# Modeling in Depth:

## What do we want our modelers to take from our course?

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United States Military Academy
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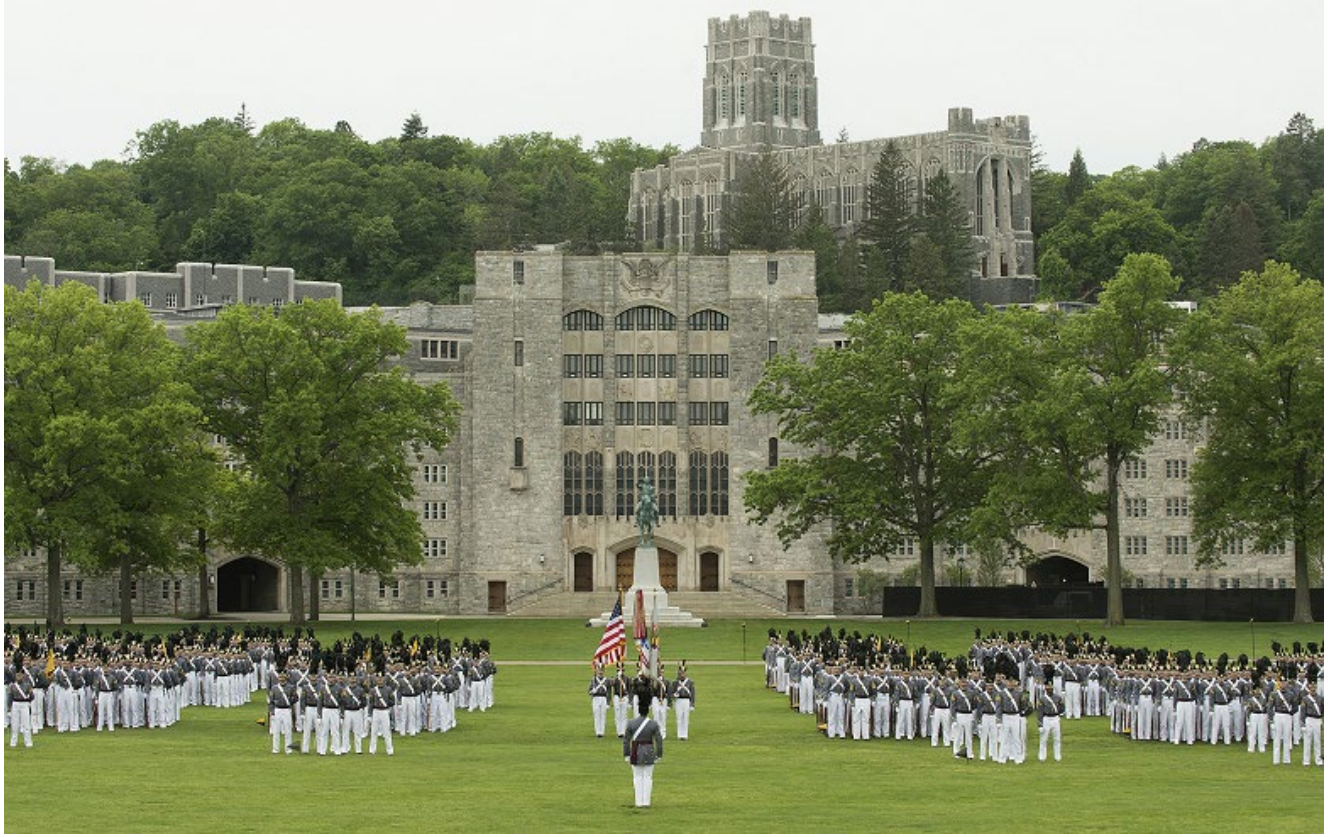
# Agenda

A modeling in depth approach allows students to reach peak modeling

- Background
- Backwards Planning
- Bloom's taxonomy
- Traditional Traps
- Parallel modeling teaching

