SOLVING A FREE-UNDERDAMPED DIFFERENTIAL EQUATION AND USING ITS COMPUTER-GENERATED GRAPH TO EXPLAIN THE ACOUSTICAL RESPONSE OF A LOUDSPEAKER

by
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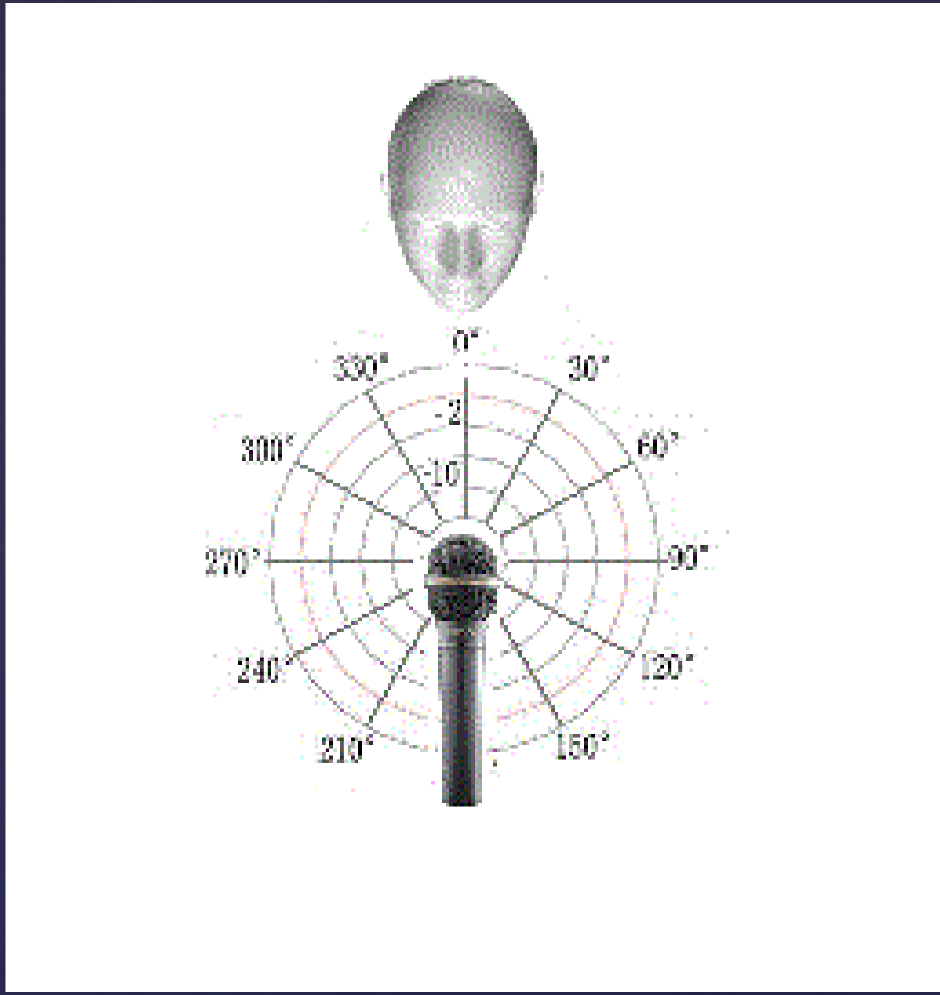
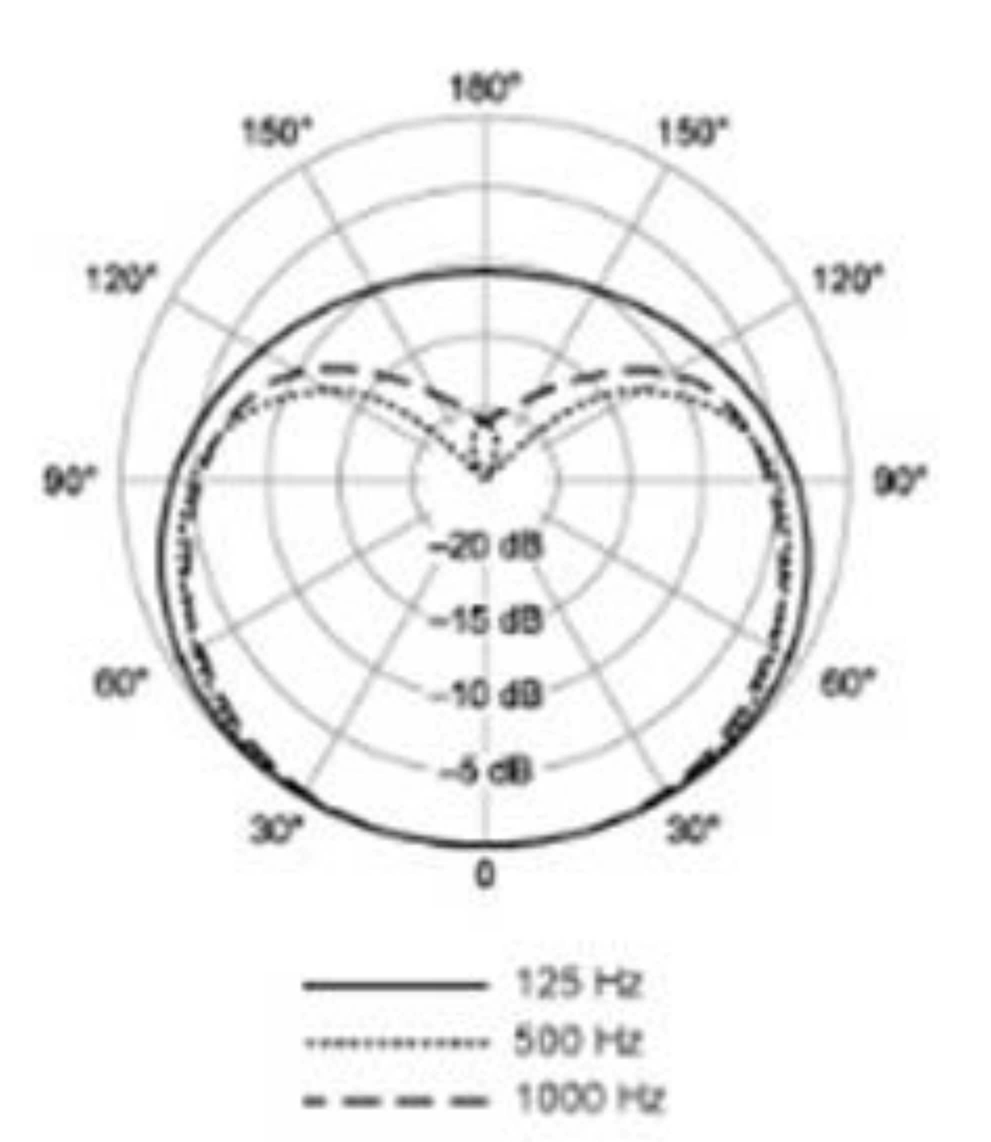
Using Graphs As Educational Tools To Study Physical Phenomena

