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How to kill several birds with one stone: Construction of a multi-criteria trajectory that is both locally and globally optimal

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Ordinary differential equations (ODE) courses are usually focused on the initial value problem (IVP) and introduction to the boundary value problem (BVP).

However, it is important to provide a broader view of the ODEs including algebraic equations and finding optimal solutions.

To achieve this goal, we suggest presenting an ODE as a means of construction of a trajectory given its properties at a point.

We demonstrate this approach using an economic problem.

Currently, all countries aim to reach a sustainable balance between economic development and environmental protection. They try to increase output, decrease energy consumption, and mitigate CO2 emissions simultaneously.

These aims are ostensibly at odds. The continuing growth of the human population and rising standards of living are contingent on higher energy consumption, and thus, greater CO2 emissions.

One of the modern economic theories claims that the only way is *degrowth*, that is decreasing consumption.

We show that a system of differential-algebraic differential equations (SDAE) allows us to resolve the problem without degrowth.

The suggested approach is multi-objective locally-and-globally optimal economic restructuring.

Publications Vaninsky (2018-2023) provide details.

We begin with the construction of three gradient vectors:

Gross Domestic Product (GDP) gradient.
Energy Consumption antigradient.
CO2 emissions antigradient.

All three are taken as their projections on an algebraic constraint stating that the total of all sectoral shares equals to one.

Feasible directions of local change lie in the cone and make acute angles with all given vectors. They are Pareto optimal

