

Three Modeling Projects in Differential Equations

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Outline

- ▶ Three modeling scenarios from SIMIODE
 - ▶ Mode of administration
 - ▶ Selection criteria
 - ▶ Adaptation
- ▶ Rubric used for grading
- ▶ Students' feedback on course evaluation

Mode of Administration & Selection Criteria

- ▶ Scenarios from SIMIODE were chosen and assigned as projects (group/individual).
- ▶ Selection Criteria
 - ▶ Fun
 - ▶ Doable = non-intimidating
 - ▶ Different tools
 - ▶ Different disciplines
 - ▶ Easy to grade

Three Modeling Scenarios

- ▶ Mixing It Up - Separable + Chemistry
- ▶ Modeling Populations with Death and Immigration Using m&ms - Linear + Life Sciences
- ▶ Modeling ICU Spread - Separable + Health Sciences

Mixing It Up

► Mixing It Up – Separable + Chemistry

- SIMIODE has it in 3 parts; I only used one = simple first project
- At time $t = 0$ a tank contains $Q(0) = 4$ lb of salt dissolved in 100 gal of water. Water containing 0.25 lb of salt per gallon is entering the tank at a rate of 3 gal/min, and the well-stirred solution leaves the tank at the same rate.
 - a) Build a differential equation for the amount of salt, $Q(t)$, in lb in the tank at time t in min.
Hint: Keep track of the amount of salt that enters and exits the tank per minute.
 - b) Find an expression for the amount of salt, $Q(t)$, in lb in the tank at time t in min and plot $Q(t)$ vs. t over time interval $[0, 200]$ min.
 - c) **Draw a graph of $Q(t)$ on Desmos and export it to your solution document**
 - d) Determine when the amount of salt doubles from the original amount in the tank
 - e) Determine when the amount of salt in the tank is 20 lb.
 - f) Determine when the amount of salt in the tank is 30 lb.
 - g) Determine the maximum amount of salt in the tank and when it occurs.
 - h) Describe the long term behavior of the amount of salt in the tank using accompanying plots to support your description.

Modeling Populations with Death and Immigration Using m&ms

Part I

- ▶ Students are given a bag of M&Ms, a plastic cup and a paper plate.
- ▶ They collect data by shaking the M&Ms in the cup onto the plate.
- ▶ M&Ms with m side up, die; set aside. M&Ms lying on the blank side live; they go back to the cup for the next iteration.
- ▶ Students are asked to answer some questions prepping them to the analysis:
 - ▶ Describe what you think will happen in each toss.
 - ▶ What are your assumptions.
- ▶ They then run the experiment tabulating the results.
- ▶ They are then asked to compare the results of the experiment with their answers to the first set of questions.

Modeling Populations with Death and Immigration Using m&ms Part II

- ▶ Students are walked through building a model for the first simulation (death only).
- ▶ They are asked to compare the results of their theoretical model with the actual data.
- ▶ Introduced to the concept of how to determine how fit a model is.

Modeling Populations with Death and Immigration Using m&ms Part III

- ▶ Students are walked through similar steps as in the previous parts but now with an immigration factor: adding 10 immigrants to the number of survivors in each iteration

ICU Spread

- ▶ Dataset of voluntary nonprofit hospitals in the US with ICUs during the period 1958 – 1974.
- ▶ Students are asked to
 - ▶ graph the dataset on Desmos
 - ▶ Analyze the graph
 - ▶ Search the internet to learn about logistic differential equations
 - ▶ Give an argument why a given logistic differential equation is a good candidate for modeling the given dataset
 - ▶ Write the IVP that represents the modeling scenario
 - ▶ Solve the differential equation and do further analysis

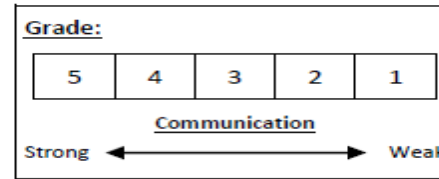
Rubric Used

Differential Equations Projects Grading Rubric

USAFA In Effect in 2009

Overview:

Your work will be graded holistically using a simple rubric (see right) consisting of a 5 point scale from 1 (for failing work¹) through 5 (for “outstanding” work). The grade will account for execution, communication, and correctness. To earn a 5, your work must be *well-executed*, *well-communicated*, and *essentially correct* as defined below. The rubric may also provide feedback (via a sliding scale) on the quality of your communication to help you improve in this area.



Definitions:

Well-Executed

- Applies a **strategy** that makes sense for the given question
- Applies appropriate mathematical concepts and processes
- Does not introduce superfluous material
- Technology is used appropriately
- Work is logical and includes a sanity check of the final answer

Well Communicated

- **Readable:** Work *stands alone* (retains context) and is neat and professional in appearance
- **Organized:** Provides a *clear logical flow* from beginning to end
- Provides sufficient supporting detail and explanation throughout
- Work is free from grammatical errors
- Mathematical composition, terminology, and notation is correct
- Results and/or conclusions are clearly annotated

Essentially Correct

- **Precision:** Performs mathematical operations correctly and derives the correct results
- Uses an appropriate degree of accuracy
- Draws correct inferences from graphical or numerical data
- Any computational or algebraic errors are trivial and isolated
- Correct units are used

Score Descriptors:

5	4	3	2	1
Outstanding (“A”)	Good (“B”)	Average (“C”)	Deficient (“D”)	Failing (“F”)
Well-executed, well-communicated, essentially correct	Generally well-executed but may have minor communication flaws or some math errors	Adequately executed but with some non-trivial errors or inconsistent communication	Flawed execution possibly with non-trivial errors or poor communication	Unsatisfactory execution and/or communication with fundamental errors

Students' Feedback

- ▶ “The examples of application to the real world using models” was an aspect of the course that students found engaging for their learning.
- ▶ “...The projects that we did at the beginning of the semester helped establish more meaning to the computations, so maybe spend a little more time in class explaining why the methods (of solutions) are useful and in what contexts we might see them again.”
- ▶ “I really enjoyed the projects. I struggled to care about many of the types of Des we had to solve until I saw how they were used in an applied area of math or science.”
- ▶ “I wish we had more project grades.”

Q & A

THANK YOU

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