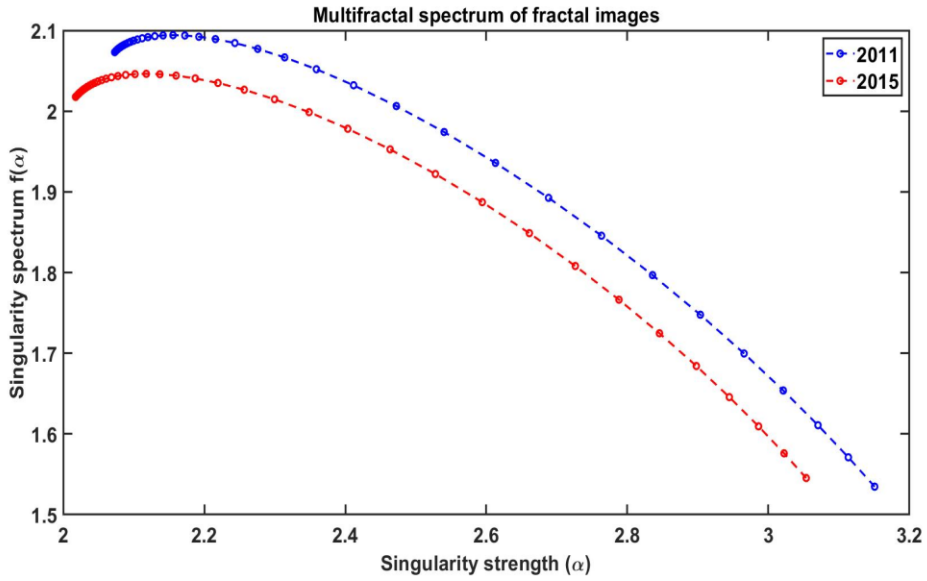
THE FRACTAL GEOMETRY OF ALZHEIMER'S DISEASE

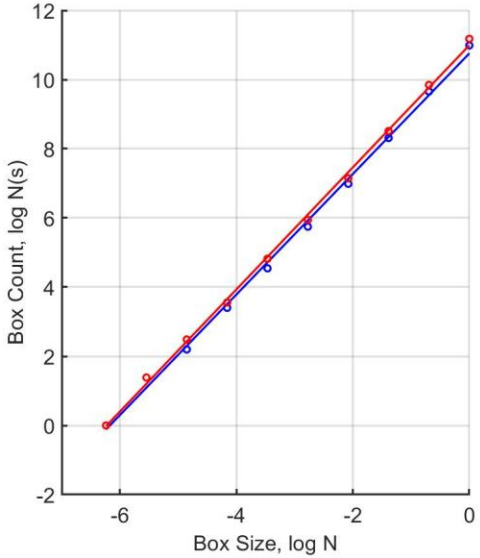
- **Human brain** is the most dynamic and varied system of the body.
- Many such **complex systems** that exist in non-linear dynamics are characterized by the fractal nature.
- The fractal dimension (FD) is a. quantitative parameter that has been extensively used to analyze the complexity of structural and functional patterns of the human brain
- **Alzheimer disease (AD)** is a progressive neurodegenerative disease that **destroys memory** and **cognitive skills**. **Aging** is the biggest risk factor for AD.
- In several pathologies of the brain such as **Alzheimer**'s, Epilepsy and Stroke, fractal dimension (FD) is altered.
- FD in combination with other features is emerging as a powerful **diagnostic approach** at the hands of a clinician.