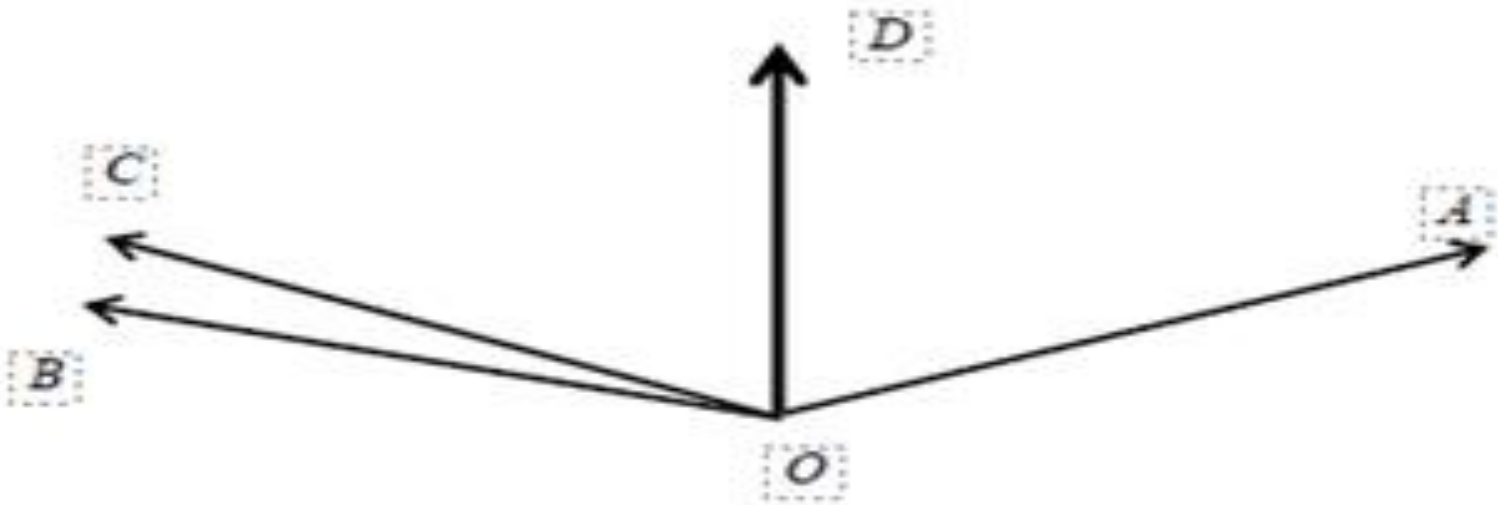
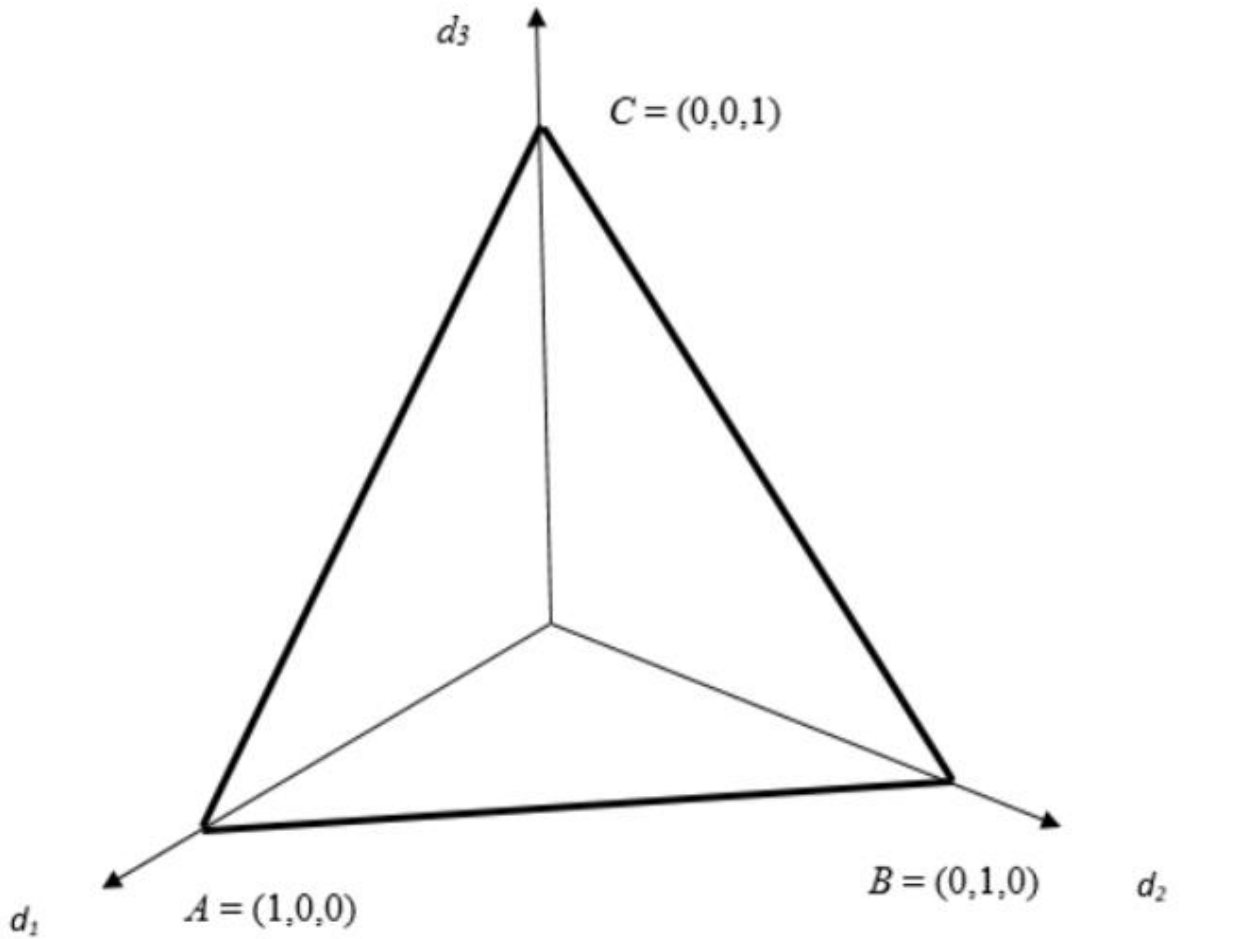


Fig. 1 Geometry of optimal economic restructuring.
$OA = d_{GDP}$, $OB = d_e$, $OC = d_c$; OD is a vector of optimal economic restructuring. Vector OD – the vector of optimal economic restructuring – constitutes acute angles with each of the vectors OA, OB, and OC.