- **Facilitate and make explanations clearer**
- **Convey the essence of a technical work and make it more understandable**
- **Serve as shortcuts to avoid talking separately about diverse cases of similar nature**

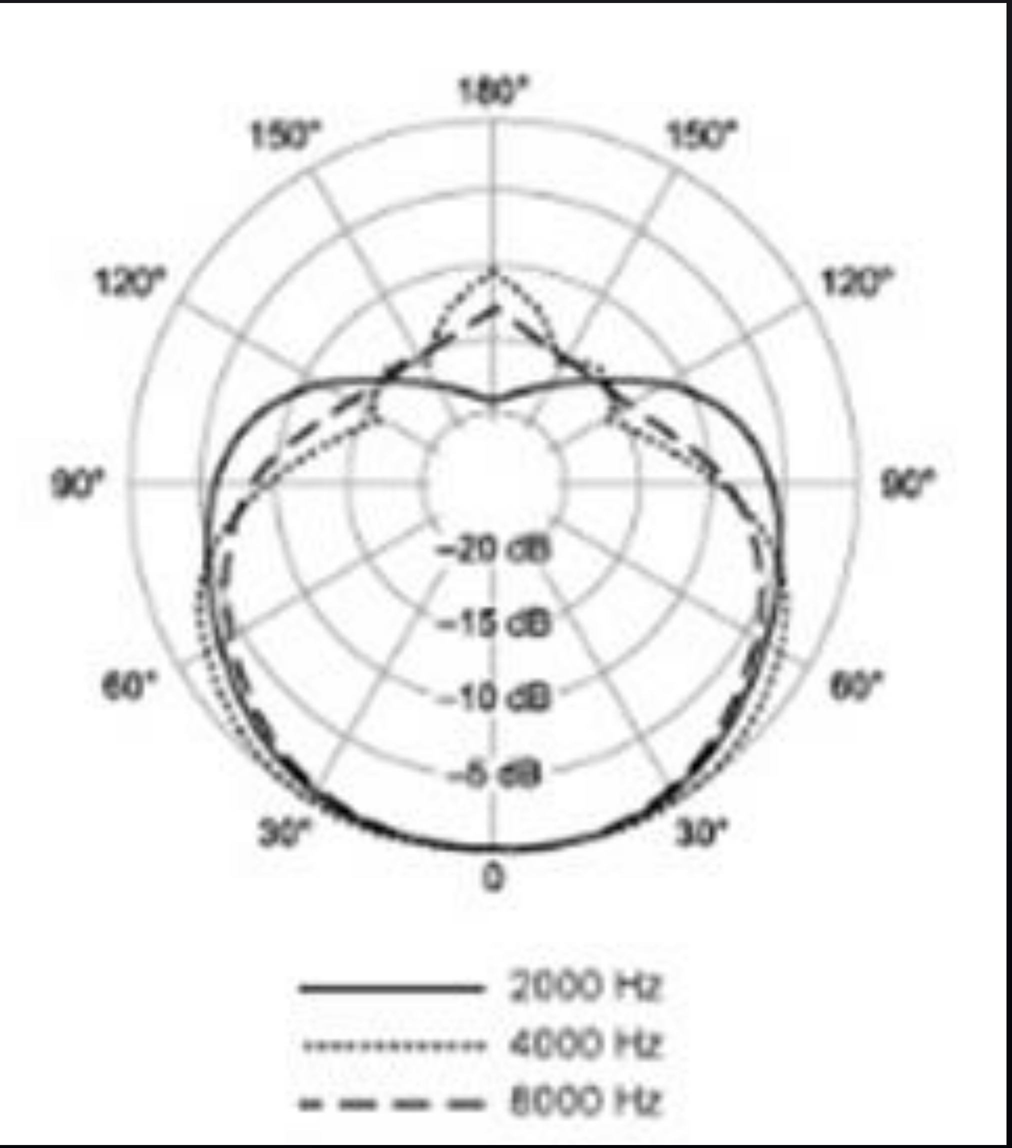
To study a microphone pattern of sensitivity audio engineers, test its output signal by measuring it at different angles.