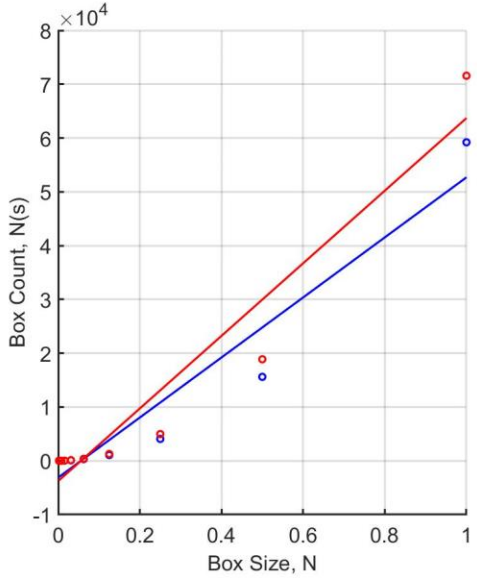




ALZHEIMER'S DISEASE

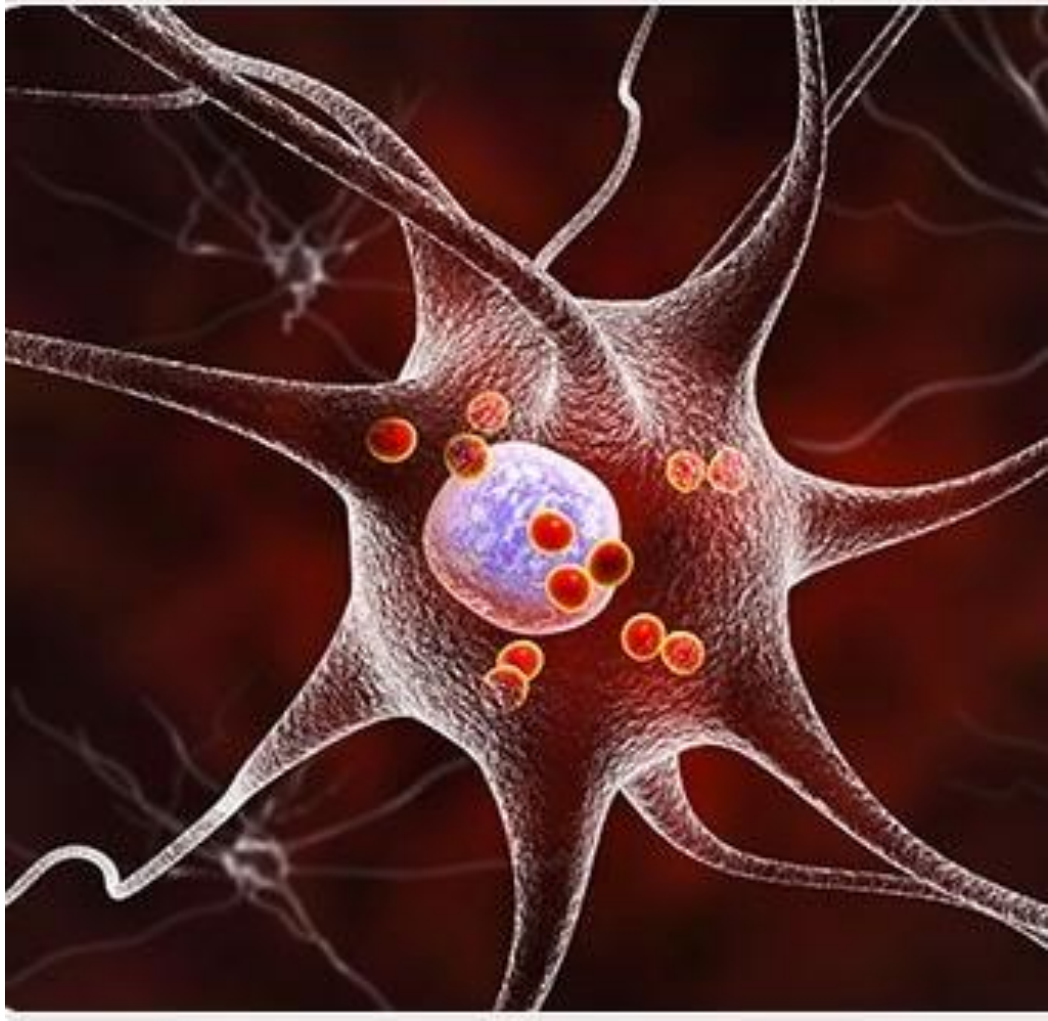






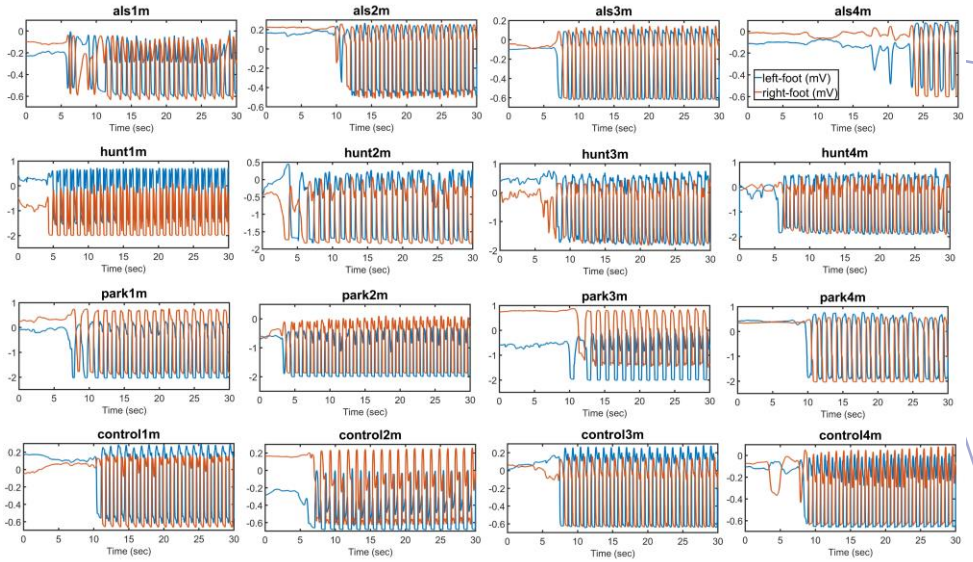
NEURODEGENERATIVE DISEASES

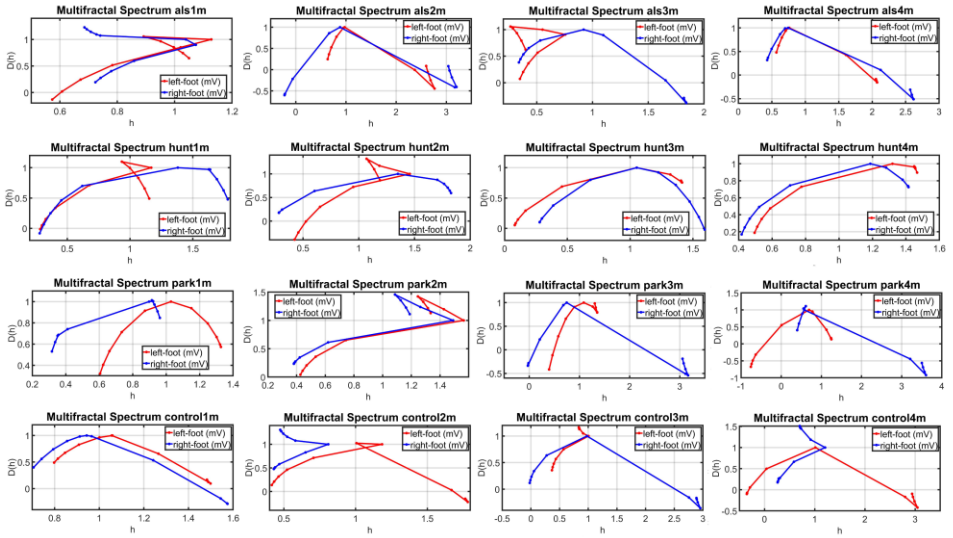
- **Neurodegenerative diseases** occur when nerve cells in the brain or peripheral nervous system **lose function** over time and ultimately **die**.
- Although treatments may help relieve some of the physical or mental symptoms associated with neurodegenerative diseases, **there is currently no way to slow disease progression and no known cures**.



