

# Strategies for Active Learning in a Math Class

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**snəwəyət̓ leləm̓.**

THE COLLEGE OF HIGHER LEARNING.

**Langara.**

THE COLLEGE OF HIGHER LEARNING.

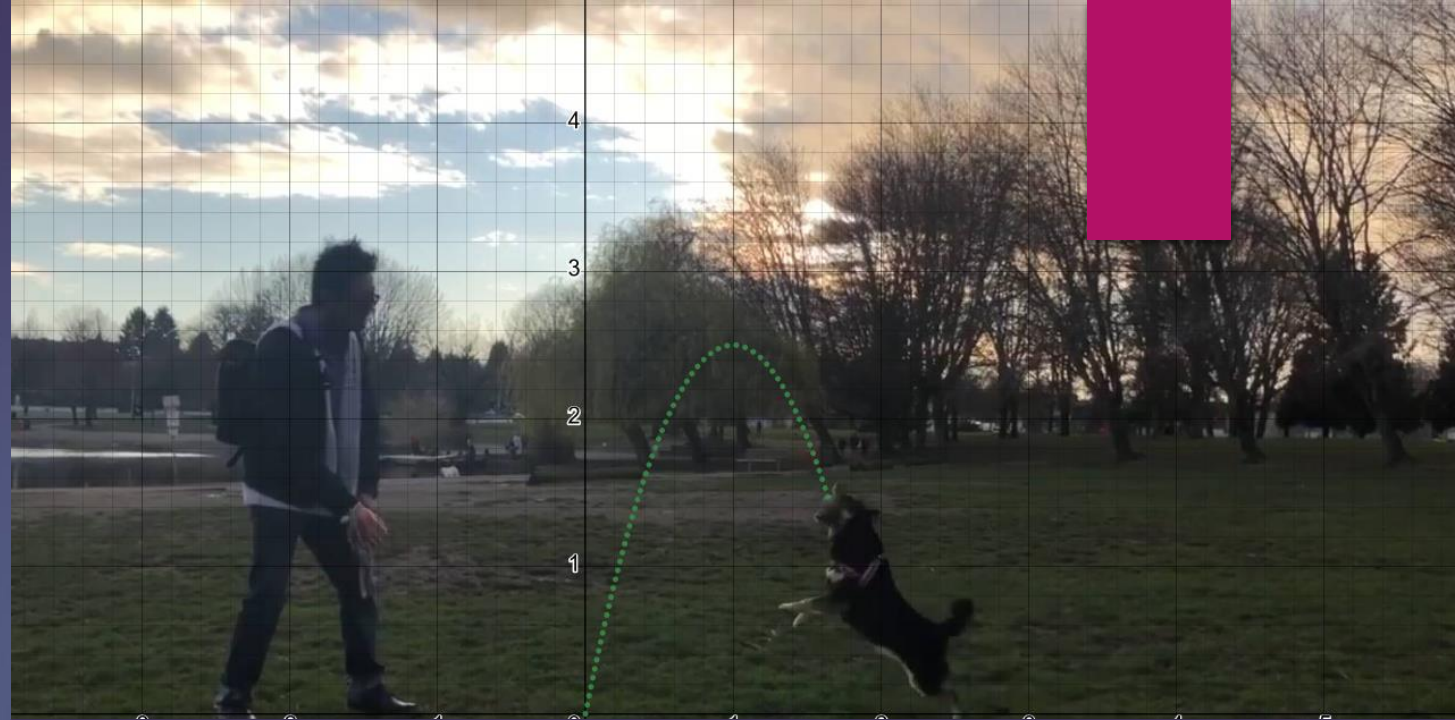


# Overview and objectives

- ▶ Introduction
- ▶ Moon crater example
- ▶ Some theory about “active learning”
- ▶ Quiz!
- ▶ Catalogue of strategies and examples

# Introduction

- ▶ Hi. I'm Jeremy
- ▶ Positions:
  - ▶ Langara College, math instructor
  - ▶ Simon Fraser University, applied math PhD student
  - ▶ St. John Ambulance, Division 998 training team
- ▶ Hobbies: dogs, sports, gardening, coffee, math, Pokémon, Magic, first aid





# Dark side of the moon

- ▶ Due to *tidal locking*, we only see one side of the moon from earth, the near side
- ▶ Question: The far side from earth has many craters more craters than the near side. Why?