# Background



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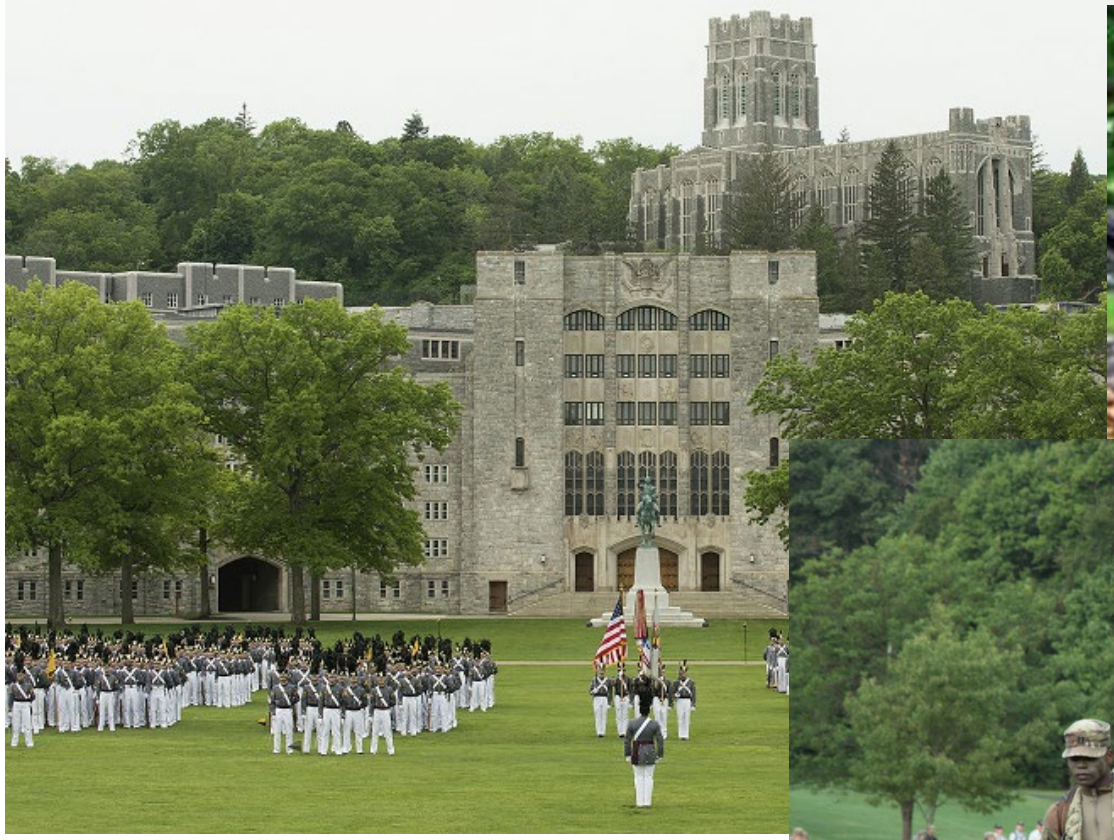

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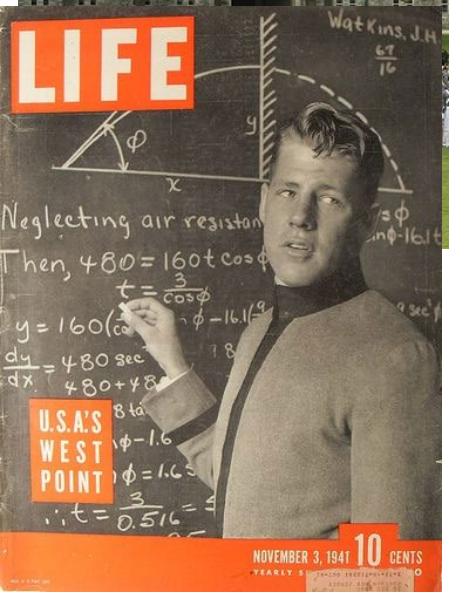
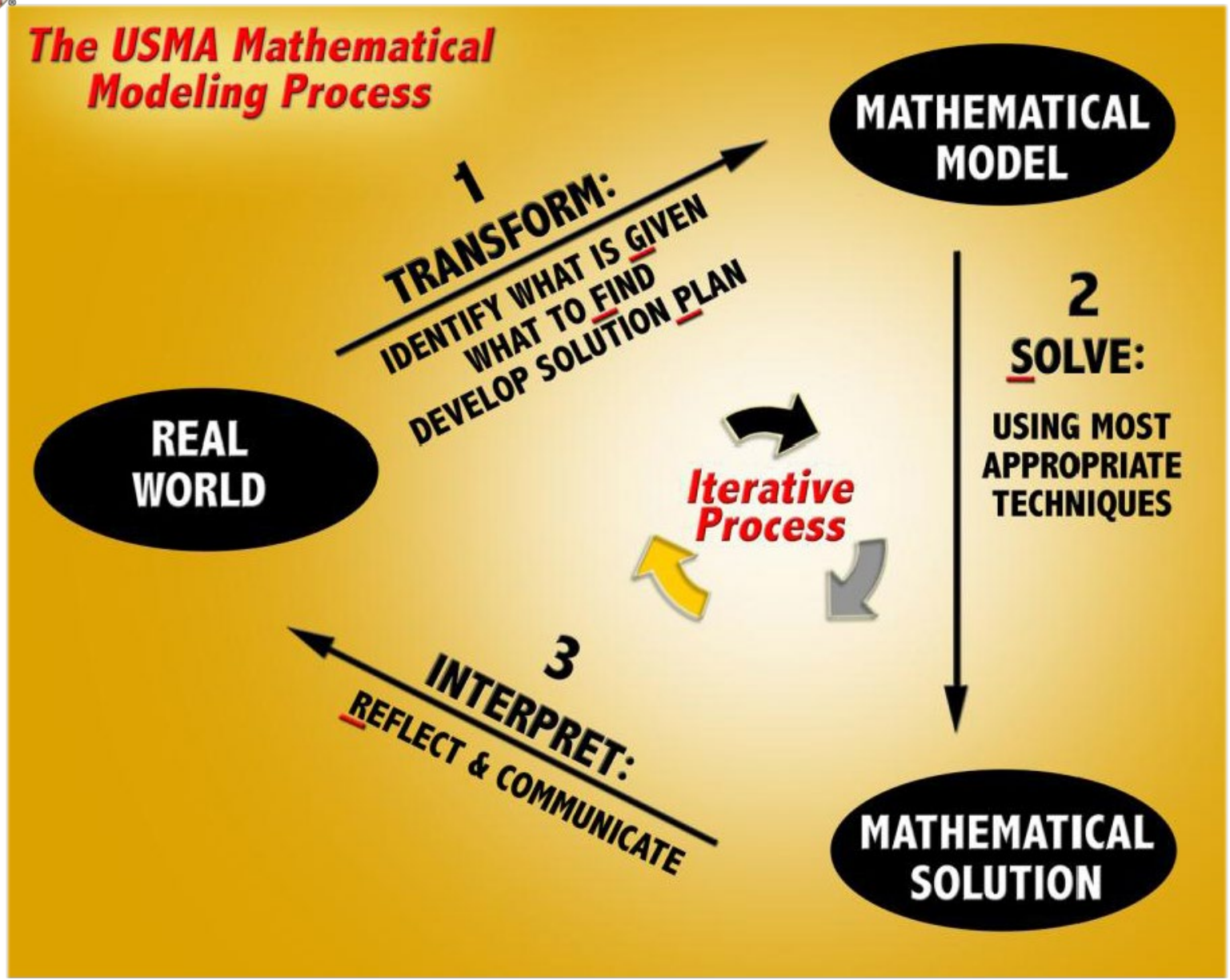