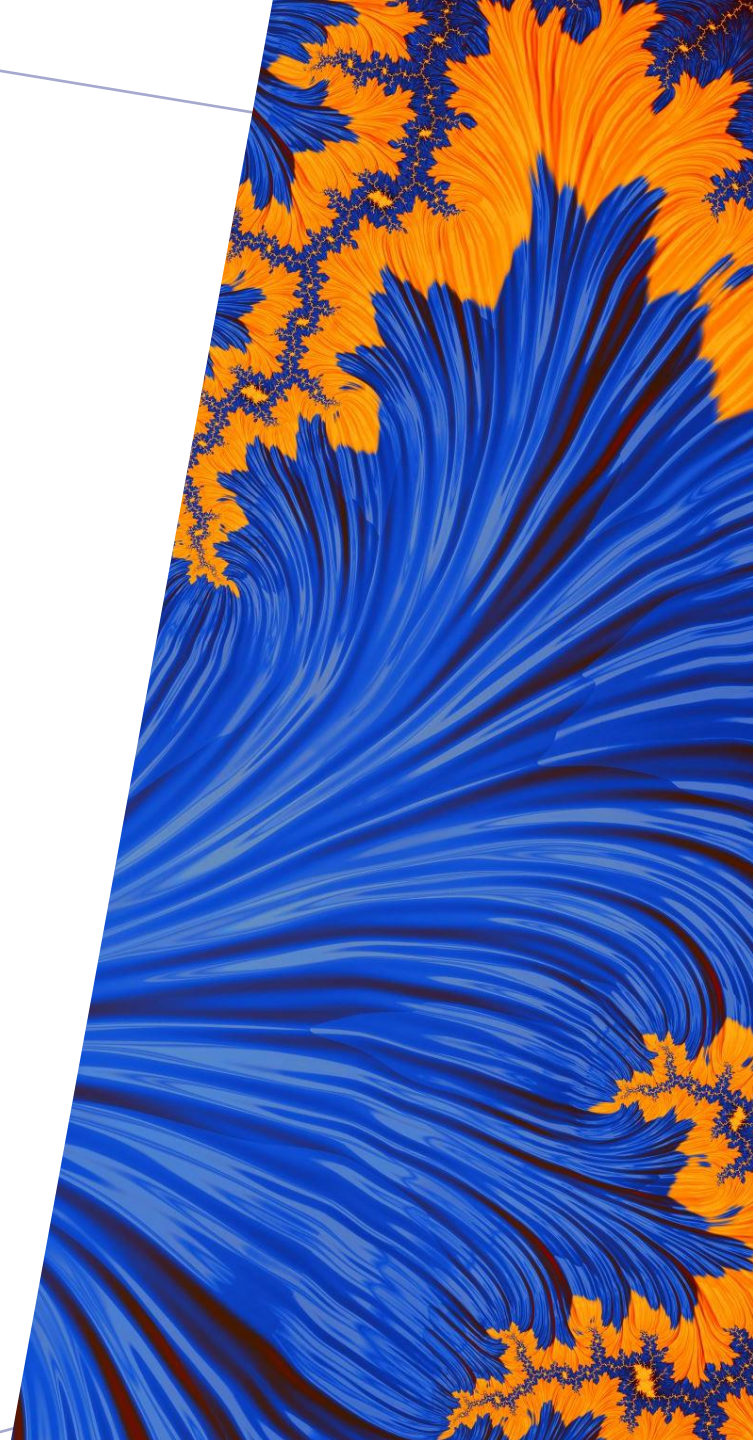
NEURODEGENERATIVE DISEASES

- **Neuro-degenerative diseases** influence significantly the **gait behavior** and the **ability to move**.
- To explore the etiology of neuro-degenerative disease, it would be useful to **characterize gait dynamics**.
- The purpose of this study is to **classify different neuro-degenerative diseases** using **fractal geometry**.
- We use Gait Dynamics in Neuro-Degenerative Disease Data Base including recordings from patients with **Parkinson's disease** (n = 15), **Huntington's disease** (n = 20), or **amyotrophic lateral sclerosis** (n = 13) and 16 healthy control subjects are also included.