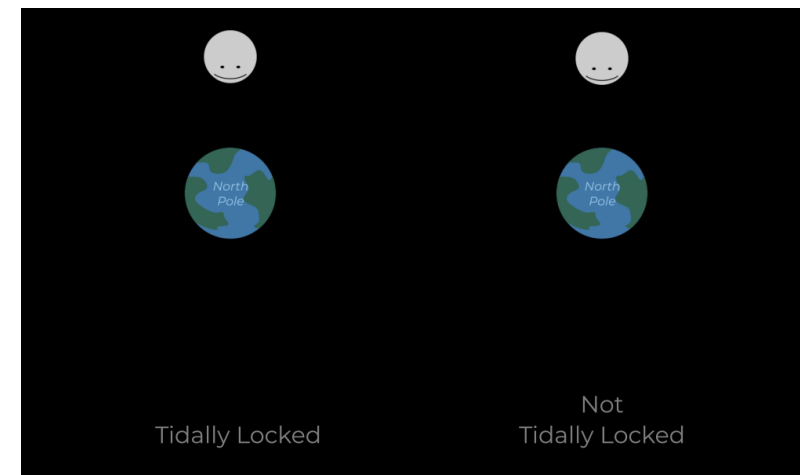


Image and animation from Nasa

# Dark side of the moon

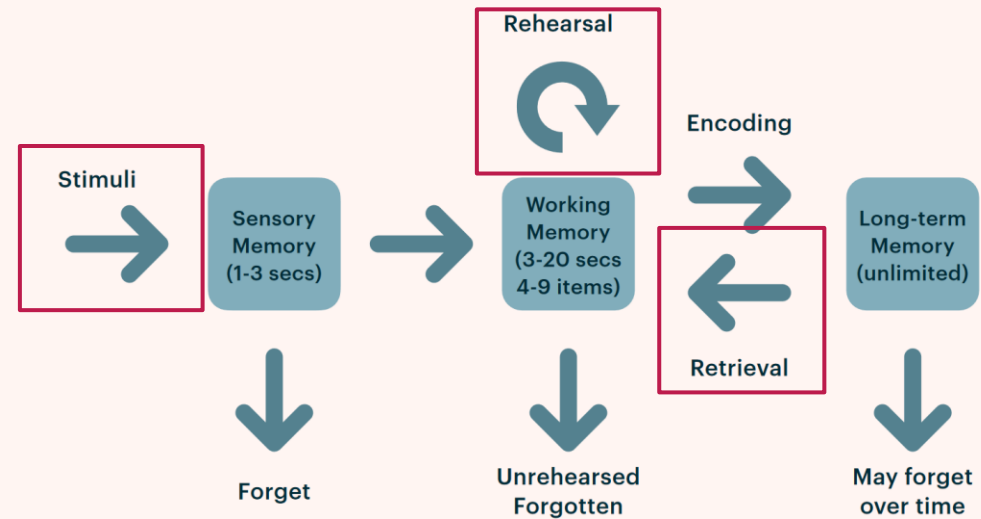
- ▶ Earth does *not* act as a “shield”
  - ▶ “Proven” using mathematical models and scientific computing
  - ▶ Earth blocks out less than 1% of moon’s night sky
  - ▶ Gravitational pull of earth may make more asteroids hit moon
- ▶ Fewer craters because lunar volcanic eruptions covered the craters on near side



To be continued...

# Atkinson-Shiffrin's Multi-Store Model

- ▶ Image taken from Anna Stokke
- ▶ "Learning is a change in long-term memory" – Kirschner, Sweller, Clark



Adapted from Atkinson, R.C. and Shiffrin, R.M. (1968). "Human memory: A Proposed System and its Control Processes". In Spence, K.W. and Spence, J.T. *The psychology of learning and motivation*, (Volume 2). New York: Academic Press. pp. 89-195.