Cardioids are used when measuring the sensitivity of a Microphone



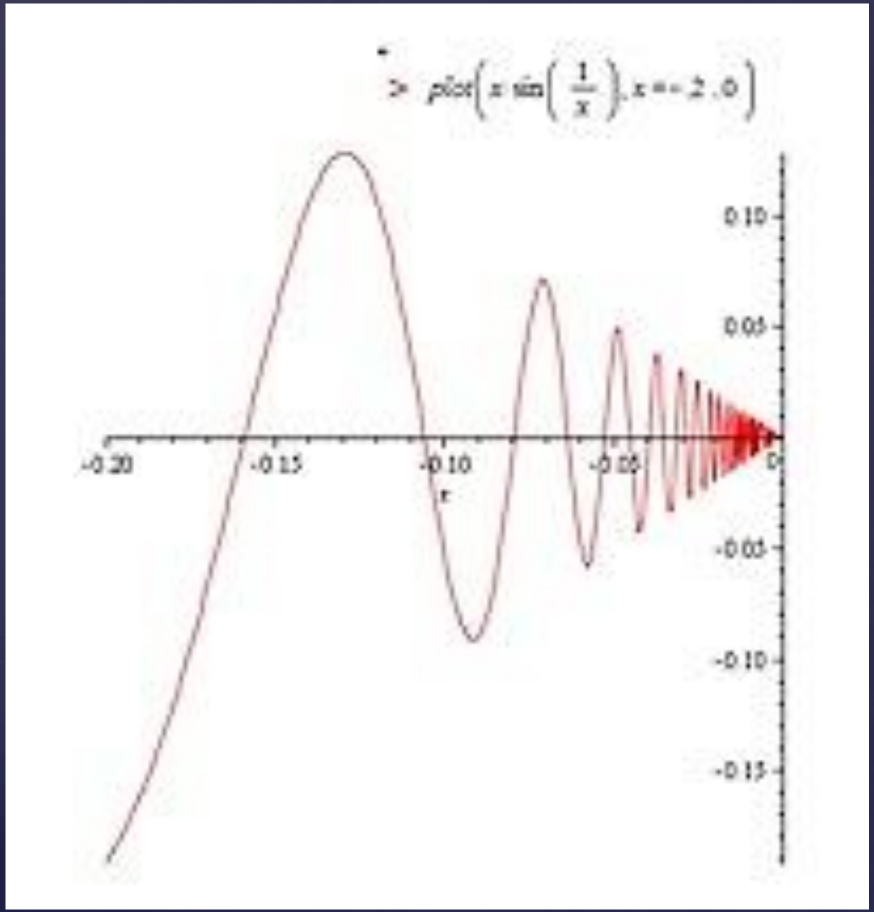
A computer-generated graph Cardioid



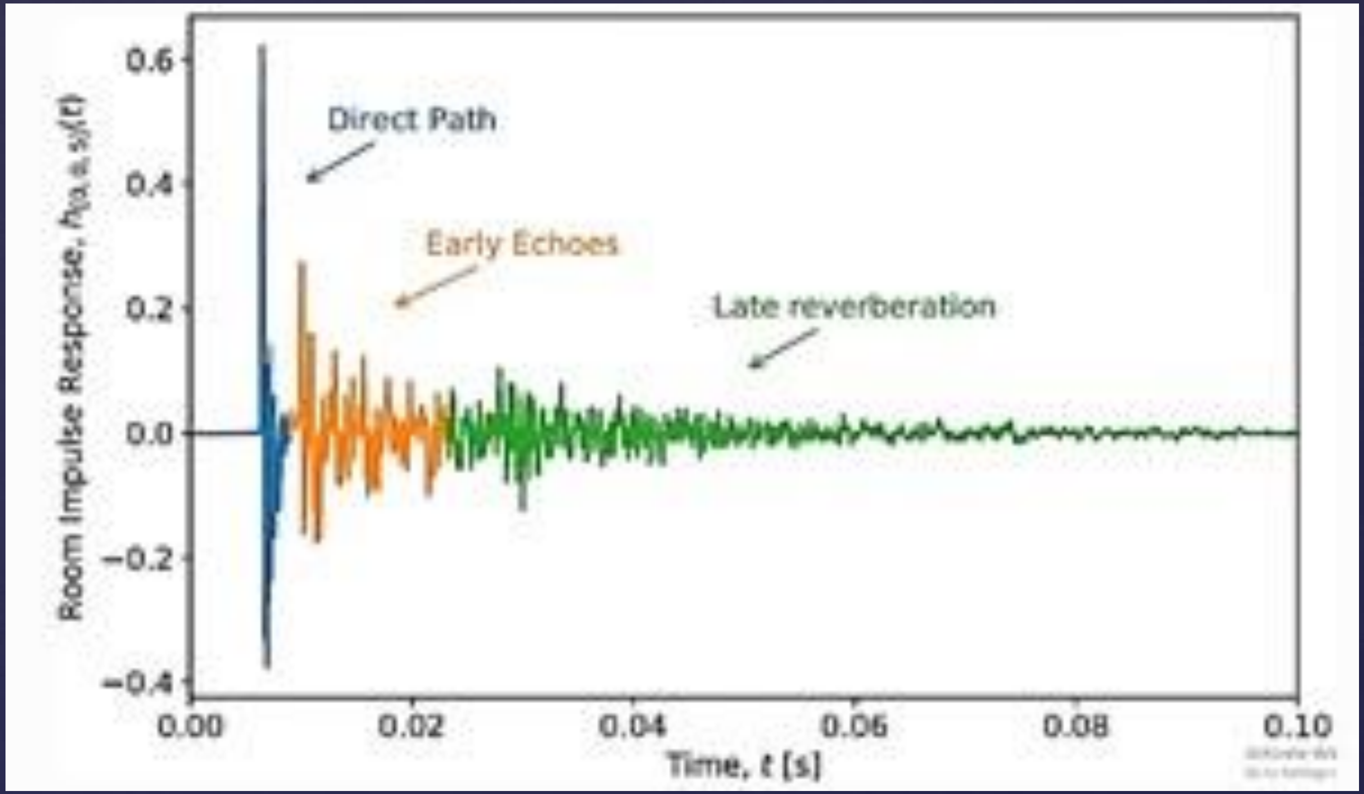
Cardioid provided by the manufacturer

Graphs of Reverberation time of a closed-model Room

Computer Generated Graph



Actual Graph of a Real Reverberation Time Test



By tweaking the parameter is possible to approximate the computer generated graph to the real one

GENERAL RESOLUTION OF A SECOND-ORDER HOMOGENEOUS DIFFERENTIAL EQUATION BY AN ALGEBRAIC METHOD

The general form of a second-order homogenous differential equation where *a*, *b*, and *c* are arbitrary constants is as follows

$$ay'' + by' + cy = 0 \qquad (1)$$

For this type of equation, the function $y = e^{mx}$ is always a solution.

The general solution obtained is as follows:

$$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$$

GENERAL SOLUTION OF A SECOND-ORDER HOMOGENEOUS DIFFERENTIAL EQUATION OF A FREE UNDERDAMPED SYSTEM

The general form of a second-order homogeneous differential equation is

$$\frac{d^2x}{td^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = 0$$

The general solution of this equation can be rewritten as

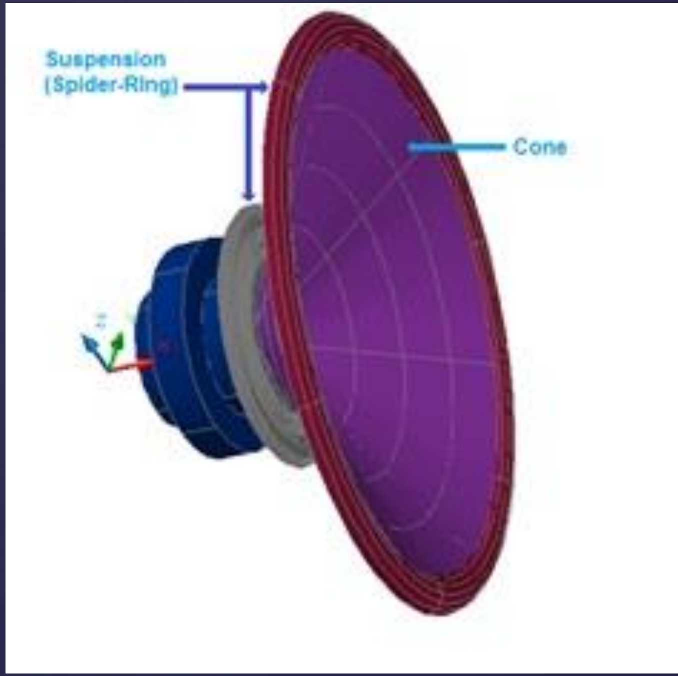
$$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$$

A LOUDSPEAKER AS AN UNDERDAMPED SYSTEM

A loudspeaker can be considered as a mass-spring-damper system.