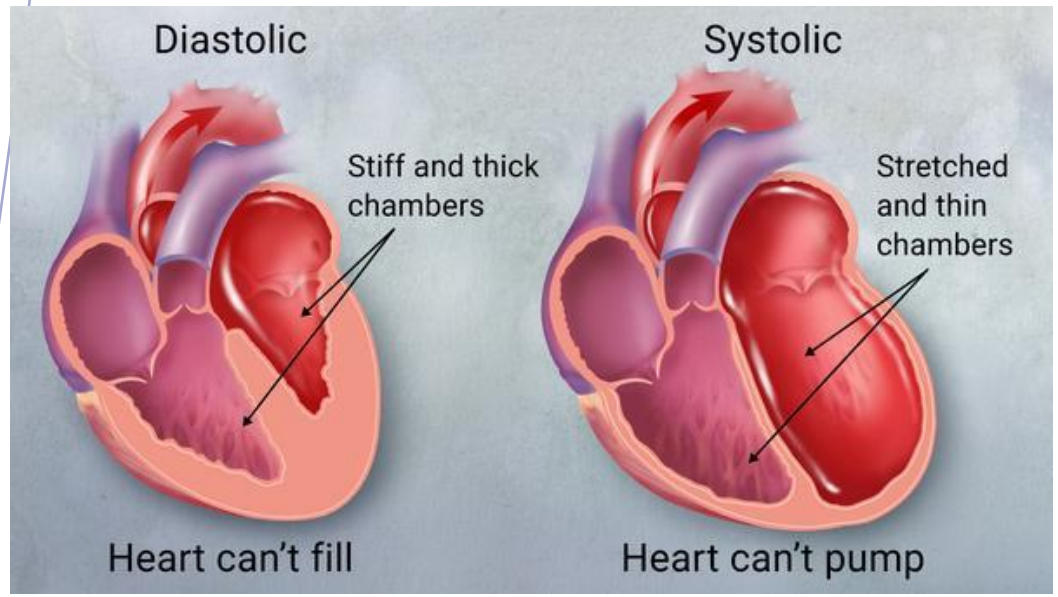




ON THE FRACTAL GEOMETRY OF DIFFERENT HEART RHYTHMS



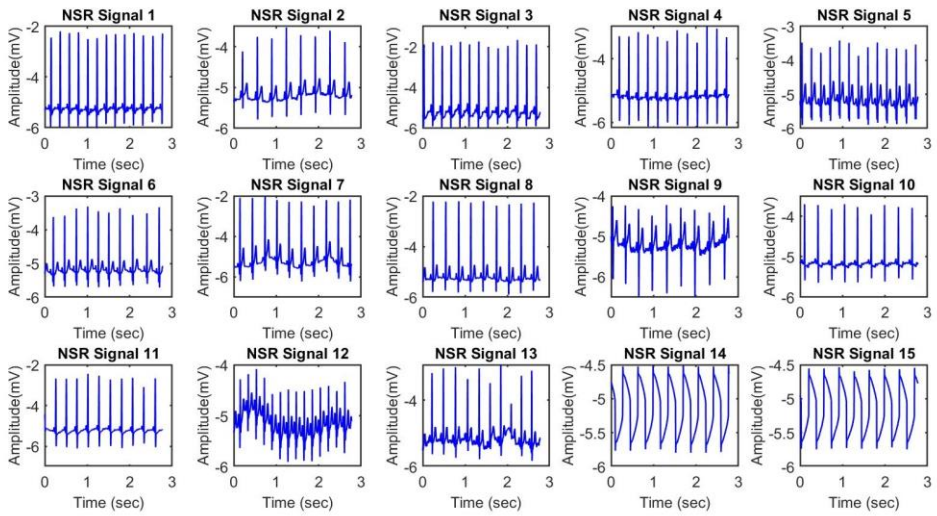
CONGESTIVE HEART FAILURE

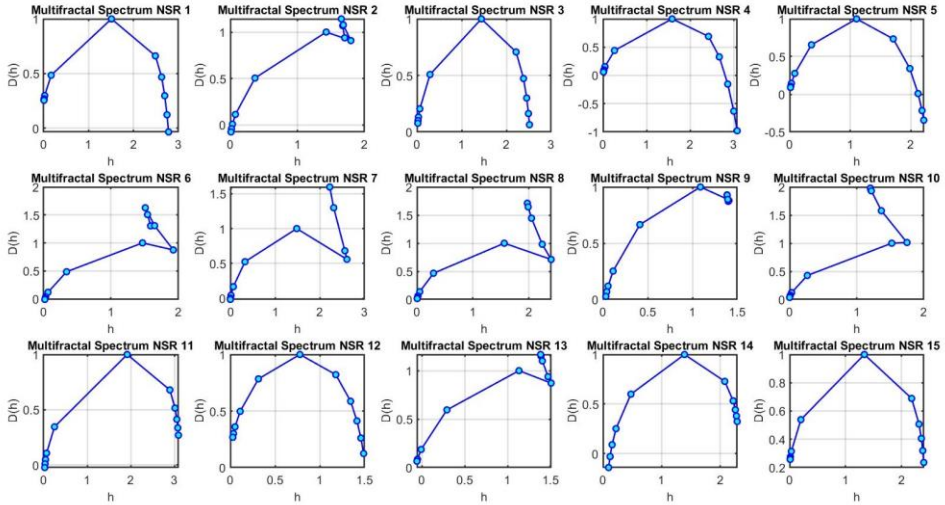


- A chronic condition in which the heart doesn't pump blood as well as it should.
- Heart failure can occur if **the heart cannot pump (systolic) or fill (diastolic) adequately**.
- **Symptoms** include shortness of breath, fatigue, swollen legs, and rapid heartbeat.
- **Treatments** can include eating less salt, limiting fluid intake, and taking prescription medications. In some cases a defibrillator or pacemaker may be implanted.

HEALTHY HEART RHYTHM

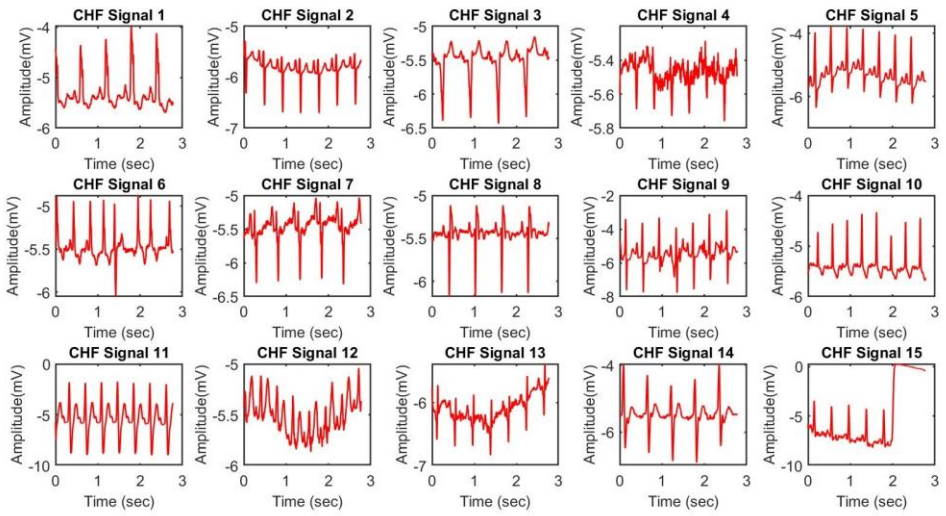
- The **multifractal spectrum** of all long-term ECG signals with normal Sinus rhythm. The heart rate of healthy humans shows a **multifractal process** with a broad range of exponents.