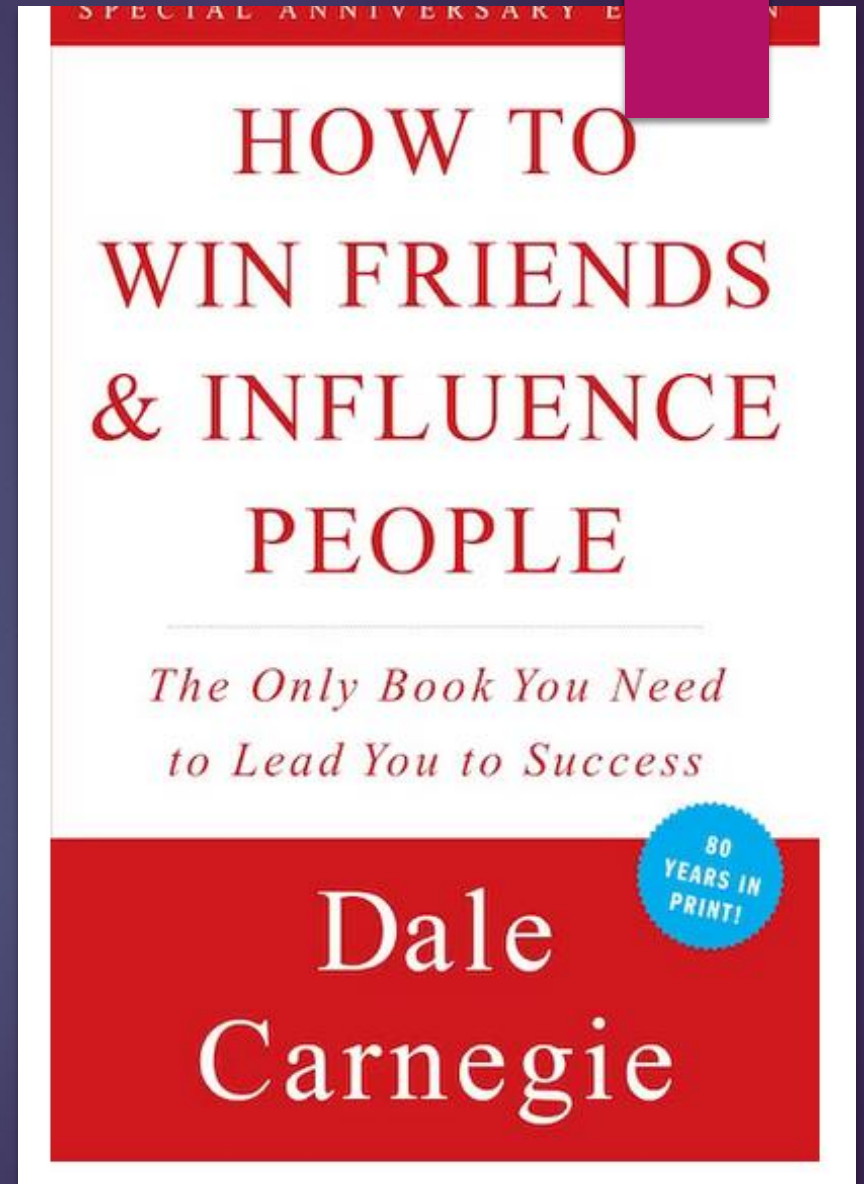
# Bonus: Influence

*“Personally I am very fond of strawberries and cream, but I have found ... fish prefer worms. So when I went fishing, ...I didn't bait the hook with strawberries and cream. Rather, I dangled a worm or grasshopper in front of the fish and said: ‘Wouldn't you like to have that?’*

*Why not use the same common sense when fishing for people?”*

- Dale Carnegie

I.e., to influence (manipulate, control) students, recognize their goals, and make their goals align with your goals



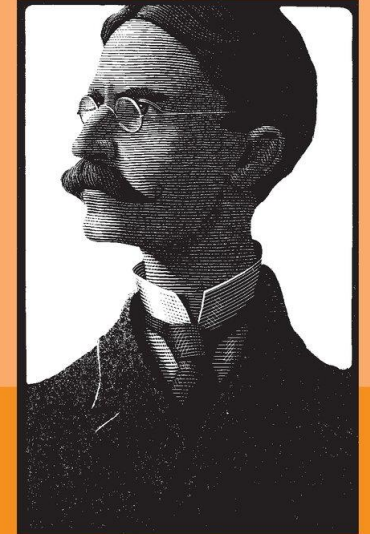


# What do students want?

- ▶ Dewey's *Fourfold Interests of Children*:
  1. conversation or communication
  2. inquiry, or finding out things
  3. making things, or construction
  4. artistic expression
- ▶ Assumption (1): Uni-students want
  - ▶ a) good grades
  - ▶ b) utility of material [can apply lessons in math class outside of math class]
  - ▶ c) enjoyment / social

JOHN DEWEY

*The  
School  
and  
Society*

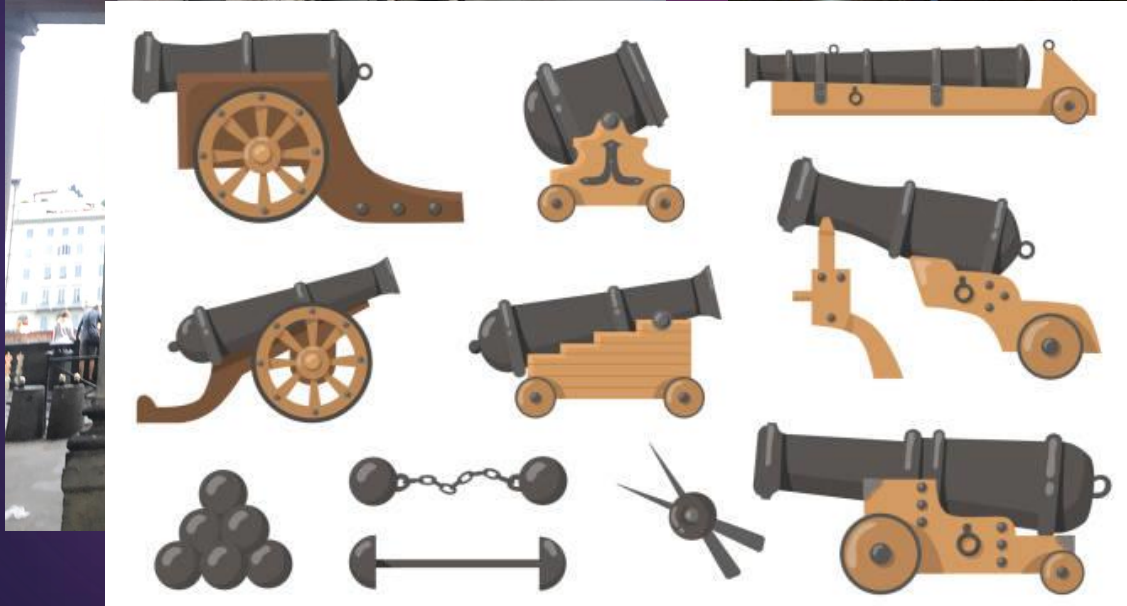
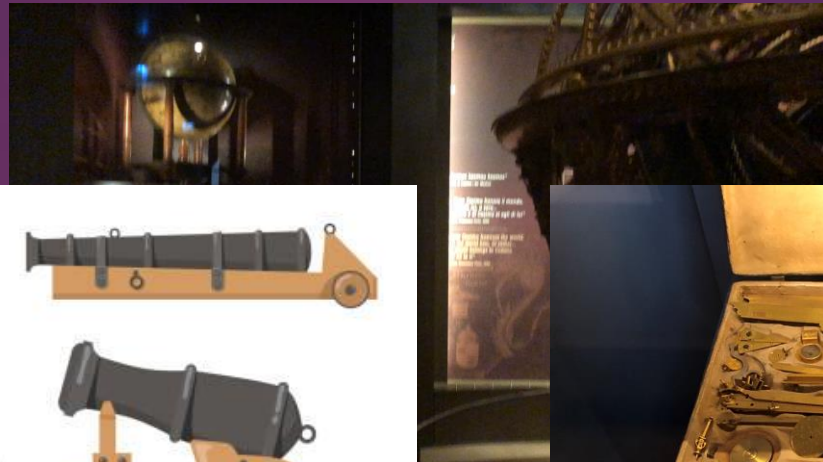