# Modeling Process



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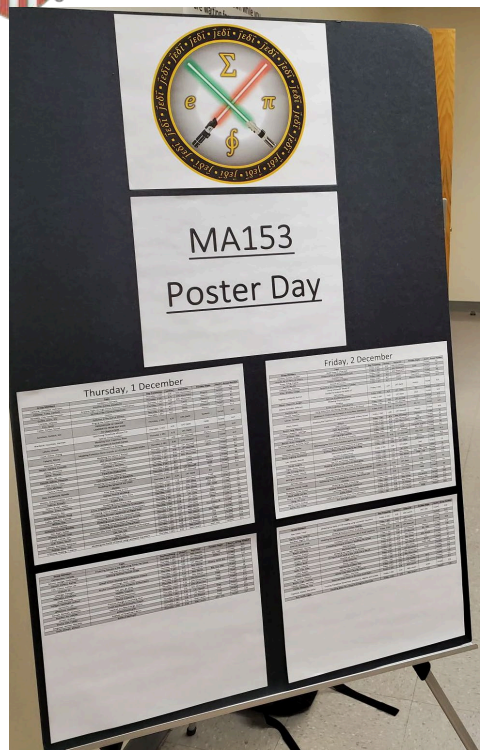
# A Couple Examples





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# Peak Modeling

Students use real world phenomena to cause them to

- Construct a model from basic assumptions or physical laws
- Make a parameter a function
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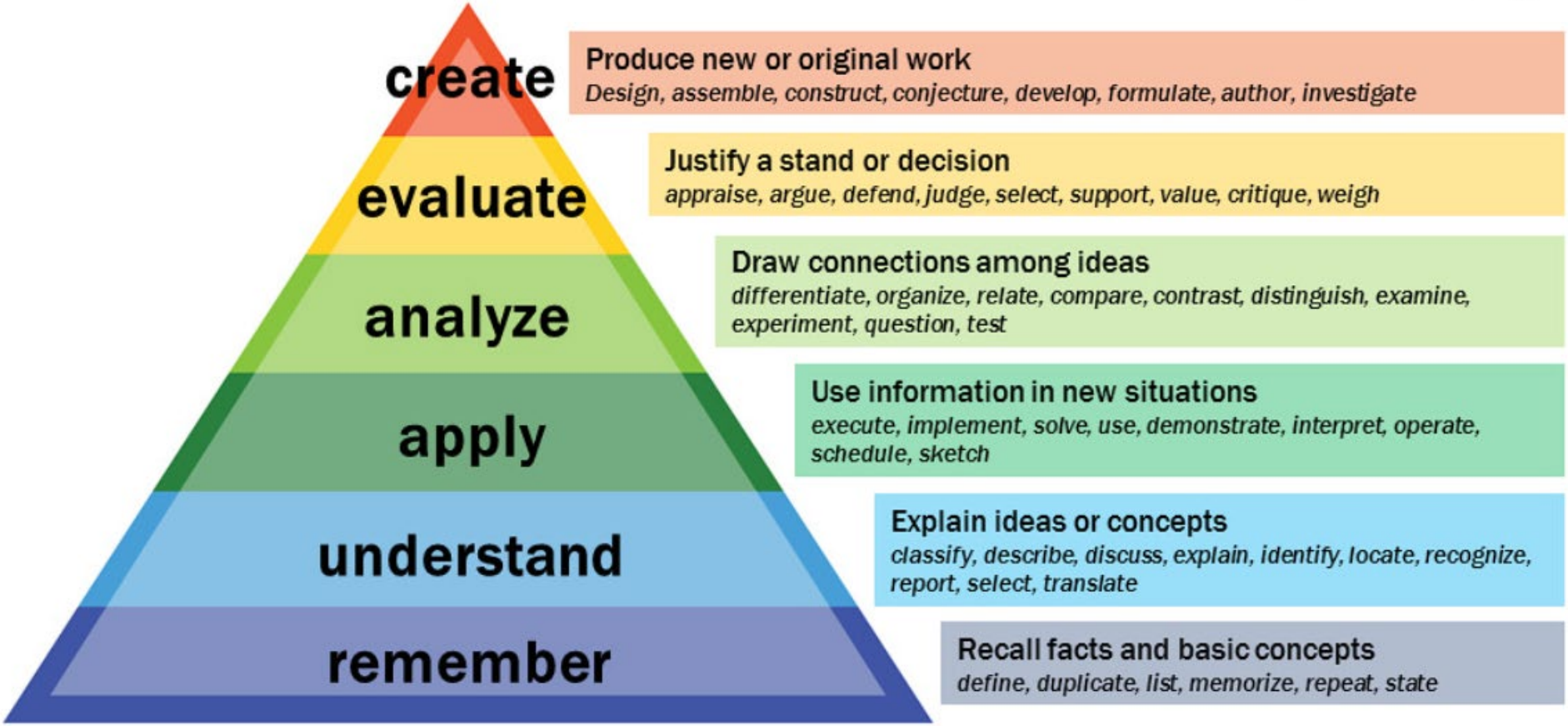
# Building Good Modelers

## Bloom's Taxonomy



# Building Good Modelers


- Create
  - Add a term to overcome short coming in model
- Evaluate
  - Acknowledge short comings in model
- Analyze
  - Solve for a parameter
- Apply
  - Create a model with same parameters as textbook
- Understand:
  - What is rate of change
  - How this is an ODE
  - Assumptions used to build the model
- Remember:
  - Given textbook models
    - $T'(t) = k(T - T_A)$
    - $\sum F = ma$

# Introducing Frameworks

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How do we teach to help students reach the top?

Traditional vs. Parallel

# A common framework

In class for lecture I:

1. Talk about how many rabbits there are/were in Australia
2. Propose or read from the book $\frac{dP}{dt} = kP$
3. Find $k$ from data
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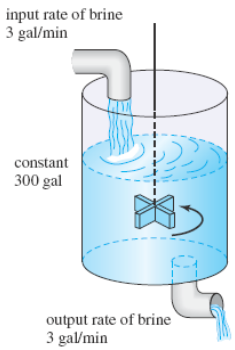
For Homework Students solve problems such as:

1. Consider a mixing tank find concentration at time $t$
2. A radioactive isotope decays how long ago was it buried?
3. A cold drink is set out at room temperature how long until it reaches 50 Fahrenheit?

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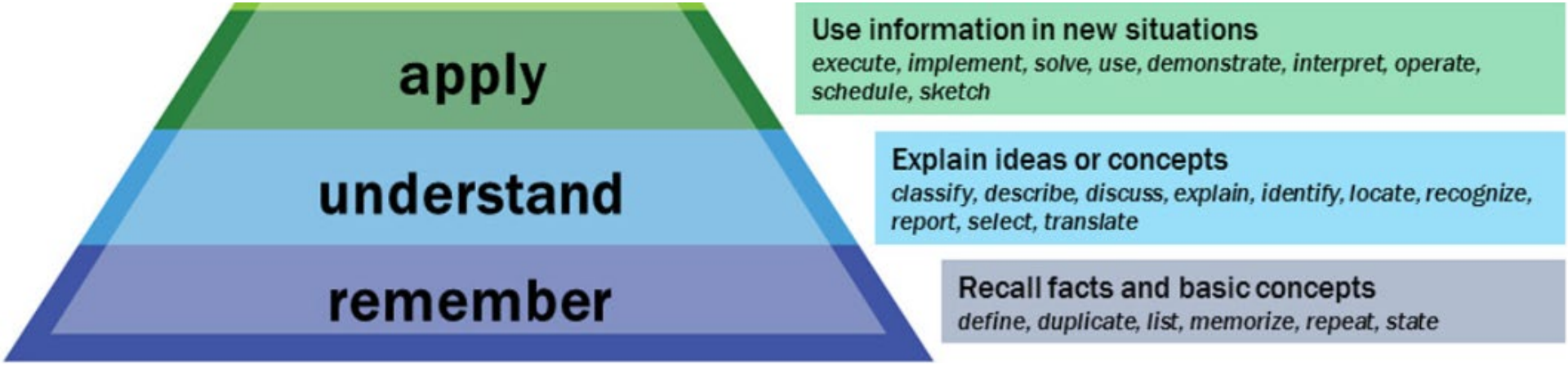

