Using Student Differential Equations Projects for General Public Education:

Towards Framing Ethical Reasoning in Mathematics

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SIMIODE EXPO 2024!

Summary

Student final projects in an ODE's course, designed to educate the public.

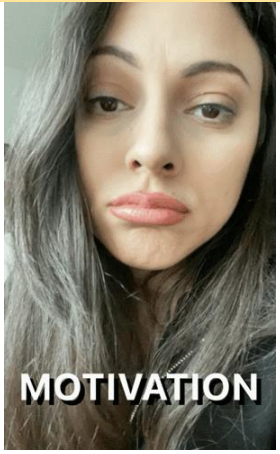
- What was the experience like?

- How can we use this avenue to frame ethical reasoning?

Motivations

Adapting assessment methods for traditional mathematics courses (forced by the pandemic)

Keeping students motivated to learn mathematics



Making mathematics accessible to the general public

Keeping up with and taking advantage of technological trends

The Course: Math 286 - Differential Equations

Spring 2021

Topics / Learning Outcomes

- Mathematical modelling through differential equations.
- First-order separable, linear, and exact differential equations.
- Second order homogeneous equations w/ constant coefficients.
- Second order nonhomogeneous equations.
- Use of Laplace transform
- First and second order initial value problems arising from modeling
- Systems of two linear first-order constant coefficient equations.

Text

- A First Course in Differential Equations with Modeling Applications, 11th Edition, by Dennis G. Zill, (Cengage 2018).

Enrollment:

- 56 students over 2 sections

The Course: Math 286 - Differential Equations

Assessment

- Homework, Quizzes, Midterm Exams, Final Exam
- Projects based on SIMIODE modeling scenarios
  - Two Term Projects
  - A ***final project*** for a general audience (the public)

SIMIODE

https://qubeshub.org/community/groups/simiode/

- SIMIODE: Systemic Initiative for Modeling Investigations & Opportunities with Differential Equations
- Peer reviewed, classroom ready Modeling Scenarios and Technique Narratives for differential equations modeling activities.

Term Project 1

Population Model Variations and MATLAB
By William (Bill) Skerbitz

- A walk-through of a detailed derivation of basic exponential and logistic population models.
- How to create variations of these models
- Exploration and visualizations of models using MATLAB, desmos, dfield

Differential Equation Classes: ODE, First, Linear, Nonlinear, Homogeneous

Analytical Techniques: Initial Value Problems, Separation of Variables

Qualitative Analysis: Stability, Phase Plane, Graphical analysis

Population Model Variations and MATLAB, William (Bill) Skerbitz (2022). 1-143-PopulationModelVariationsMATLAB-ModelingScenario. SIMIODE, QUBES Educational Resources. doi:10.25334/8PE1-0732

Exponential Model of Population Growth

- Suppose a population changes only via births and deaths (no migration or other external influences), where $\beta(t)$ is the number of births per unit of population per unit of time at time t and $\delta(t)$ is the number of deaths per unit of population per unit of time at time t.
- The numbers of births and deaths would be $\beta(t)\cdot P(t)\cdot\Delta t$ and $\delta(t)\cdot P(t)\cdot\Delta t$, respectively, and the incremental change in population would be $\Delta P = \{\text{births}\} - \{\text{deaths}\} \approx \left[\beta(t)-\delta(t)\right]\cdot P(t)\cdot\Delta t$, so in the end, we would have $\frac{\Delta P}{\Delta t} \approx \left[\beta(t)-\delta(t)\right]P(t)$.
- If the birth and death rates ($\beta(t)$ and $\delta(t)$) are constant, then as $\Delta t \to 0$, we have $\frac{dP}{dt} = (\beta-\delta)P$, or in other words, $\frac{dP}{dt} = kP$, where $k = \beta - \delta$.
- This leads to the familiar exponential solution $P(t) = P(0)e^{kt}$, which is often called the *Malthusian model* (after the English clergyman and political economist Thomas Malthus (1766-1834)). Note that if births outpace deaths, i.e. if $\beta > \delta$, then $k > 0$ and we have exponential population growth, whereas if $\beta < \delta$, then $k < 0$ and we have population decline (exponential decay).