*The  
Child  
and the  
Curriculum*

An Expanded Edition with a  
New Introduction by Philip W. Jackson



# Museo Galileo at Florence, Italy



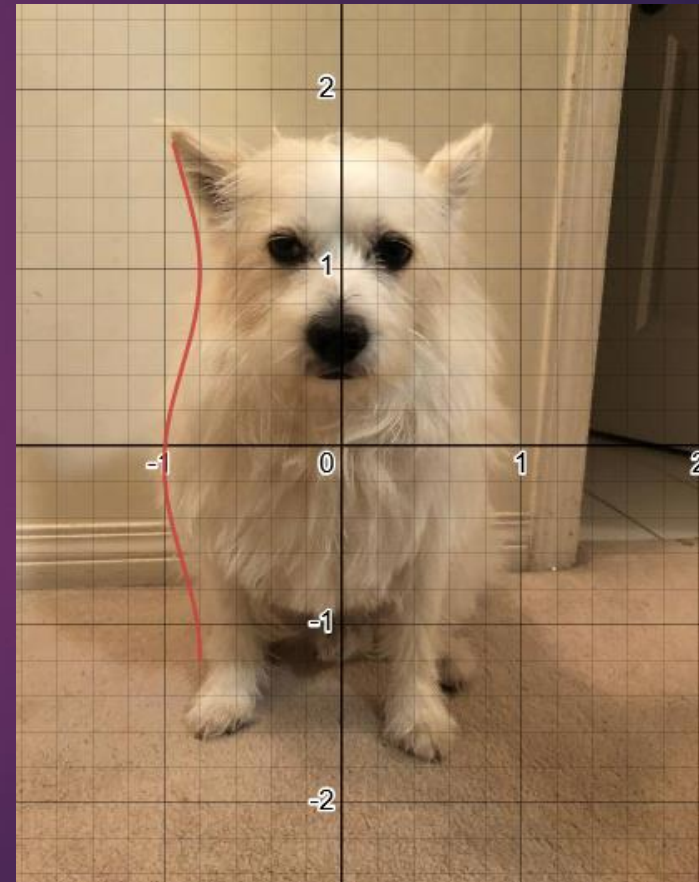
# Quiz

- ▶ Match following physical problems with the most appropriate math concept(s)
- ▶ Bonus: match the problem to the most appropriate scientist(s) too
- ▶ Bonus 2: what are typical units?
- ▶ Note: vertical and horizontal line test will fail

Problem	Math concept	Quantitative Scientist
1. Basic reproduction number of a pathogen	a) Arithmetic with negative numbers	i. Roy Anderson
2. Investments and credit card debt	b) Differential equations	ii. Marie Curie
3. Ionic bonds of elements	c) Exponential functions	iii. Leonhard Euler
4. Radioactive decay	d) Solids of revolution	iv. Joseph Fourier
5. Tidal locking	e) Trigonometric functions	v. Johannes Kepler
6. Volume of my dog		vi. Dr. Dog McRuffson
		vii. John Napier
		viii. Sir Isaac Newton

# Volume of my dog: Benji of revolution

- ▶ Solid of revolution
- ▶ Dr. Dog McRuffson and Johannes Kepler; litres
- ▶ Let students explore transformations [[DESMOS](#)]
- ▶ Assuming Benji is rotationally symmetric, I calculated he is 7.2L.  
Question: How can we sanity check this answer?





