Three Modeling Projects in Differential Equations

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Caroline Maher-Boulis

Outline

- Three modeling scenarios from SIMIODE
  - Mode of administration
  - Selection criteria
  - Adaptation
- Rubric used for grading
- Students' feedback on course evaluation

Mode of Administration & Selection Criteria

- Scenarios from SIMIODE were chosen and assigned as projects (group/individual).
- Selection Criteria
  - Fun
  - Doable = non-intimidating
  - Different tools
  - Different disciplines
  - Easy to grade

Three Modeling Scenarios

- Mixing It Up – Separable + Chemistry
- Modeling Populations with Death and Immigration Using m&ms – Linear + Life Sciences
- Modeling ICU Spread – Separable + Health Sciences

Mixing It Up

- Mixing It Up – Separable + Chemistry
  - SIMIODE has it in 3 parts; I only used one = simple first project
- At time t = 0 a tank contains Q(0) = 4 lb of salt dissolved in 100 gal of water. Water containing 0.25 lb of salt per gallon is entering the tank at a rate of 3 gal/min, and the well-stirred solution leaves the tank at the same rate.

a) Build a differential equation for the amount of salt, Q(t), in lb in the tank at time t in min.
Hint: Keep track of the amount of salt that enters and exits the tank per minute.
b) Find an expression for the amount of salt, Q(t), in lb in the tank at time t in min and plot Q(t) vs. t over time interval [0, 200] min.
c) Draw a graph of Q(t) on Desmos and export it to your solution document
d) Determine when the amount of salt doubles from the original amount in the tank
e) Determine when the amount of salt in the tank is 20 lb.
f) Determine when the amount of salt in the tank is 30 lb.
g) Determine the maximum amount of salt in the tank and when it occurs.
h) Describe the long term behavior of the amount of salt in the tank using accompanying plots to support your description.

Modeling Populations with Death and Immigration Using m&ms Part I

- Students are given a bag of M&Ms, a plastic cup and a paper plate.
- They collect data by shaking the M&Ms in the cup onto the plate.
- M&Ms with m side up, die; set aside. M&Ms lying on the blank side live; they go back to the cup for the next iteration.
- Students are asked to answer some questions prepping them to the analysis:
  - Describe what you think will happen in each toss.
  - What are your assumptions.
- They then run the experiment tabulating the results.
- They are then asked to compare the results of the experiment with their answers to the first set of questions.

Modeling Populations with Death and Immigration Using m&ms
Part II

- Students are walked through building a model for the first simulation (death only).
- They are asked to compare the results of their theoretical model with the actual data.
- Introduced to the concept of how to determine how fit a model is.

Modeling Populations with Death and Immigration Using m&ms
Part III

- Students are walked through similar steps as in the previous parts but now with an immigration factor: adding 10 immigrants to the number of survivors in each iteration

ICU Spread

- Dataset of voluntary nonprofit hospitals in the US with ICUs during the period 1958 – 1974.
- Students are asked to
  - graph the dataset on Desmos
  - Analyze the graph
  - Search the internet to learn about logistic differential equations
  - Give an argument why a given logistic differential equation is a good candidate for modeling the given dataset
  - Write the IVP that represents the modeling scenario
  - Solve the differential equation and do further analysis

Rubric Used

Differential Equations Projects Grading Rubric

USAFA In Effect in 2009

Overview:

Your work will be graded holistically using a simple rubric (see right) consisting of a 5 point scale from 1 (for failing work[1]) through 5 (for "outstanding" work). The grade will account for execution, communication, and correctness. To earn a 5, your work must be *well-executed*, *well-communicated*, and *essentially correct* as defined below. The rubric may also provide feedback (via a sliding scale) on the quality of your communication to help you improve in this area.

Grade:

5	4	3	2	1

Communication

Strong ← → Weak

Definitions:

Well-Executed

- Applies a **strategy** that makes sense for the given question
- Applies appropriate mathematical concepts and processes
- Does not introduce superfluous material
- Technology is used appropriately
- Work is logical and includes a sanity check of the final answer

Well Communicated

- **Readable:** Work *stands alone* (retains context) and is neat and professional in appearance
- **Organized:** Provides a *clear logical* flow from beginning to end
- Provides sufficient supporting detail and explanation throughout
- Work is free from grammatical errors
- Mathematical composition, terminology, and notation is correct
- Results and/or conclusions are clearly annotated

Essentially Correct

- **Precision:** Performs mathematical operations correctly and derives the correct results
- Uses an appropriate degree of accuracy
- Draws correct inferences from graphical or numerical data
- Any computational or algebraic errors are trivial and isolated
- Correct units are used

Score Descriptors:

5	4	3	2	1
Outstanding ("A")	Good ("B")	Average ("C")	Deficient ("D")	Failing ("F")
Well-executed, well-communicated, essentially correct	Generally well-executed but may have minor communication flaws or some math errors	Adequately executed but with some non-trivial errors or inconsistent communication	Flawed execution possibly with non-trivial errors or poor communication	Unsatisfactory execution and/or communication with fundamental errors

Students' Feedback

- "The examples of application to the real world using models" was an aspect of the course that students found engaging for their learning.
- "…The projects that we did at the beginning of the semester helped establish more meaning to the computations, so maybe spend a little more time in class explaining why the methods (of solutions) are useful and in what contexts we might see them again."
- "I really enjoyed the projects. I struggled to care about many of the types of Des we had to solve until I saw how they were used in an applied area of math or science."
- "I wish we had more project grades."

Q & A

THANK YOU

cmaherboulis@leeuniversity.edu