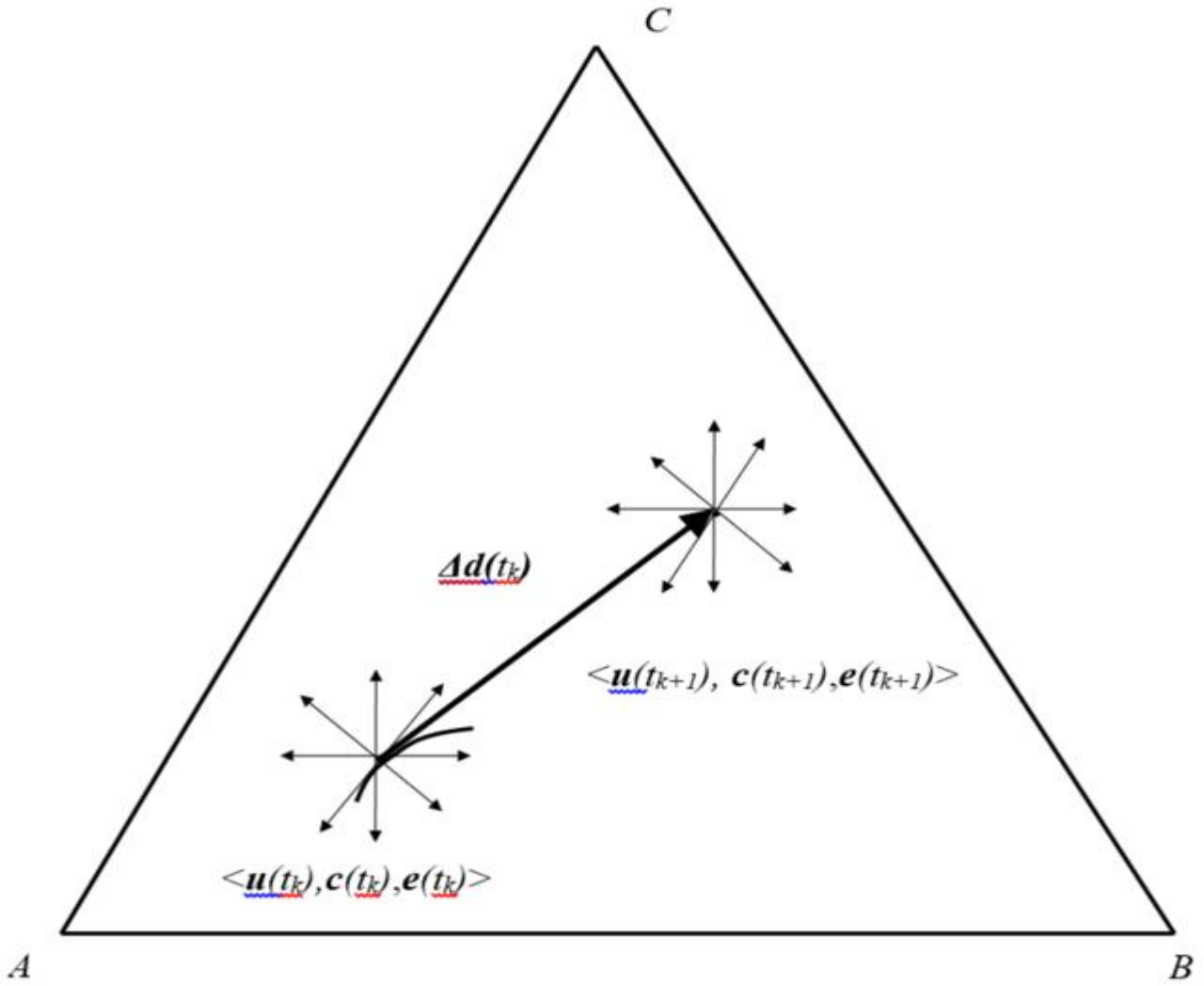
Source: Vaninsky (2018b)