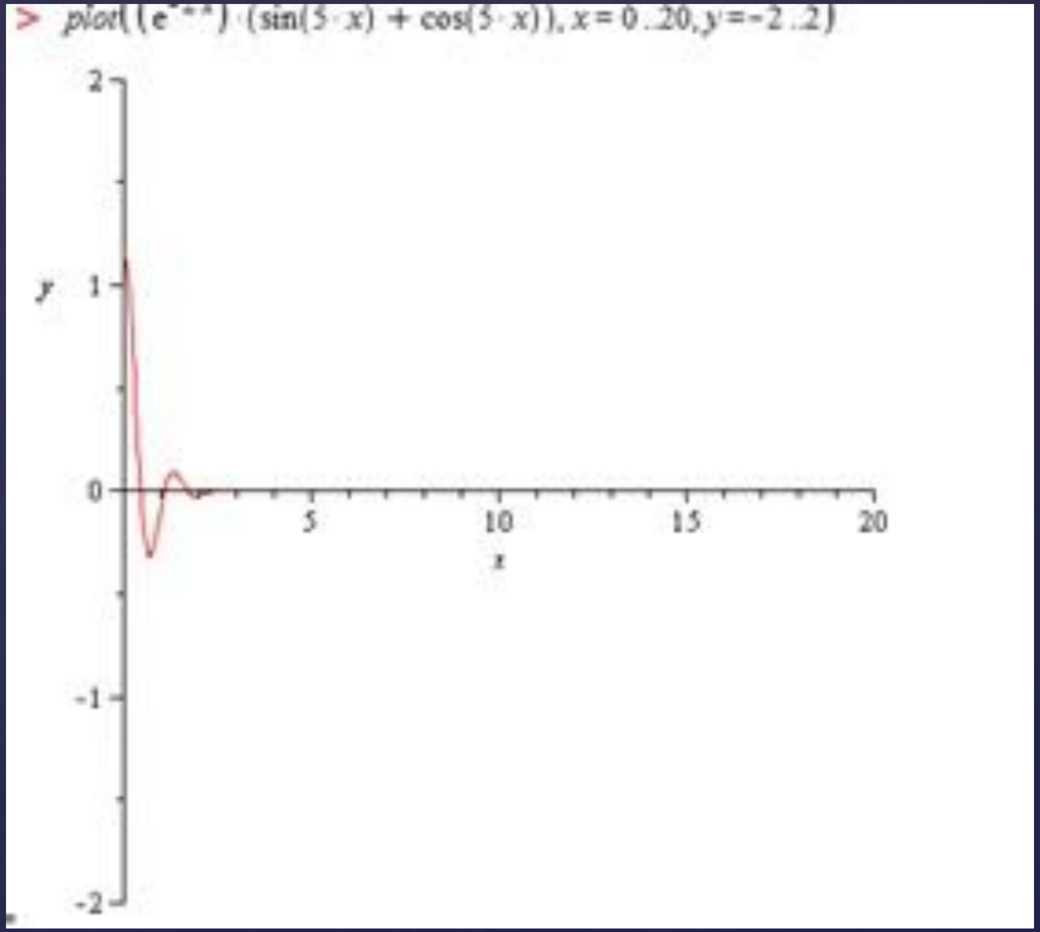
The figure shows a simplified view of a loudspeaker.

The cone (the body) with the voice coil represents the moving mass. The suspension, formed by the spider and the ring surround, keep the moving parts in place (with a degree of freedom) and provides the rigidity, mechanical damping, and resistance of the system.