# A parallel framework

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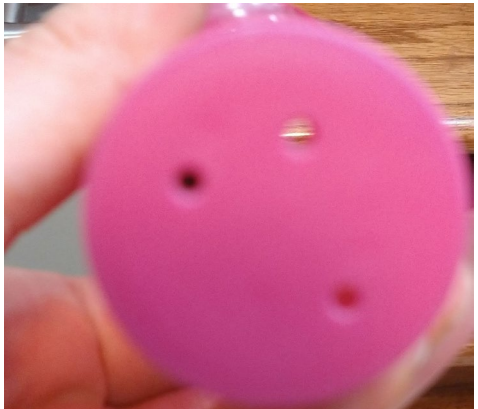

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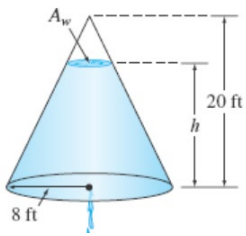
3. Make a $A_w$ a function $h$ (re-transform)

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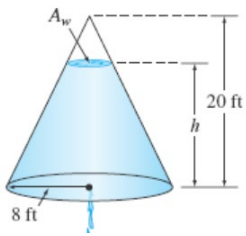


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For Homework Students emulate this process:

1. Consider the temperature of a lake: derive Newton's cooling
2. Find a coefficient that approximates data
3. Make the ambient temperature a function of time
4. Add a function to account for:
   a. The ground temp under the lake
   b. Evaporation
   c. Inflow of cold water
   d. The sun heating the water

# Comparing the Frameworks

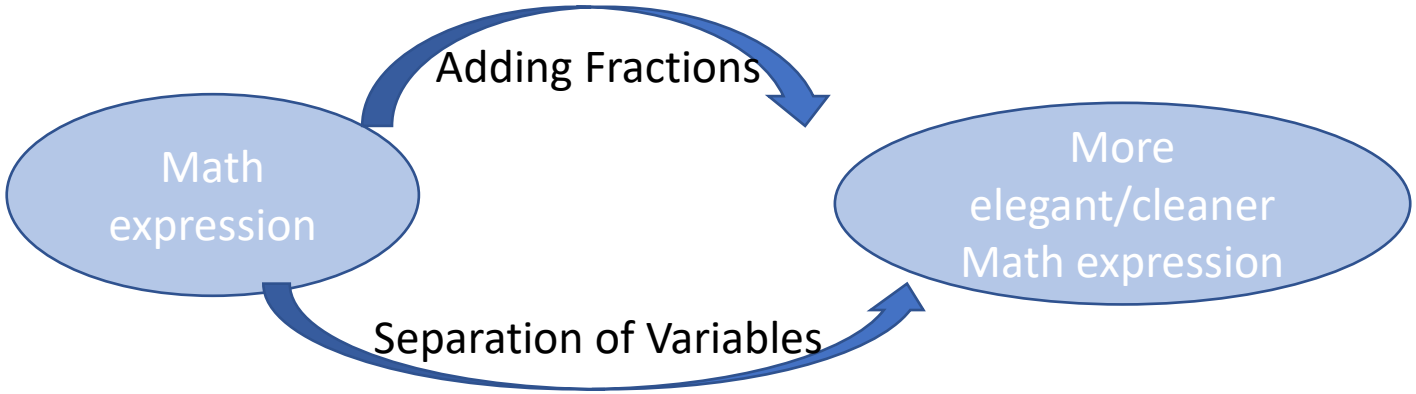
Parallel

- Teaches how to make an ODE model "from scratch"
- Teaches the flexibility of model making


