Difficult Choices in Planning to Teach Introductory Differential Equations

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Springfield College Differential Equations

- A gentle introduction for future teachers
- Traditional and non-technology based
- Has worked well- but should we change ?
- Perhaps other instructors have faced a similar conundrum

Subsequent slides describe the Springfield College course…

Course Summary:

This course serves as a gentle introduction to the fundamental concepts of elementary differential equations. Students will learn the basic theory behind ordinary differential equations and explore simple techniques for solving them. Emphasis will be placed on developing mathematical intuition and problem-solving skills in a technology-free environment.

Prerequisites:

Basic understanding of calculus (differentiation and integration)
Calculus I and II ; Calculus II preferred

Textbook:

- "Elementary Differential Equations”

by Earl D. Rainville and Phillip E. Bedient

Or other choices

Unit 1 : Introduction to Differential Equations

- Definition of a differential equation
- Classification of differential equations: ordinary vs. partial
- Understanding the order of a differential equation
- Basic terminology and notation

Unit 2 : First-Order Differential Equations

- Separable equations
- Linear equations
- Exact equations
- Applications to simple real-world problems

Unit 3: Linear Second-Order Differential Equations

- Homogeneous equations with constant coefficients
- Method of undetermined coefficients

Unit 4 : Introduction to Higher-Order Differential Equations

- General theory of nth-order linear equations
- Homogeneous equations with constant coefficients
- Introduction to characteristic equations

$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$

A Possible Application: Logistic Equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

$$\frac{dP}{P(1 - \frac{P}{K})} = r\,dt$$

A Possible Application: Logistic Equation

$$\frac{dP}{P(1-\frac{P}{K})} = r\,dt$$

$$\int \frac{dP}{P(1-\frac{P}{K})} = \int r\,dt$$

$$\int \left(\frac{1}{P} + \frac{1}{K-P}\right) dP = \int r\,dt$$

A Possible Application: Logistic Equation

$$\int \left(\frac{1}{P} + \frac{1}{K-P}\right) dP = \int r dt$$

$$\ln|P| - \ln|K-P| = rt + C$$

$$\ln\left|\frac{P}{K-P}\right| = rt + C$$

A Possible Application: Logistic Equation

$$\ln\left|\frac{P}{K-P}\right| = rt + C$$

$$\left|\frac{P}{K-P}\right| = e^{rt+C}$$

$$P = \frac{KC_1e^{rt}}{1+C_1e^{rt}}$$

Should Springfield College change to a more modelling and technology-intensive course ?

Teaching a differential equations course with an emphasis on modeling can be highly beneficial for several reasons…

Should Springfield College change to a more modelling and technology-intensive course ?

- 1. Real-world relevance: Differential equations are fundamental in modeling various phenomena across science, engineering, economics, and more. Teaching with a focus on modeling allows students to see the practical applications of the mathematical concepts they're learning.

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- 2. Interdisciplinary understanding: Modeling with differential equations bridges the gap between mathematics and other disciplines. Students gain insight into how mathematical tools can be used to understand and solve problems in diverse fields, fostering interdisciplinary thinking.

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- 3. Critical thinking skills: Modeling requires students to analyze complex systems, identify relevant variables, and formulate mathematical descriptions. This process encourages critical thinking and problem-solving skills, which are valuable beyond mathematics.

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- 4. Creativity and innovation: Modeling often involves simplifying real-world scenarios to create tractable mathematical representations. Encouraging students to engage in this process fosters creativity and innovation as they explore different approaches to representing and solving problems.

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- 5. Preparation for research and industry: Many careers in STEM fields involve modeling and solving differential equations. Teaching with a modeling emphasis equips students with skills and knowledge that are directly applicable in research, industry, and academia.

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- 6. Connection to modern technology: Advances in computational tools have made it easier to solve and simulate differential equations. Teaching modeling with differential equations can include exposure to computational methods, preparing students to leverage modern technology in their future work.

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- 7. Long-term understanding: By focusing on modeling, students may develop a deeper understanding of the underlying principles behind differential equations. This understanding can help them retain knowledge and apply it effectively in future studies and careers.

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- 8. Engagement and motivation: Modeling presents students with tangible, real-world problems, which can increase engagement and motivation. Seeing the practical relevance of the material can inspire students to delve deeper into the subject and pursue further studies in mathematics or related fields.

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9. Preparation for advanced study: Differential equations with modeling serves as a foundation for more advanced courses in applied mathematics, engineering, physics, and other fields. By teaching modeling early on, students are better prepared for advanced studies that build upon these concepts.

MIT’s Undergraduate Differential Equations Classes

“Shoot for the moon. Even if you miss, you’ll land among the stars”

SC cannot realistically plan to emulate MIT’s curriculum.

Still, it’s interesting to look at the “gold standard”.

MIT's Undergraduate Differential Equations Classes

18.03 Differential Equations

Study of differential equations, including modeling physical systems. Solution of first-order ODEs by analytical, graphical, and numerical methods. Linear ODEs with constant coefficients. Complex numbers and exponentials. Inhomogeneous equations: polynomial, sinusoidal, and exponential inputs. Oscillations, damping, resonance. Fourier series. Matrices, eigenvalues, eigenvectors, diagonalization. First order linear systems: normal modes, matrix exponentials, variation of parameters. Heat equation, wave equation. Nonlinear autonomous systems: critical point analysis, phase plane diagrams.

18.032 Differential Equations

Covers much of the same material as 18.03 with more emphasis on theory. The point of view is rigorous and results are proven. Local existence and uniqueness of solutions.

MIT's Undergraduate Differential Equations Classes

18.152 Introduction to Partial Differential Equations

Introduces three main types of partial differential equations: diffusion, elliptic, and hyperbolic. Includes mathematical tools, real-world examples and applications, such as the Black-Scholes equation, the European options problem, water waves, scalar conservation laws, first order equations and traffic problems.

MIT’s Undergraduate Differential Equations Classes

18.303 Linear Partial Differential Equations: Analysis and Numerics

Provides students with the basic analytical and computational tools of linear partial differential equations (PDEs) for practical applications in science and engineering, including heat/diffusion, wave, and Poisson equations. Analytics emphasize the viewpoint of linear algebra and the analogy with finite matrix problems. Studies operator adjoints and eigenproblems, series solutions, Green's functions, and separation of variables. Numerics focus on finite-difference and finite-element techniques to reduce PDEs to matrix problems, including stability and convergence analysis and implicit/explicit timestepping. Some programming required for homework and final project.

MIT's Undergraduate Differential Equations Classes

18.306 Advanced Partial Differential Equations with Applications

Concepts and techniques for partial differential equations, especially nonlinear. Diffusion, dispersion and other phenomena. Initial and boundary value problems. Normal mode analysis, Green's functions, and transforms. Conservation laws, kinematic waves, hyperbolic equations, characteristics shocks, simple waves. Geometrical optics, caustics. Free-boundary problems. Dimensional analysis. Singular perturbation, boundary layers, homogenization. Variational methods. Solitons. Applications from fluid dynamics, materials science, optics, traffic flow, etc.

Thanks for attending this presentation:

"Difficult Choices in Planning to Teach Introductory Differential Equations".

Questions?