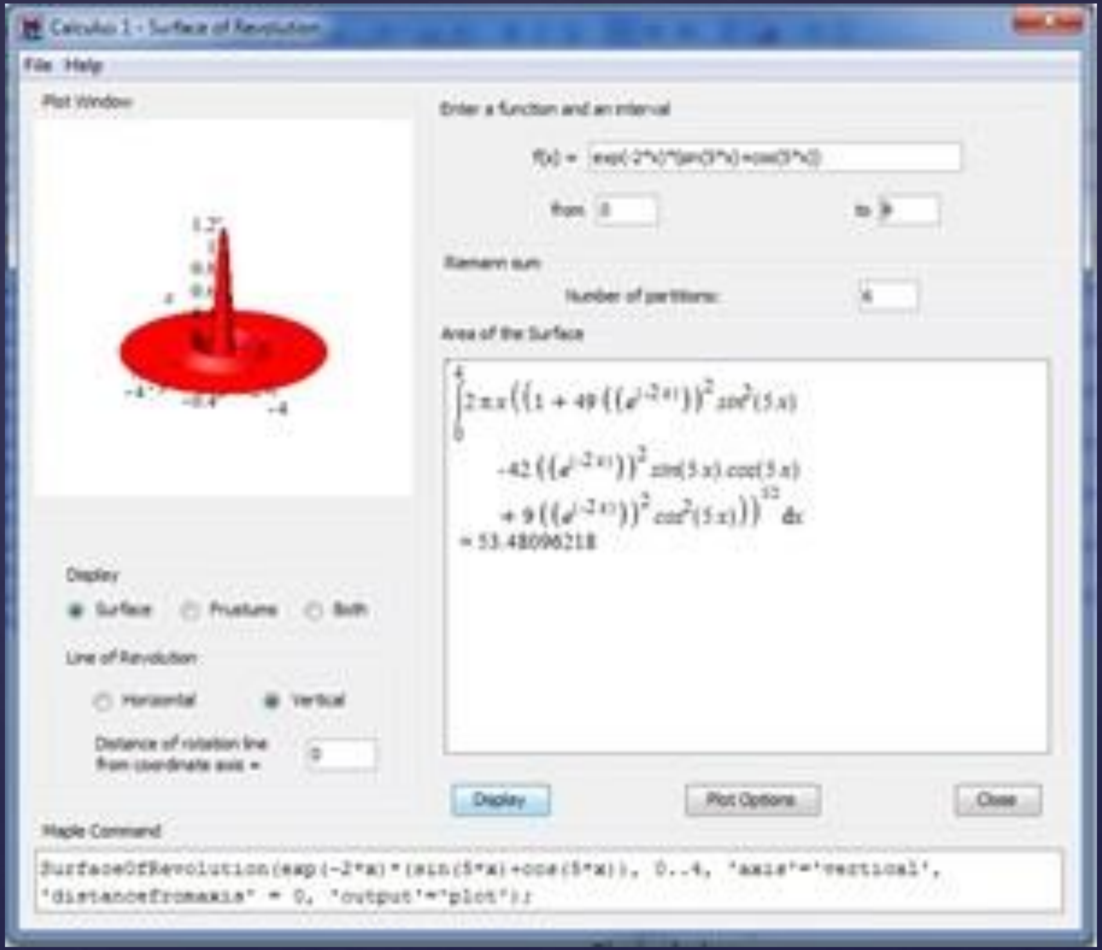
Plot the equation with Maple

We use the function e-$^{.9x}$ to plot to generate to resemble a high frequency response