Breadth

- Makes Homework more "mathy"
- Demonstrates breadth of ODE applications

# Why Teach Deep Modeling



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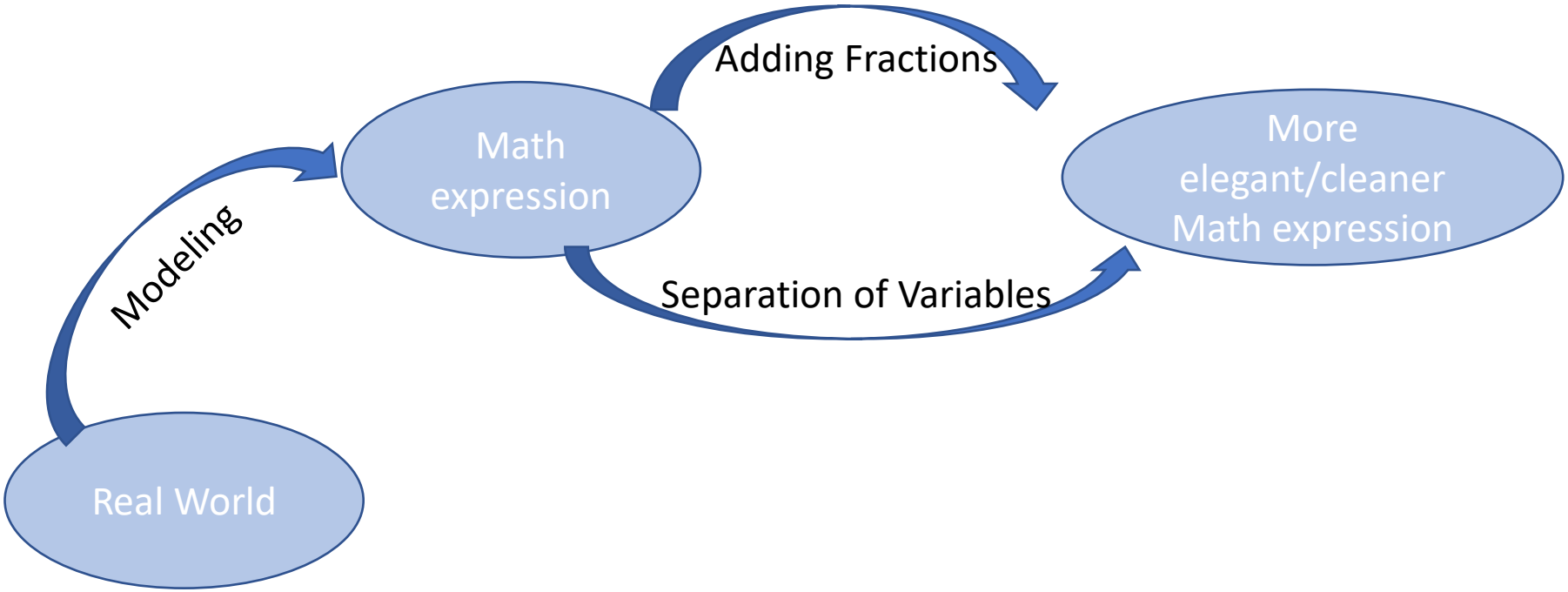