Feasible direction vectors lie in *n*-dimensional simplex



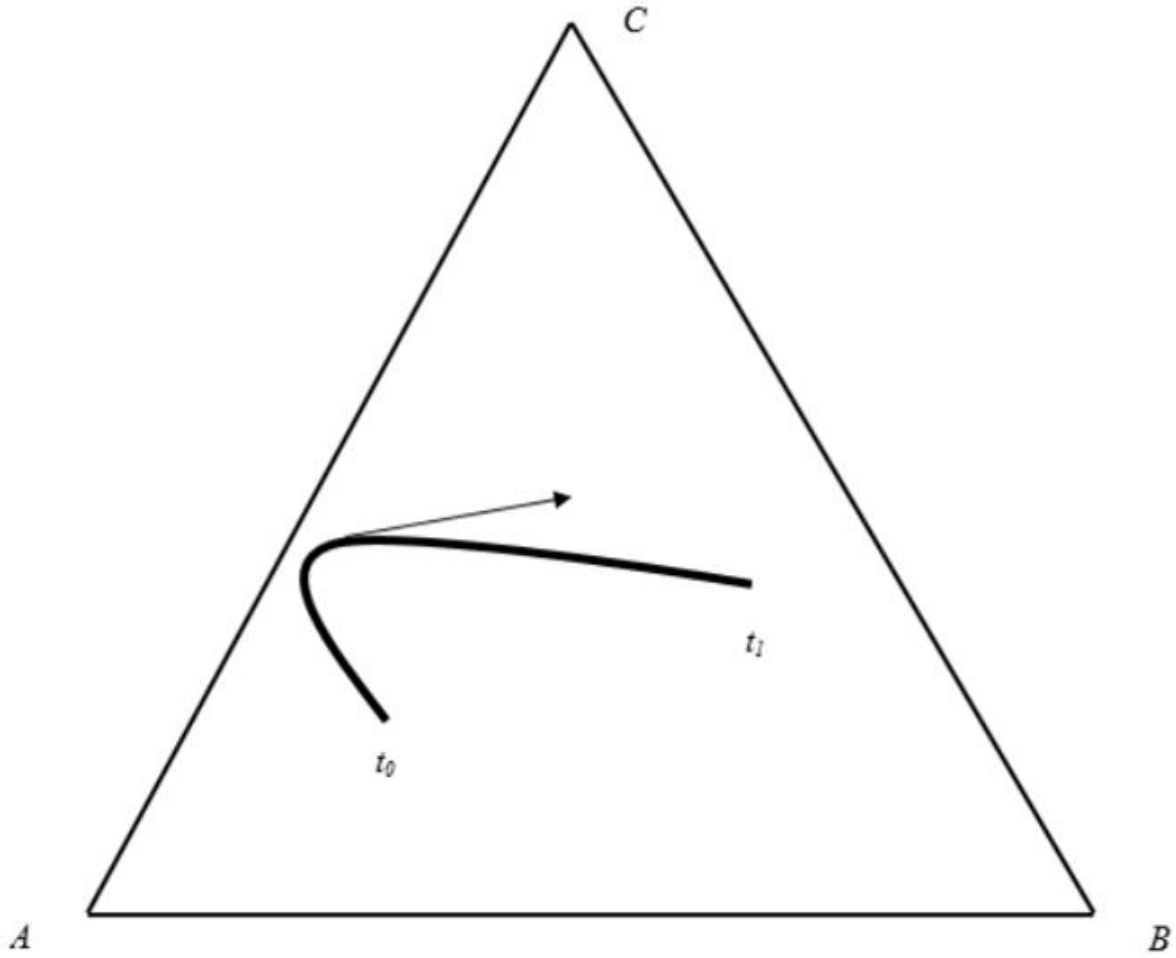
Source: Vaninsky (2018a)