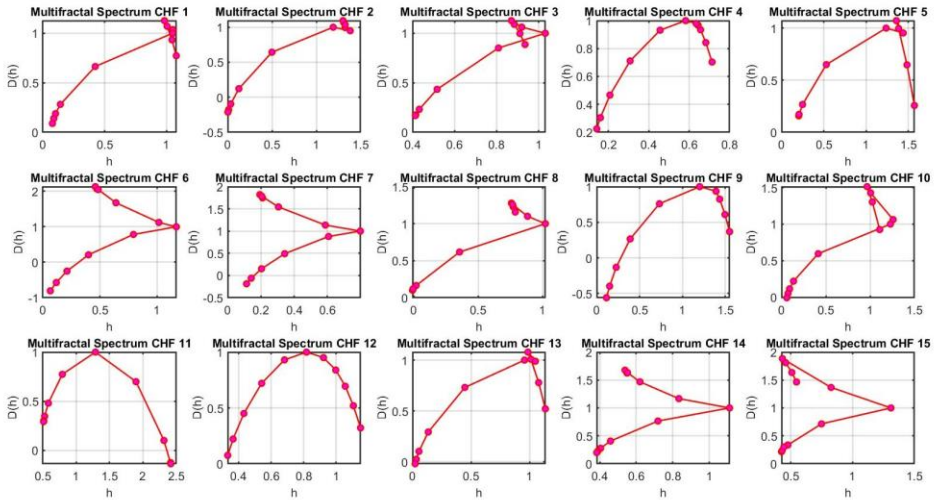




CONGESTIVE HEART FAILURE

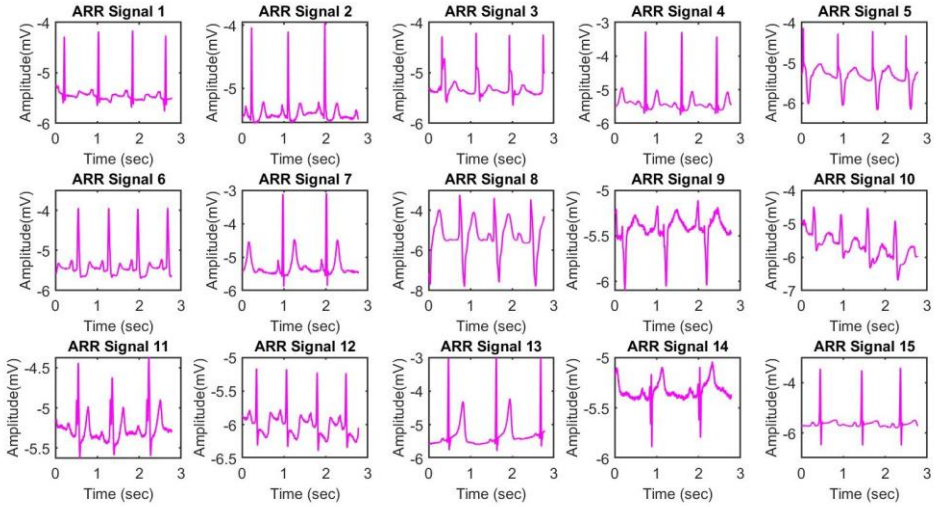
- However, **monofractal dimension** is not able to fully characterize complex scaling behaviors of many irregular objects in the real world.
- That's why to study **irregular objects** like **ECG signals** one may need to apply the **multifractal algorithm**.
- **The multifractal analysis used a spectrum of singularity exponents to provide a detailed and local description of complex scaling behaviors.**