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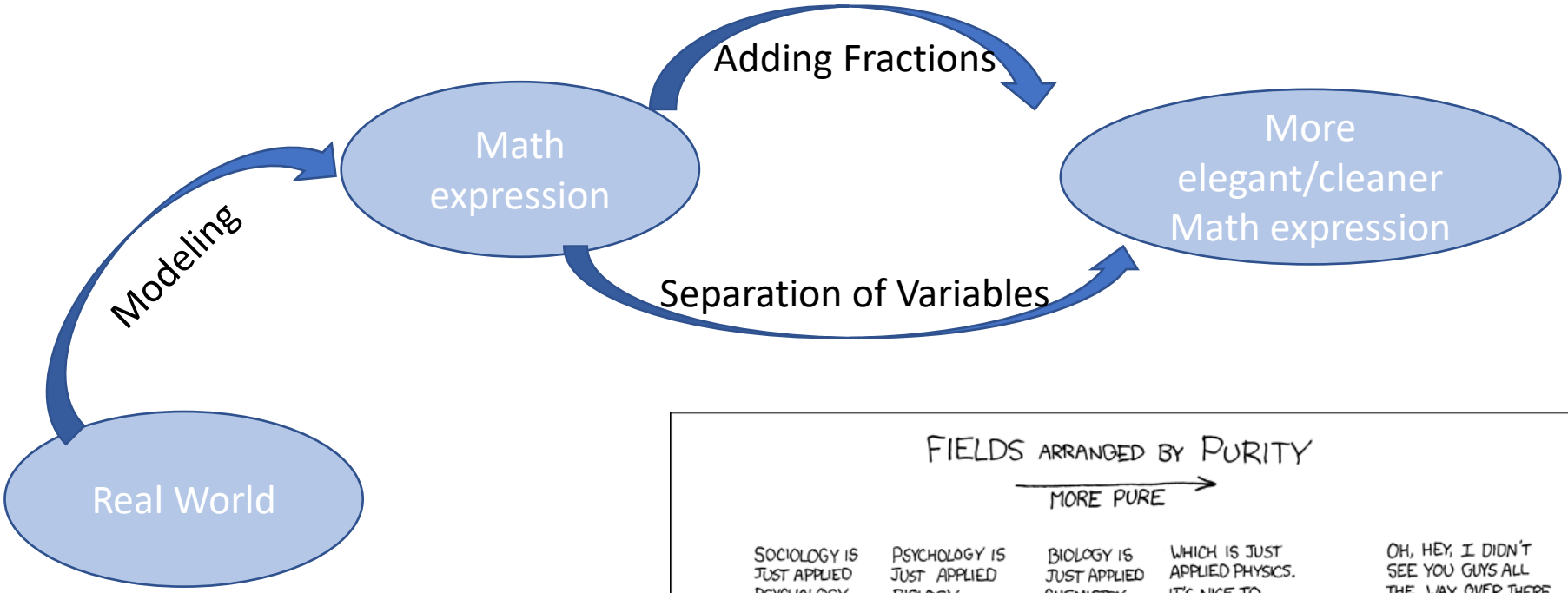