## Basic reproduction number

- ▶ Exponential functions, differential equations
- ▶ Roy Anderson; infections/infection (dimensionless)

## Mortgages and credit card debt

- ▶ Exponential functions
- ▶ John Napier, Euler; %/year

## Radioactive decay

- ▶ Exponential functions, differential equations
- ▶ Marie Curie; becquerel (Bq) or curie (Ci) or "disintegrations / second"

"Mathematics is the art of giving the same name to different things." - Henri Poincaré

11. **Example**  
interest

3. **Motivating problem.** You lend Jeremy \$1 on January 1. You are a loan shark and charge Jeremy 100% interest, to be paid on December 31.

- (a) How much money will Jeremy owe you on December 31? *per annum*  
(b) You are greedy and want to make more money off Jeremy. *legally* How do you do this?

MATH 1152

CHAPTER 6

13. **Example. Radiocarbon Dating Egyptian Mummies.** (Numbers have been simplified in this historically significant example.) In 1949, Willard Libby found that the tombs of two Egyptian kings, Zoser and Sneferu, had about 55% of its remaining radiocarbon 14 isotopes. Assuming that the half-life of radiocarbon is 5568 years, estimate the age of the Egyptian tombs. Compare the estimate to the archaeologically estimated age of  $2675 \pm 75$  BCE.

$$N(t) = N_0 e^{kt}$$

"half-life is 5568"

"in 1949, 55% remains"

"estimate age of mummies"

quantify goal & givens

$N$ : remaining # isotopes [%]

$t$ : time after mummies died [years]

( $t$ : time after 1949

or time after 0 CE)

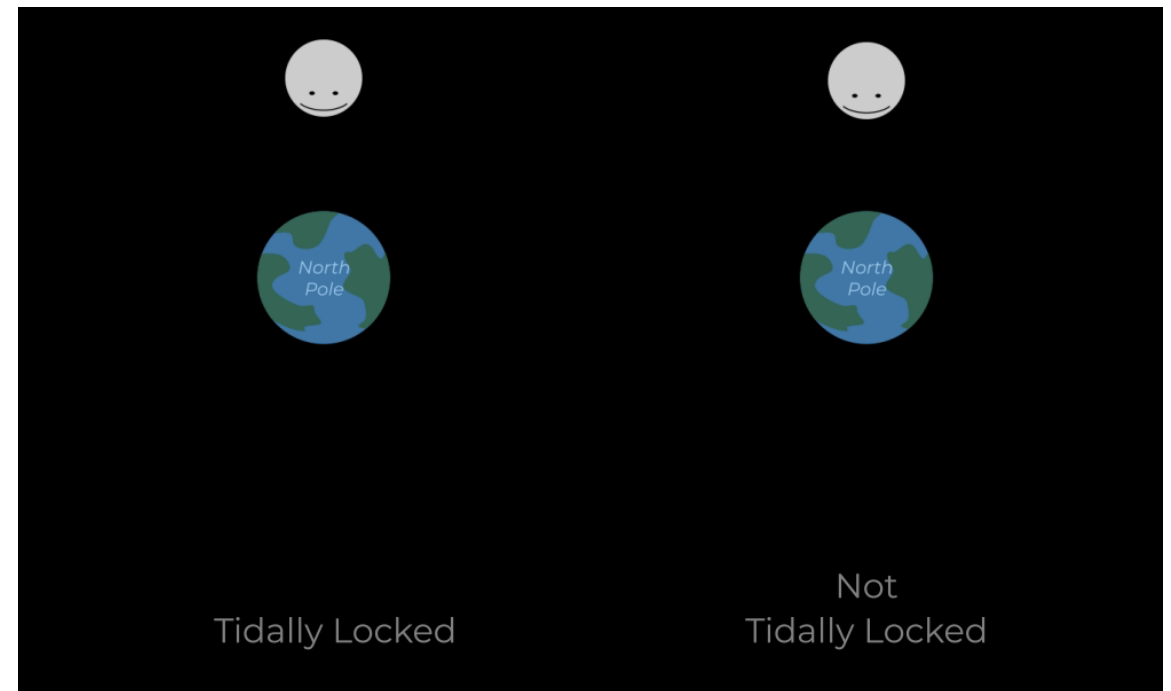
→ mummies died at  $t=0$





# Think-[pair]-share

- ▶ Goal: create a mathematical model to simulate *tidal locking*.
- ▶ Exercise 1: brainstorm [mathematical] models that describe any aspect of celestial motions.
- ▶ Exercise 2: create a math question based on what you produced in Exercise 1.
- ▶ Exercise 3: develop some hints or questions to drive discussion / thinking.
- ▶ Hint: think about astronomical numbers, then use it in an equation or function



## Tidal Locking

1. **Problem.** Describe the direction the earth is facing  $\theta$  as a function of time  $t$ .