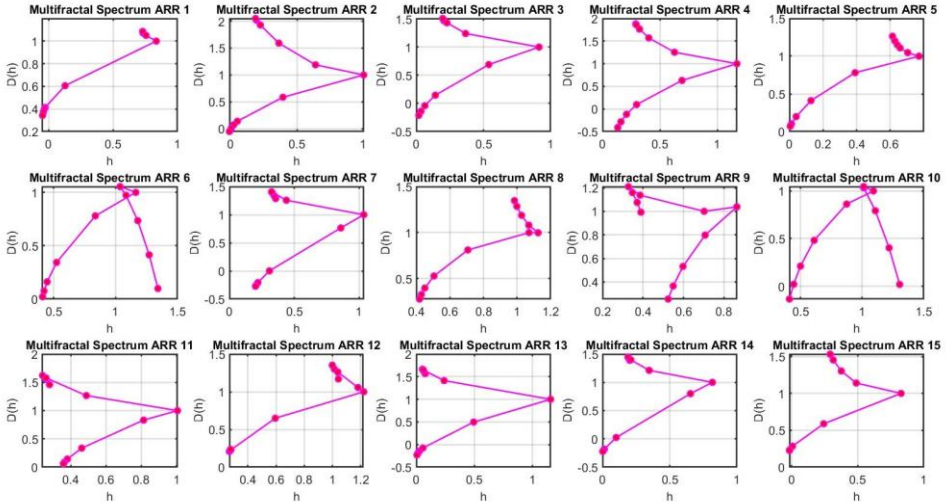




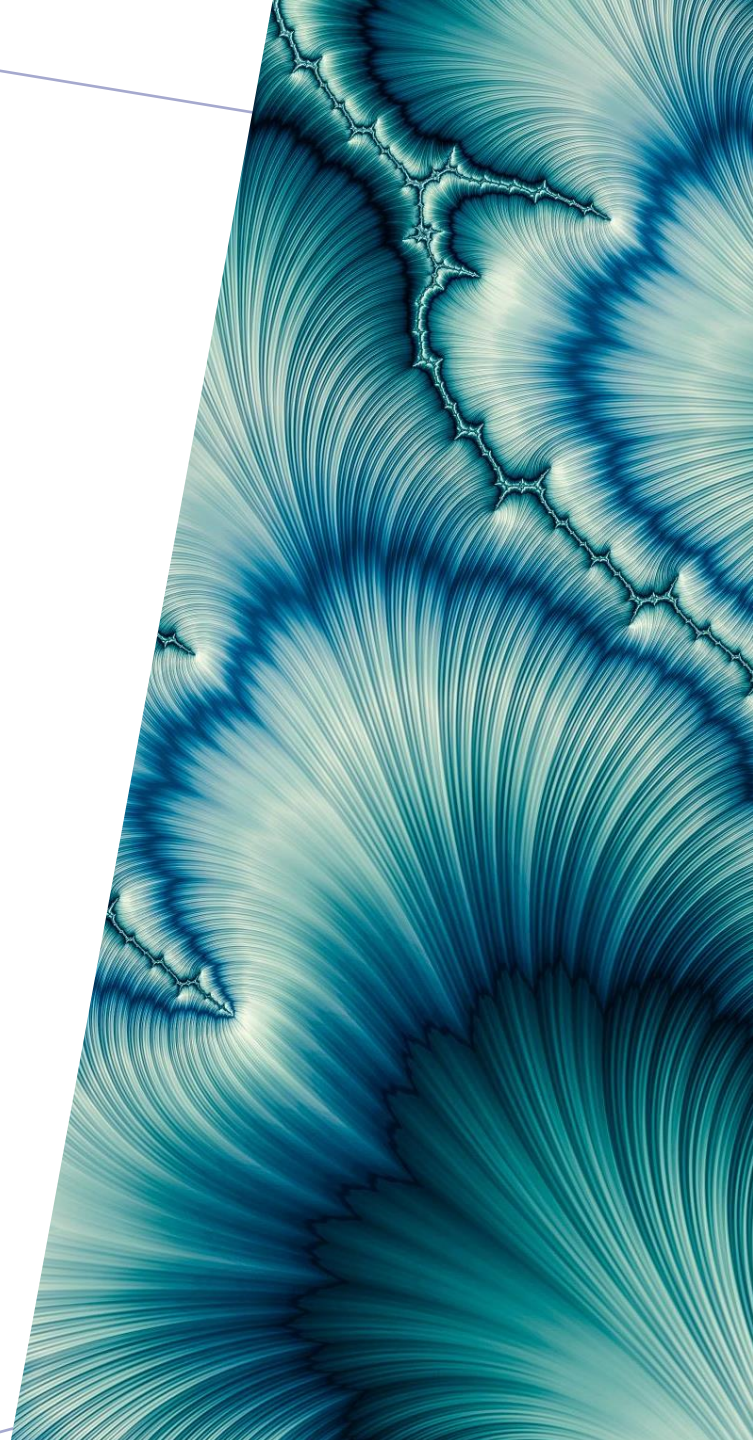
ARRHYTHMIA

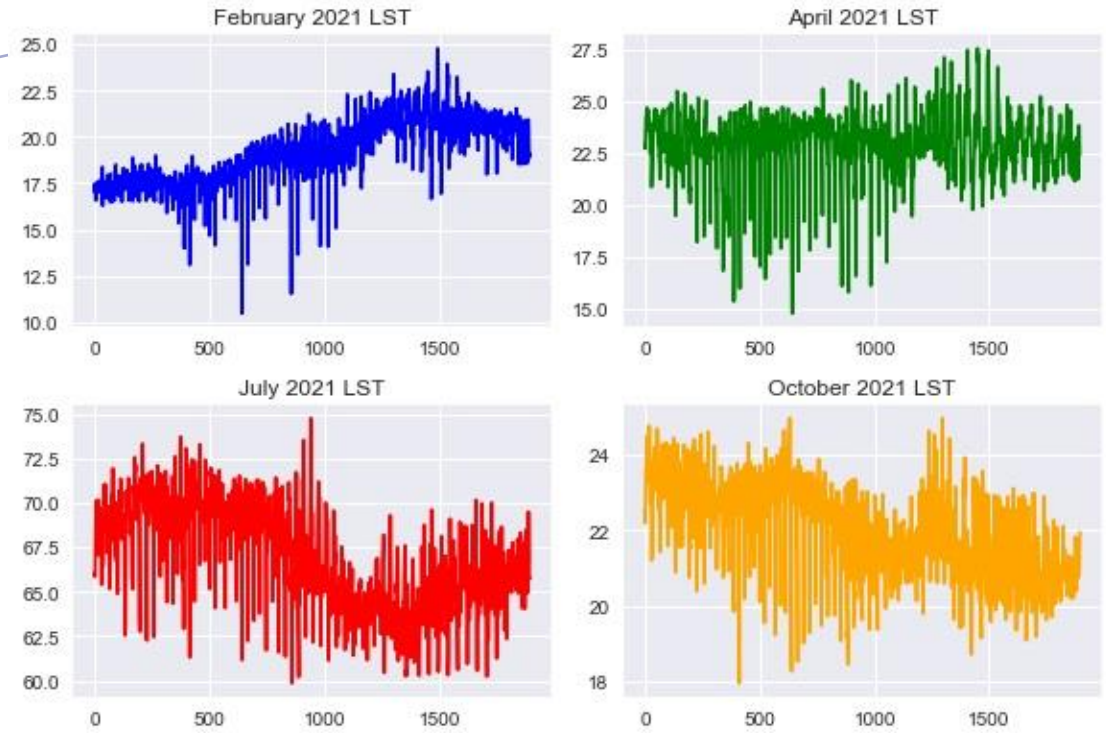
- Unlike **healthy subjects**, the signals with **congestive heart failure** and **arrhythmia** show a clear **loss of multifractality** and both types of heart diseases are **homogeneous** and **monofractal** since **their spectrum displays a narrow width of scaling exponent**.
- In these two cases, recordings demonstrate **similar scaling features throughout the signal and they can be characterized by only a single global exponent**.
- These results also revealed that the **arrhythmia signals have narrower range of exponents compared to congestive heart failure signals and this important finding may be used to differentiate these two classes of heart disease as well**.