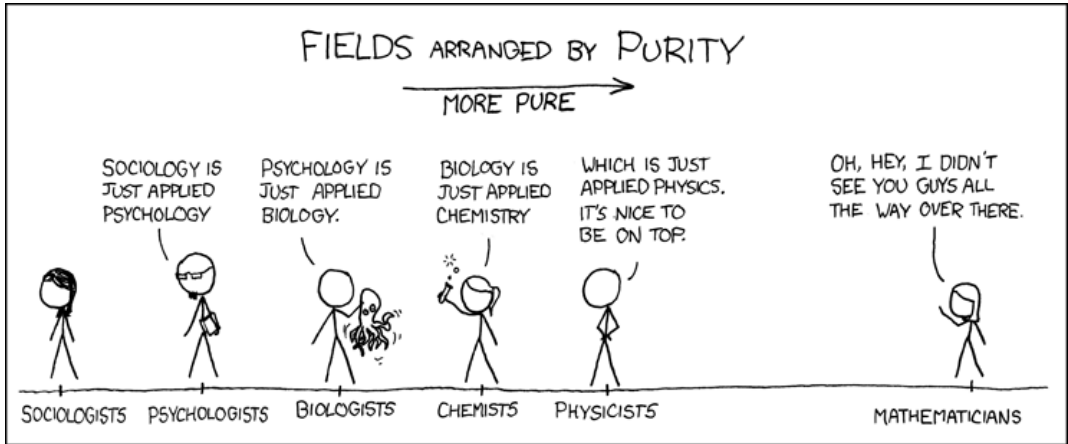
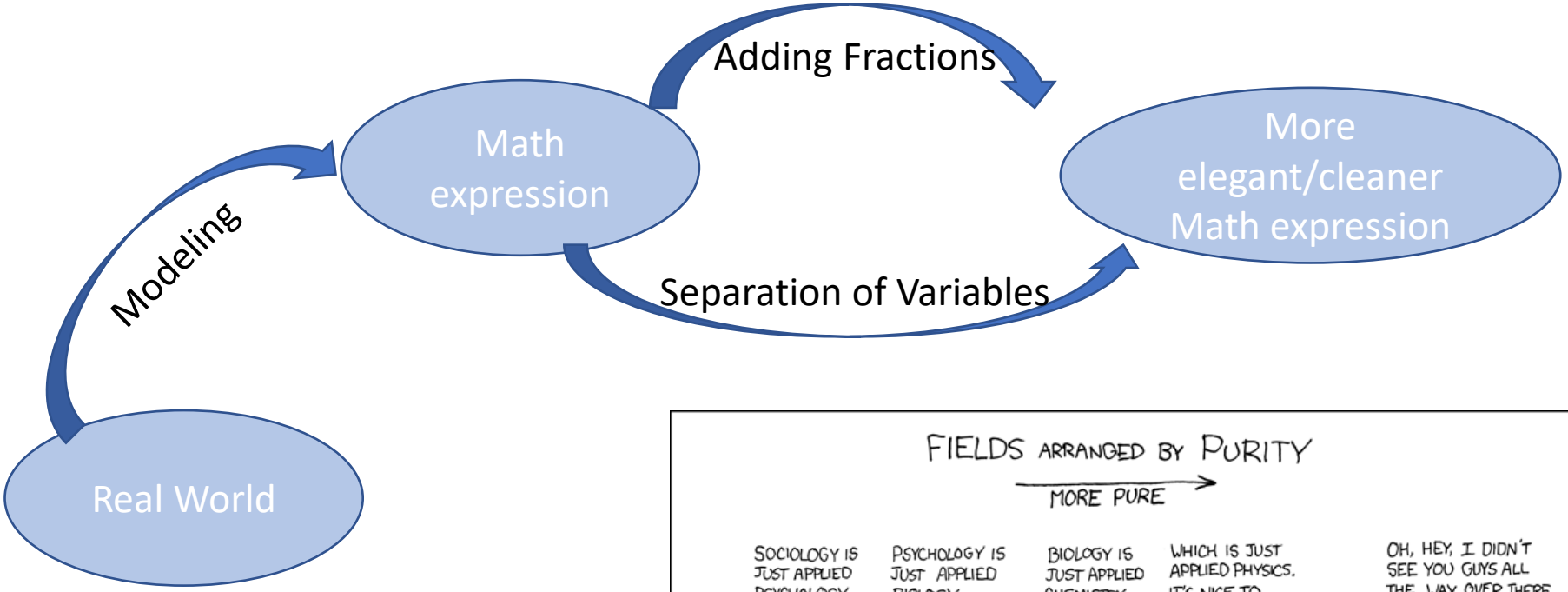
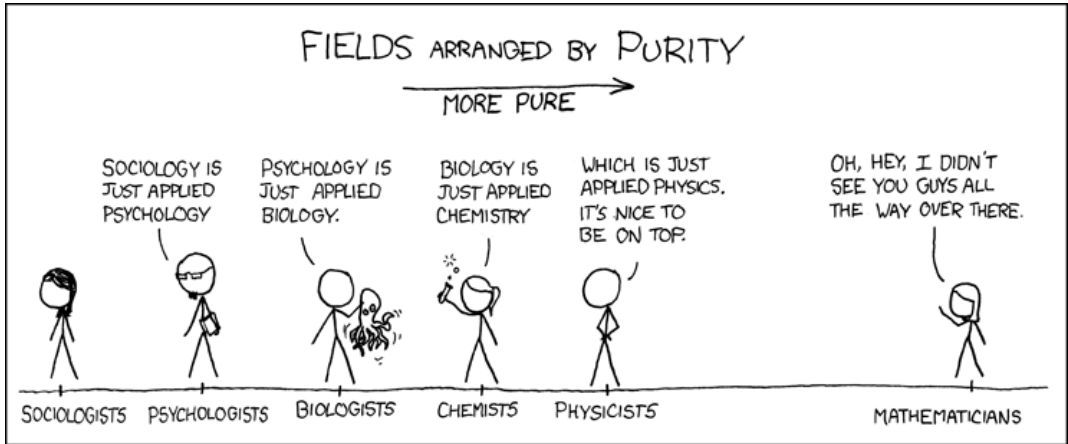