### Hints.

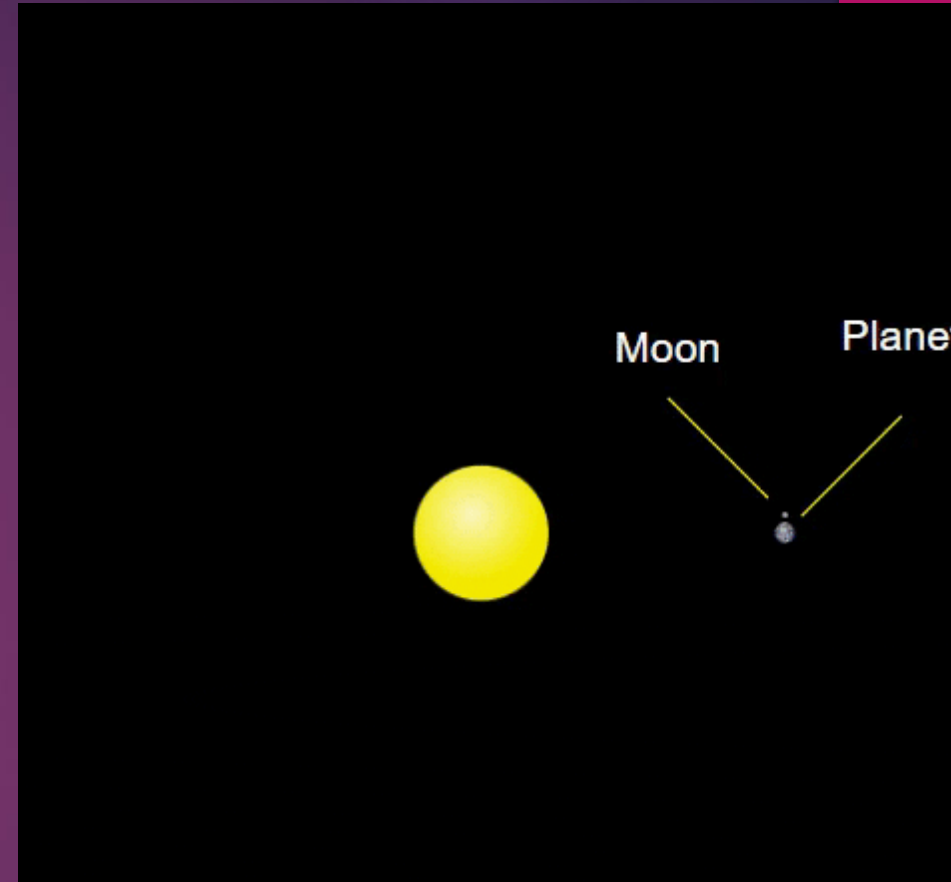
- (a) What are appropriate units of  $t$ ? How would your unit choice affect later calculations?
  - (b) What is the *prime meridian*? Why might this be useful to define  $\theta$ ?
  - (c) Let  $\theta = ct$ , then use the fact there are 24-hours in a day to find  $c$ .
2. **Problem.** Using the sun as the origin, describe where the earth is as a function of time.

### Hints.

- (a) What is an *astronomical unit*?
  - (b) How many days are there in a year?
  - (c) Assume earth has a circular orbit.
  - (d) Determine earth's elliptic orbit (hard!).
3. **Challenging problem.** Find functions that describe the moon's position and orientation.

### Hints.

- (a) Assume the origin is the earth. Then change the origin to the sun.
- (b) The moon orbits the earth every 27 days.
- (c) What is the rotational rate of the moon?
- (d) It takes 29.5 days to change from New Moon. How would you extend your model to incorporate this?



Next, catalogue of active learning strategies (and some theory).

# Scaffolding

- ▶ Create a gradient of difficulty among questions
  - ▶ resources to help below
- ▶ Lenticular cards from game design: “Lenticular cards are simple to the beginner but more advanced to the experienced player.” – Mark Rosewater, head designer of Magic the Gathering
- ▶ Opinion: I believe exams should be scaffolded, in the sense there is a mix of easy to hard questions.
- ▶ “Let the D student demonstrate they are a D student.”



# Polling and quizzing

- ▶ In-class polling (clicker questions)
  - ▶ Cornell's *GoodQuestions* <https://pi.math.cornell.edu/~GoodQuestions/materials.html>
  - ▶ McGill clicker database <https://www.mcgill.ca/tpulse/clickers/questions>
  - ▶ Simon Fraser University, for numerical analysis <https://summit.sfu.ca/item/20569>

# Solicit questions and feedback

- ▶ Questions?
- ▶ Feedback?





# Gameification

Game: guess the key concepts from each section.

- ▶ Collaborative cheat-sheet making.
- ▶ Direct students to course outline.
- ▶ Low-effort review session.

Game: guess the marking scheme.