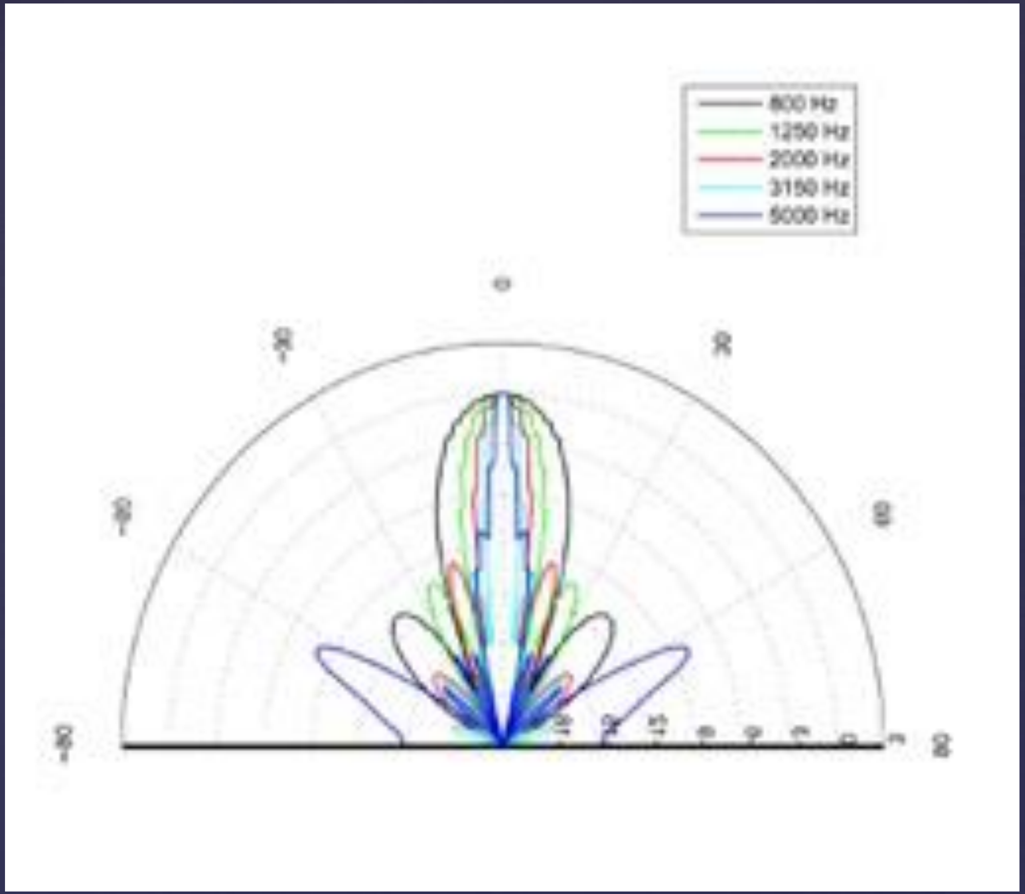


Solid of Revolution

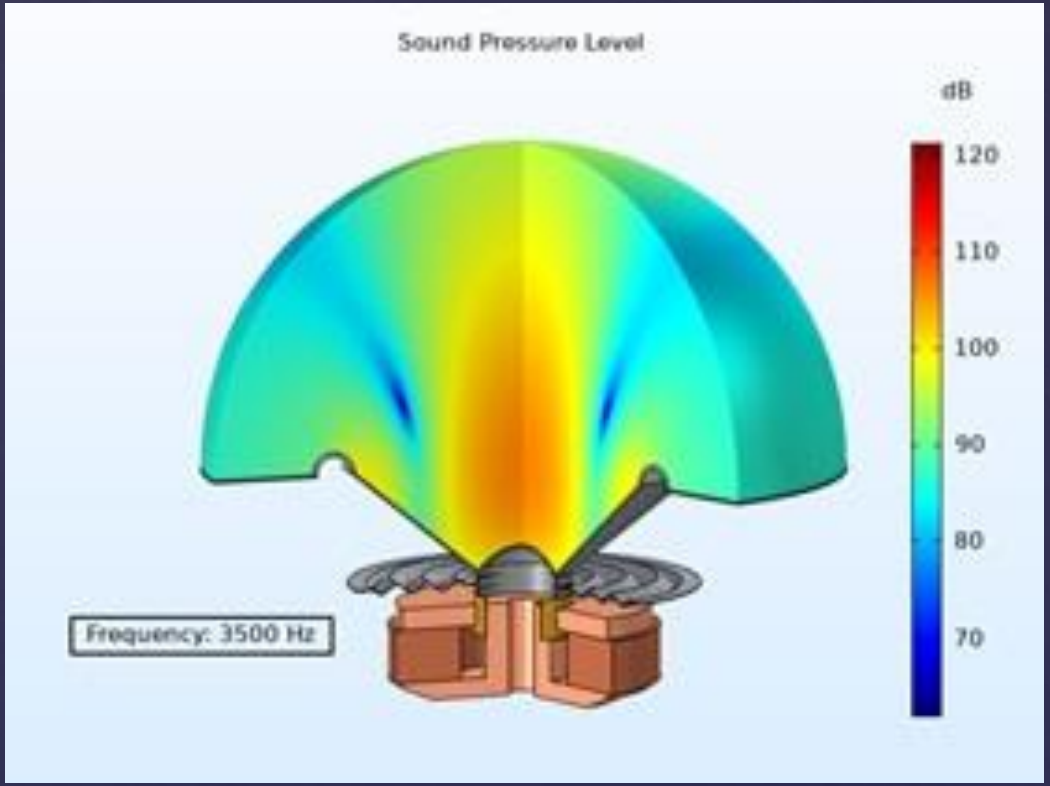
The tutorial solid of revolution in Calculus 1 of Maple allowed us to obtain the solid of revolution



This graph of the real spectrum and the 3D of the cone of revolution are similar. Notice