# Why Teach Deep Modeling

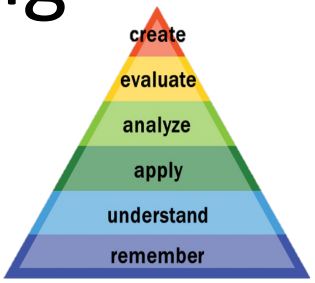
Just as students take our Math skills to other classes, so they should take our modeling skills



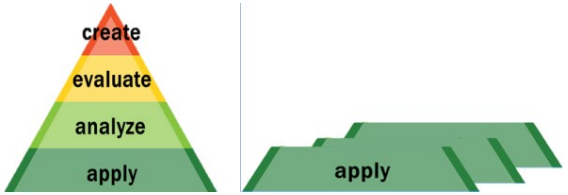



# A modeling in depth approach allows students to reach peak modeling

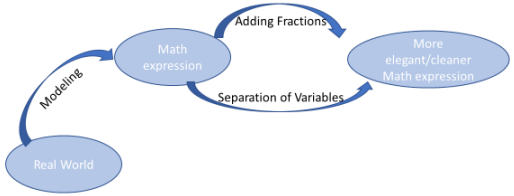
- Used Bloom's taxonomy to analyze better teaching of modeling




- Compared a traditional modeling approach to a parallel modeling approach




- Abstracting the idea of modeling
- Modeling is often considered trivial to the more mathematically experienced



# References

Slide 3-5 Photos: Benjamin Potter completes cadet training at West Point (poconorecord.com)

https://midhudsonnews.com/wp-content/uploads/2021/08/Beast-Barracks-2021-1024x683.jpg

Bloom's taxonomy pyramid taken from: Bloom's Taxonomy | Center for Teaching | Vanderbilt University

Slide 16: I used the following website to help derive Torricelli's Law: Torricelli's Law | Neutrium

Slide 19: Conical Tank picture taken from Zill Differential Equations with Boundary-Value Problems Chapter 3.2 page 103

Slide 24: XKCD comic **https://xkcd.com/435/**