Bifurcations

Professor Donnay
Bryn Mawr College

Team Number: _____ Group Members:

1. _____
2. _____
3. _____
4. _____

Goal: To study the effect of various levels of harvesting on the steady state fish population

$$\frac{dP}{dt} = P(1-P) - C = -P^2 + P - C$$

gotten from including a fishing term in the logistic equation.
The value of C denotes how many fish per year are caught. When C = 0, no fish are caught and the model reduces to the standard logistic model

$$\frac{dP}{dt} = rP(1 - \frac{P}{N})$$

with r=1 and N =1.

Directions: Each person in the group will examine the equation for a different value of C. The group will break up and we will re-group people from other groups together who are working with the same C values.

Person 1: C = 0, C=.25

Person 2: C = .05, C=.4

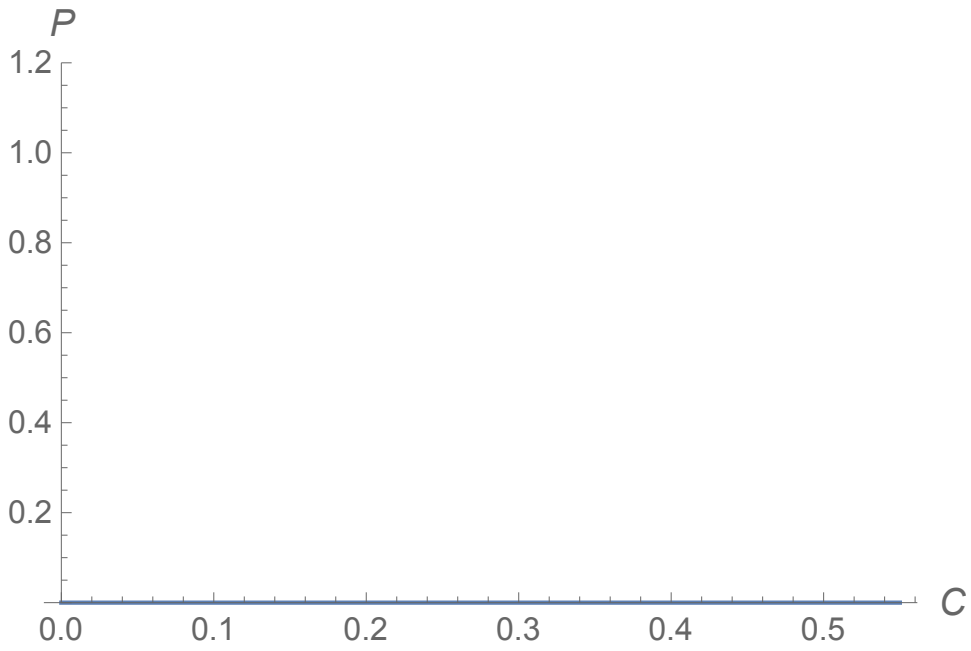
Person 3: C = 1/8, C = .3

Person 4: C = .2, C= .5

For each c value, draw the graph of the slope function $f(P, C) = -P^2 + P - C$ and then draw the phase line diagram. Use the worksheets at the end of this packet.

When you return to your group, you will combine all your results into one picture. For each c value, you will plot the phase line for that c value as a vertical line in the (c, y) plane. The union of all these phase line pictures will help us understand how the fish populations vary as we vary the harvesting amount C.

Go in order of increasing c values. One at a time, each person adds their phase line picture to the overall picture. Explain what equilibrium points you calculated and describe their type.



Group Questions:

1. "Connect the dots" that represent the equilibrium values so you have a (rough) estimate of the equilibrium values for ***all*** C values, rather than just the few you have calculated exactly. You have created the **bifurcation diagram** for the system.

2. What happens to the fish population over the long term if the fishing level C is high?

3. What happens to the fish population over the long term if the fishing level C is low to moderate?

4. Recommendation: The government Department of Fisheries in partnership with business and environmental groups has set up a Fisheries Commission. The goal of the Commission is to set a quota for how many fish can be caught each year. As an expert on the mathematics of fish populations, you are being called to testify before the Fisheries Commission. Based on your mathematical analysis, what recommendation will you give to the Commission?

5. What is the critical fishing level below which the fish population will survive and above which the fish population will die out? This value is called a **tipping point** (or in math terminology, a **bifurcation point).**

6. For $$\frac{dP}{dt} = P(1-P) - C = -P^2 + P - C = f(P)$$

a. Determine the equilibrium points. Your answer should be a function involving C.

b. Using the formulas for the equilibrium points, give the formulas for the two curves that you drew in the bifurcation diagram.

Name: Team Number: _____ Person Number: _____

First C Value: C = _____ $f(P) = - P^2 + P - $ ____

Equilibrium points:

Make a sketch of f(P):

Draw the Phase line and next to it draw the solution space (t, y) with several different solutions.

Second C Value: C = _____ $f(P) = - P^2 + P - \_\_\_\_$

Equilibrium points:

Make a sketch of f(P):

Draw the Phase line and next to it draw the solution space (t, y) with several different solutions.