- ▶ Have students split into groups of 3-4.
- ▶ Display an old midterm.
- ▶ Each group should guess the marking scheme and/or solve the exam.
- ▶ Award prizes each round of 2-3 questions.
- ▶ Memoryless property: erase scores each round so every team always has a chance to win
- ▶ Idea: if students know what we're expecting, they're more likely to include it in their solution

3a  
 3: Volume integral. 1 for "dy", 1 for pi \* (something^2 - something ^2), and 1 for correct inner radius / outer radius  
 1: switch from functions of x to functions of y  
 1: Attempt to find intersection pts by setting  $\sqrt{y} = 1/2 y$ , OR  $2x = x^2$

3a continued:  
 1: get a=0 and b=4 exactly  
 1: Final answer  $8/3 \pi$

[7] 3. (a) Consider the area bounded by  $y = 2x$  and  $y = x^2$ . If we rotate this about the  $y$ -axis, we get a bowl-like solid of revolution. Find its volume.

Find a & b:  
 $f(y) = \sqrt{y}$

$g(y) = \frac{1}{2}y$

hence

$$V = \int_a^b \pi (f(y)^2 - g(y)^2) dy$$

# Teach virtues alongside skills

Recall Assumption 1a: Uni-students want good grades.

- ▶ Code et al. (2016) developed MAPS “to characterize students’ perceptions of mathematics in authentic educational settings”
  - ▶ “Confidence subscale is the most highly correlated with course grades.”
  - ▶ “Persistence and Interest are also important predictors of course grades across all groups.”
- ▶ I explicitly teach introspection

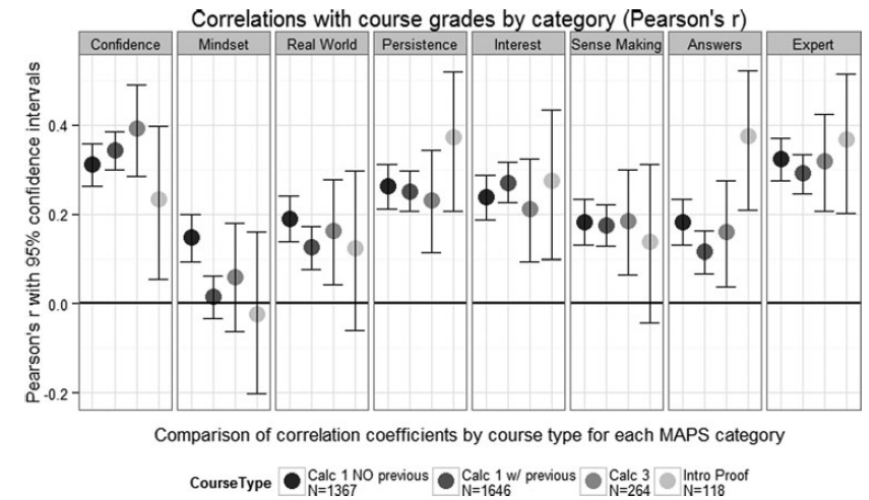
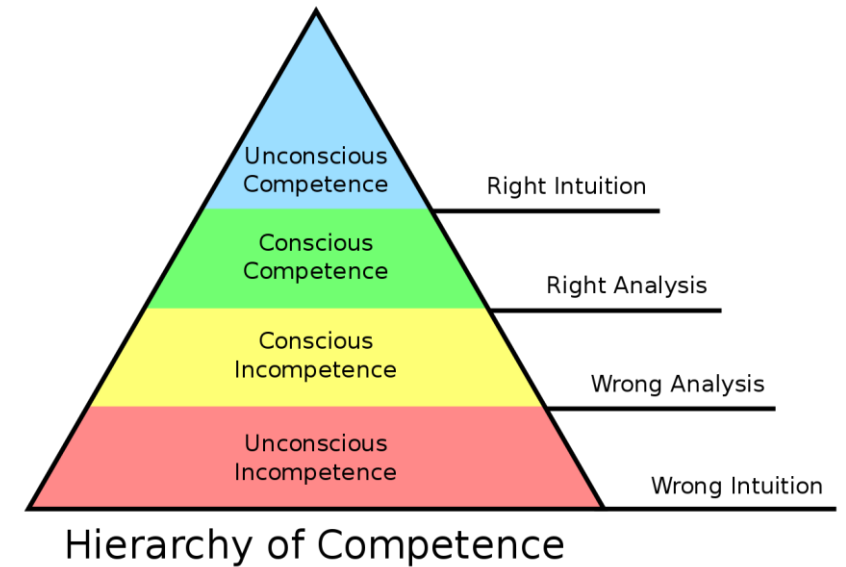


Figure 3. Correlations between MAPS sub scales and student course grades.