Locally optimal direction of the movement depends on the assigned weights



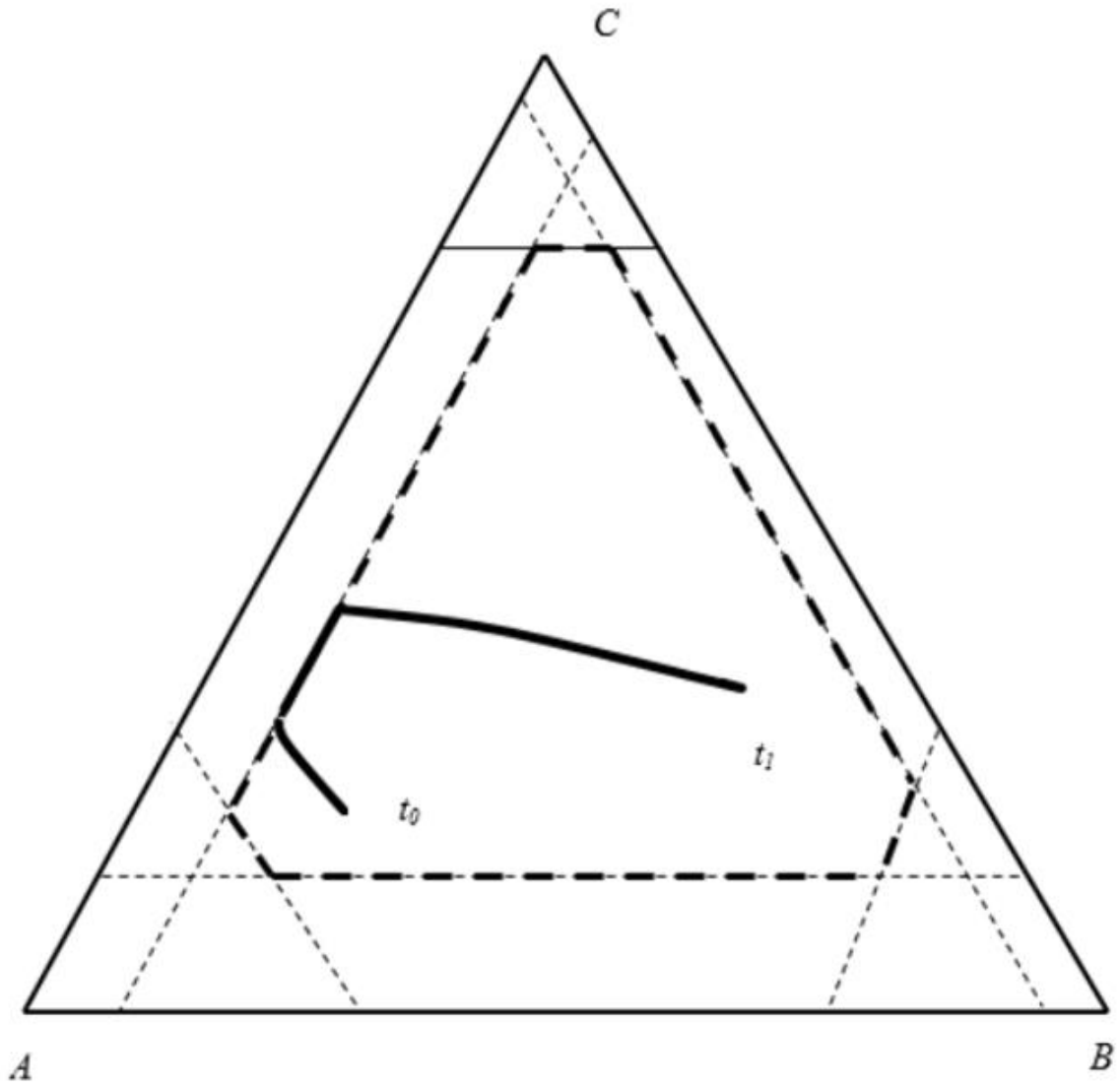
Source: Vaninsky (2018a)

The system develops following the locally optimal direction vectors



Source: Vaninsky (2018a)

Additional constraints may be imposed on the boundary values



Source: Vaninsky (2018a)

Adjustment of the local-optimization parameters allows us to achieve global optimum.

The Nelder-Mead algorithm is a convenient tool of the global optimization in these settings.

In this example, it is revealed how the short-term politically-motivated goals are related to the long-term nationally important goals of sustainability.

Presentation of the differential equations in this way increases students' understanding of the subject and their involvement in active study and research projects.

AI may be helpful in the formulation of long-term objectives.

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Thank you!

Questions ?

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