the shape in blue of the spectrum and the shape of the "spike" of the cone.



Real Spectrum

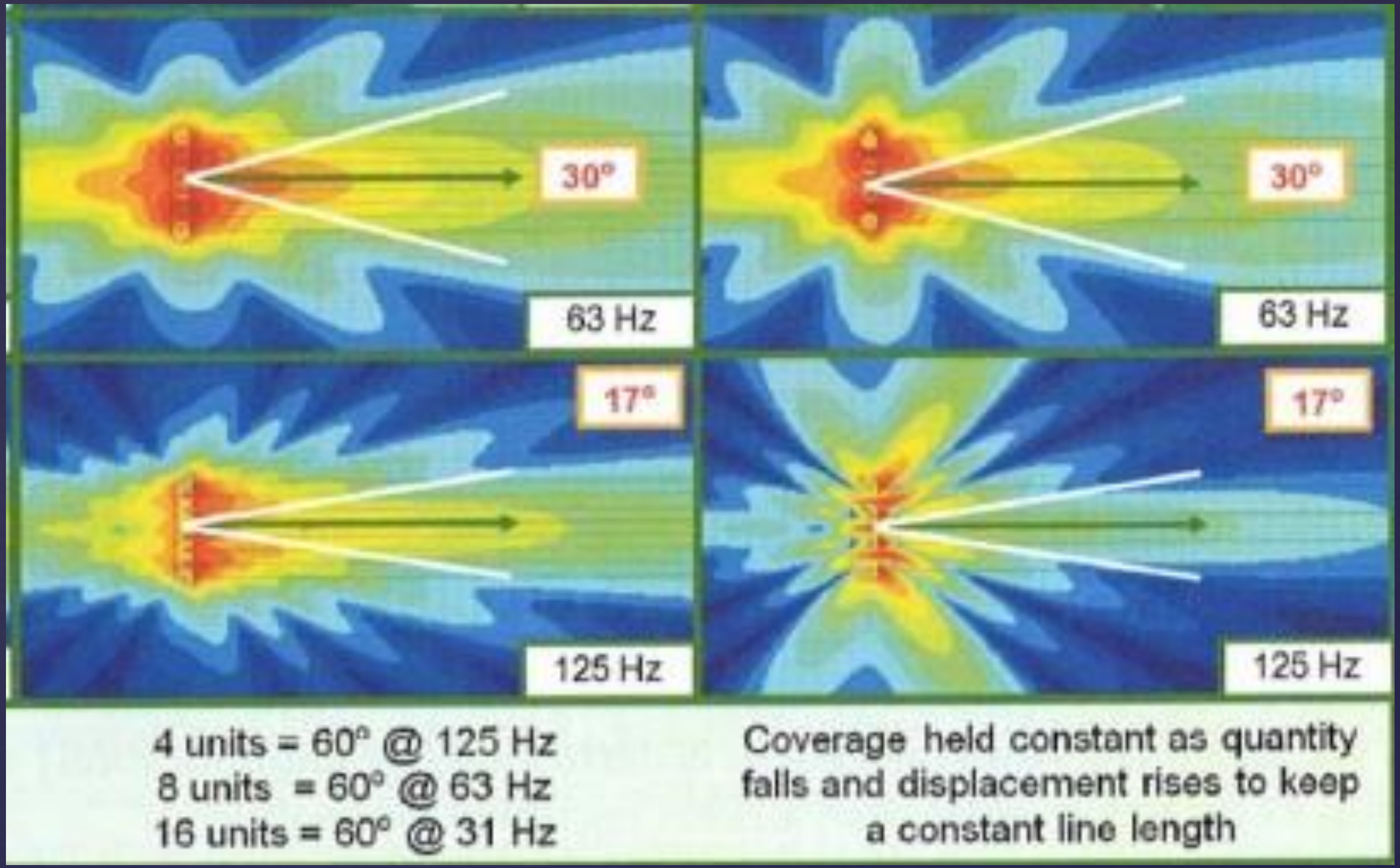


The center of this 3D graph resembles the "thin red cone of the red solid of revolution shown in slide 10 and figure at the center of the figure at slide No. 10