# Resources

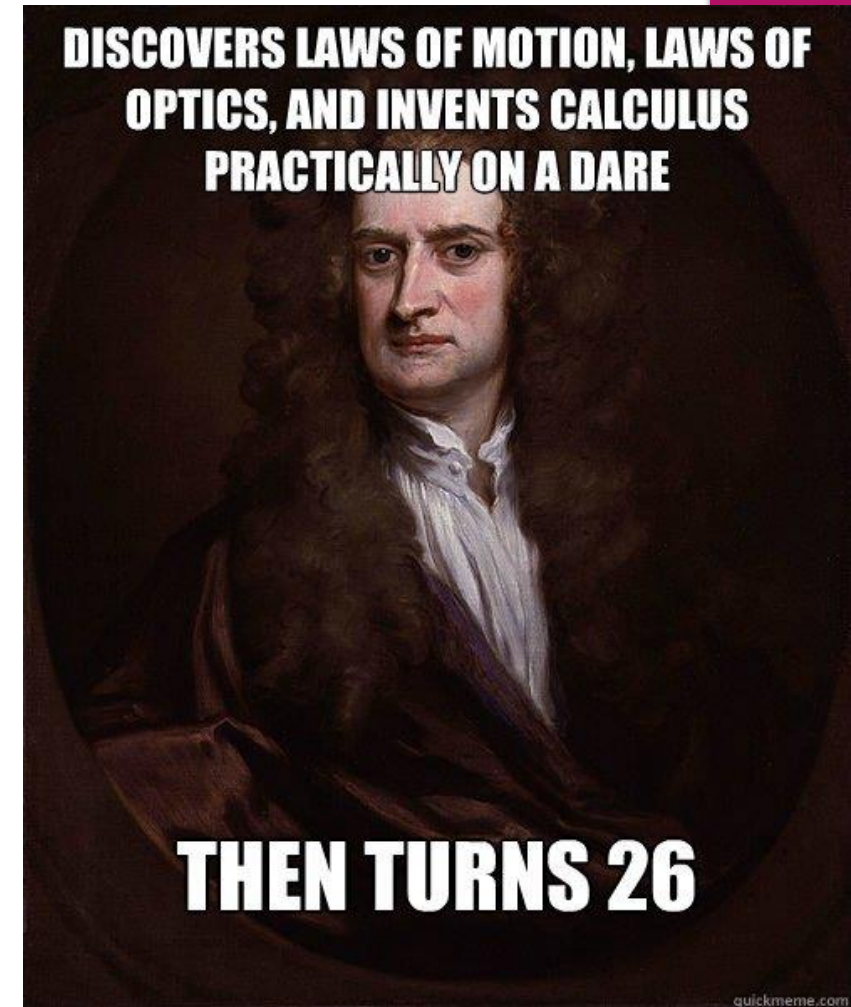
- ▶ Instructional Skills Workshop
- ▶ Peter Liljedahl's [\*Building Thinking Classrooms\*](#)
  - ▶ Furniture, when/where/how tasks given, forming groups...
  - ▶ Outreach idea: become a [\*MatheMagician\*](#) and workshop magic





# Summary

- ▶ Active learning strategies:
  - ▶ Think-pair-share
  - ▶ Open-ended, exploratory questions
  - ▶ Scaffolding (resources available)
  - ▶ Talking Circles
  - ▶ Quizzes
  - ▶ Gamification
- ▶ Some theory:
  - ▶ *Rehearsal and retrieval* key to learning
  - ▶ *Sensory memory* is first stage to learning
  - ▶ Foster virtues like confidence, persistence, interest, problem-solving, and introspection



# Conclusion

- ▶ Special thanks: CMS Online Education organizing committee
- ▶ Contact:
  - ▶ Email: [jeremychiu@langara.ca](mailto:jeremychiu@langara.ca)
  - ▶ IG: <https://www.instagram.com/terrier.benji/>

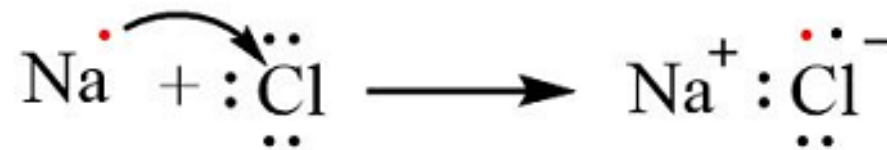
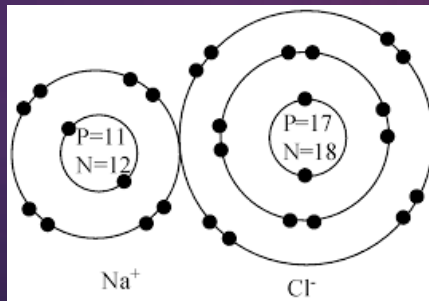


## Solicit more questions and feedback

- ▶ Questions?
- ▶ Feedback?

# Arithmetic with Negative Numbers

- ▶ Credit card debts
- ▶ John Napier; %/year or \$/month
- ▶ Ionic Bonds of Elements
- ▶ Marie Curie; charge (?)
- ▶ (trigonometric functions and Joseph Fourier ?)





# Tidal locking

## What:

- ▶ Trigonometric functions, “solid of revolution”
- ▶ Johannes Kepler and Joseph Fourier; radians and days

## How / why:

- ▶ Differential equations, ...
- ▶ Johannes Kepler and Sir Isaac Newton; ...