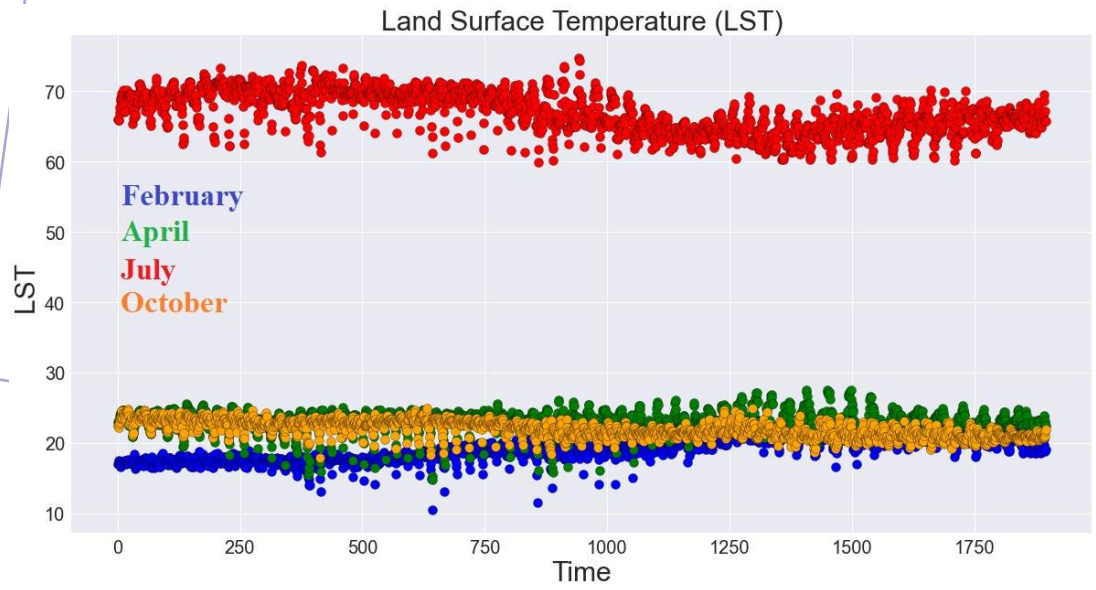




FRACTAL GEOMETRY AND CLIMATE DATA

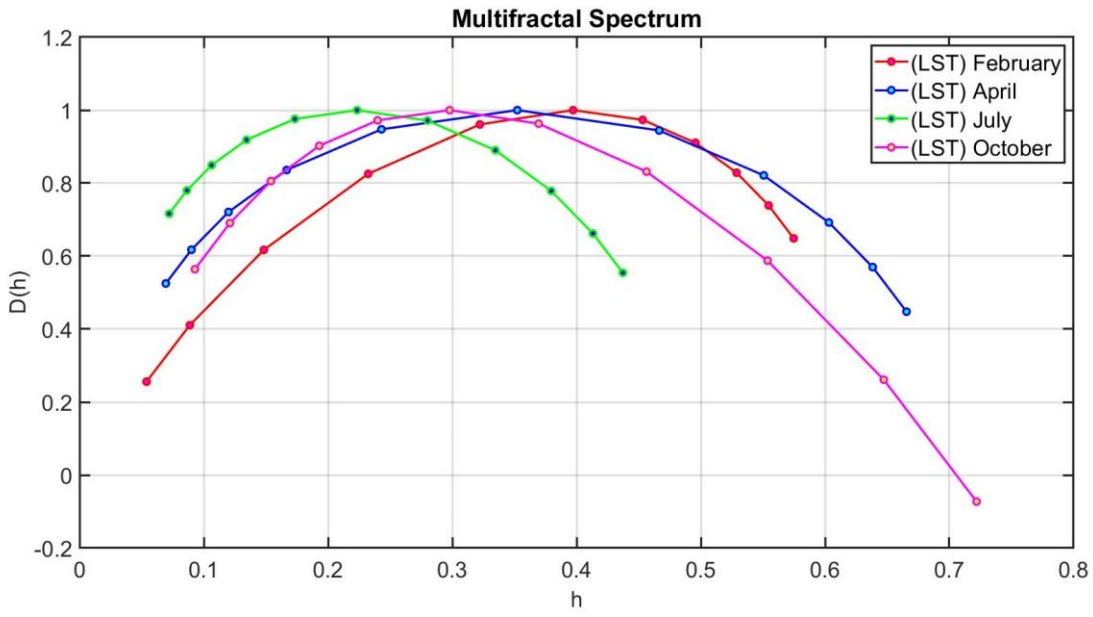


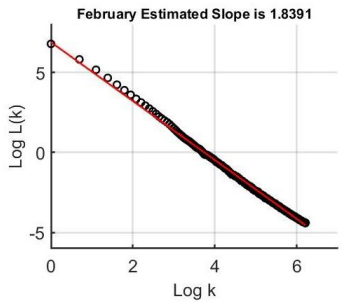


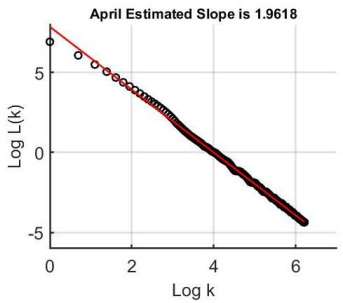


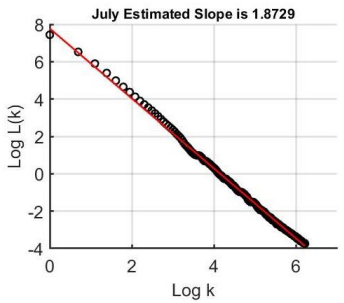
NON-INTEGER DIMENSION OF SEASONAL LAND SURFACE TEMPERATURE (LST)

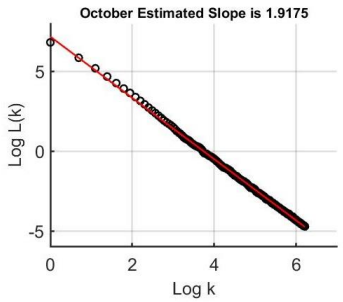
- For the first time, Mandelbrot and Wallis in 1969 applied **fractal analysis** to some climatic records.
- Here, we particularly want to estimate the **non-integer dimension** of the land surface temperature, and we conduct our research within the framework of fractal geometry.
- One of the **main advantages** of this study providing a strong comparison framework for variety of climate data sources in **different units** and on **different time scales**.

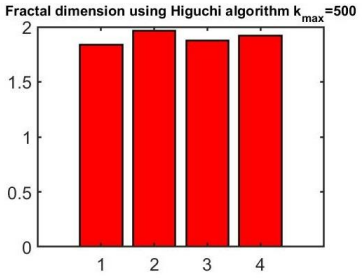








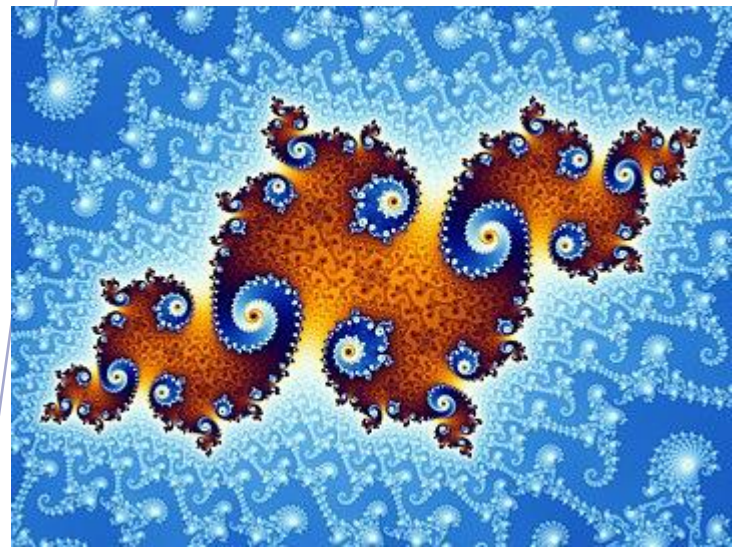


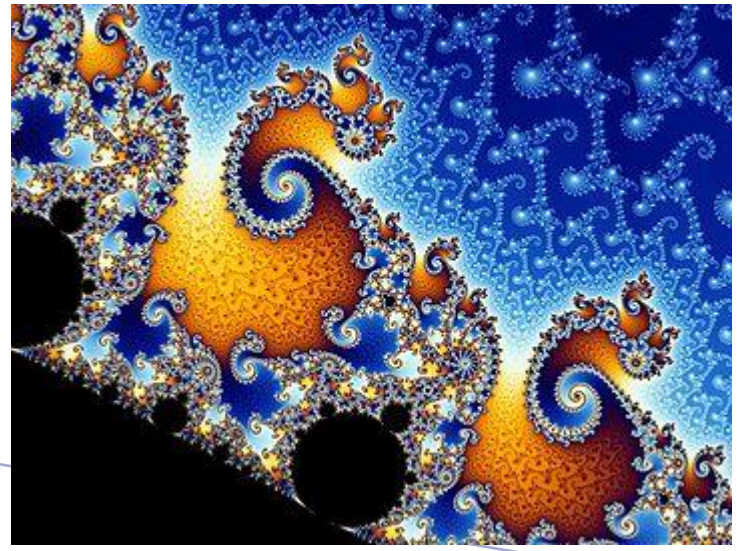


MULTI-FRACTAL ANALYSIS AND DISCRETE WAVELET TRANSFORM (DWT)

- The multi-fractal spectrum analysis of the Land Surface Temperature (LST) of Riley County during February, April, July and October (2021) shows occurrence of **mono-fractal process** with a **narrow range of exponents** for all four groups of (LST) time series data.
- The results of the present study show that the Land Surface Temperature (LST) is **mono fractal**.
- However, further studies on different time scales and locations are needed to support such a hypothesis.

The Fractal Geometry of Nature





Why is geometry often described as cold and dry? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line...

...Nature exhibits not simply a higher degree but an altogether different level of complexity. The number of distinct scales of length of patterns is for all purposes infinite.

The existence of these patterns challenges us to study those forms that Euclid leaves aside as being formless, to investigate the morphology of the amorphous. Mathematicians have disdained this challenge, however, and have increasingly chosen to flee from nature by devising theories unrelated to anything we can see or feel.

---Benoit Mandelbrot (1984)

THANKS VERY MUCH TO ALL ORGANIZERS OF SIMIODE2023

THANK YOU SO MUCH FOR YOUR ATTENTION