Real 3D Spectrum

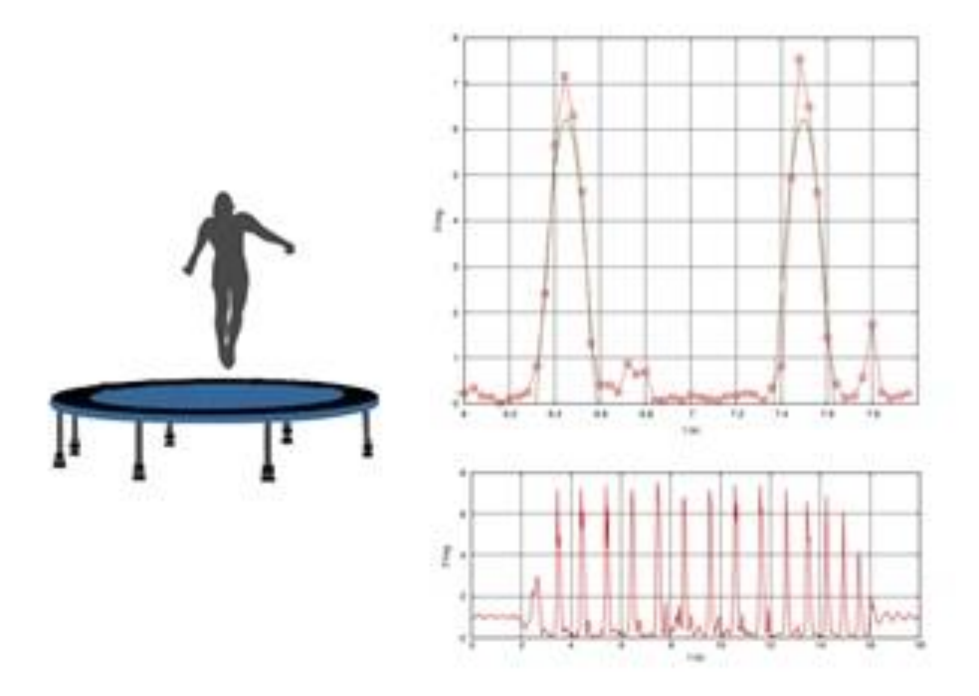
Line Array

Traditional

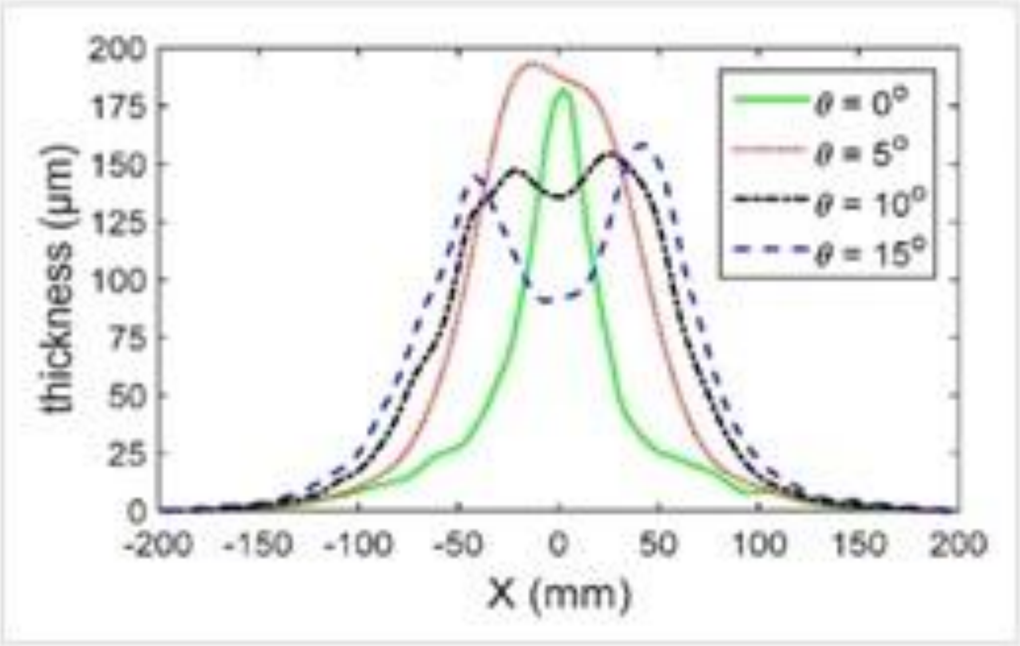


The Line Array Spectrum at low frequencies showing the steering of a coupled line source arrays of loudspeakers.

The use of differential equations of the type shown in this work can be extended to study the mechanics of a trampoline.



Trampolin



Application of the method to a fluid event

This graph shows the Coating thickness profiles along the x-axis for different values of L.

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