PROBLEM 2 – Suppose an alligator population consisted of 12 alligators in 2008 and two dozen in 2018. Also suppose that the population grows as in the exponential model above, except that the birth rate actually increases by a factor of $k > 0$ as the population increases (i.e. the birth rate is proportional to the population and is *not* constant). In addition, without any natural predators and a typically long lifespan, we'll consider the death rate to be zero. Hence the model we end up with in this case is

$$\frac{dP}{dt} = (\beta-\delta)P = (kP-0)P = kP^2, \quad \text{with } P(0)=12.$$

MATLAB usually assigns output to the variable "**ans**," and each subsequent command overwrites what used to be defined as "**ans**." So a useful practice is to name your variables with convenient names so that you can easily recall them later. With that in mind, perform the following steps in MATLAB to solve this initial value problem.

Logistic Growth In environments with limited resources, it is reasonable to assume that the birth rate actually *decreases* as the population increases. If we assume the birth rate is a decreasing linear function of the population and the death rate remains constant, we have $\frac{dP}{dt} = \left((\beta-\beta_1 P)-\delta\right)P$. To make this more familiar to us, note that this is a "logistic differential equation" $\frac{dP}{dt} = kP(M-P)$ with $k=\beta_1$ and $M = \frac{\beta-\delta}{\beta_1}$. From

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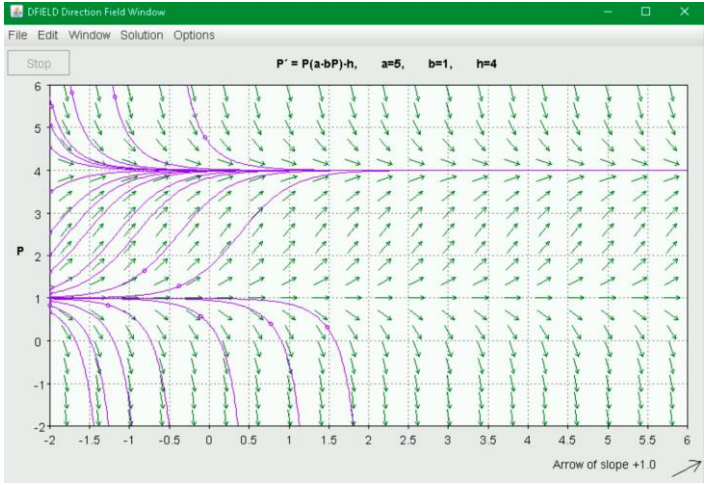


Term Project 2

Modeling Car Suspensions
By Therese Shelton, Brian Winkel

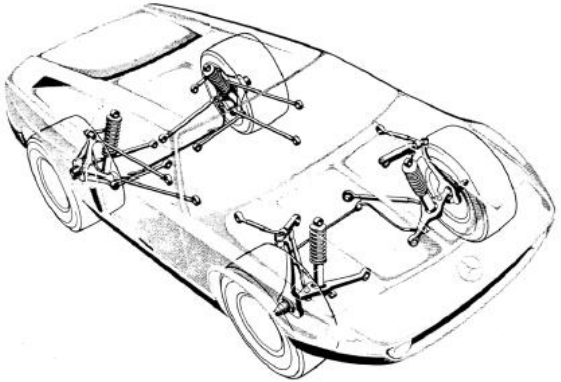
- Spring-mass phenomenon of a car suspension
- How is ride related to parameter values
- Effect of changing angle of installation

Differential Equation Classes: ODE, 2nd Order, Linear, Nonlinear, Homogeneous

Analytical Techniques: Initial Value Problems Separation of Variables

Qualitative Analysis: Stability, Phase Plane, Graphical analysis

Therese Shelton, Brian Winkel (2023). 3-034-CarSuspension-ModelingScenario. SIMIODE, (Version 2.0). QUBES Educational Resources. doi:10.25334/JYG7-5953



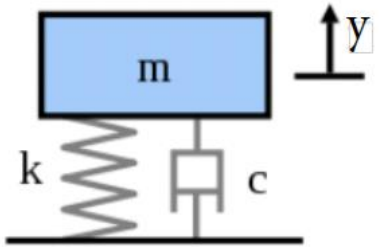


Figure 1. (a) Suspension (b) Spring Assembly (c) Diagram

external forces, such as for a car on a smooth road.

$$M \cdot y''(t) + c \cdot y'(t) + k \cdot y(t) = 0\,, \quad y(0) = y_0\,, \quad y'(0) = 0\,. \tag{1}$$

Final Project for a General Audience

Goal: Explore a phenomenon that can be explained using differential equations.

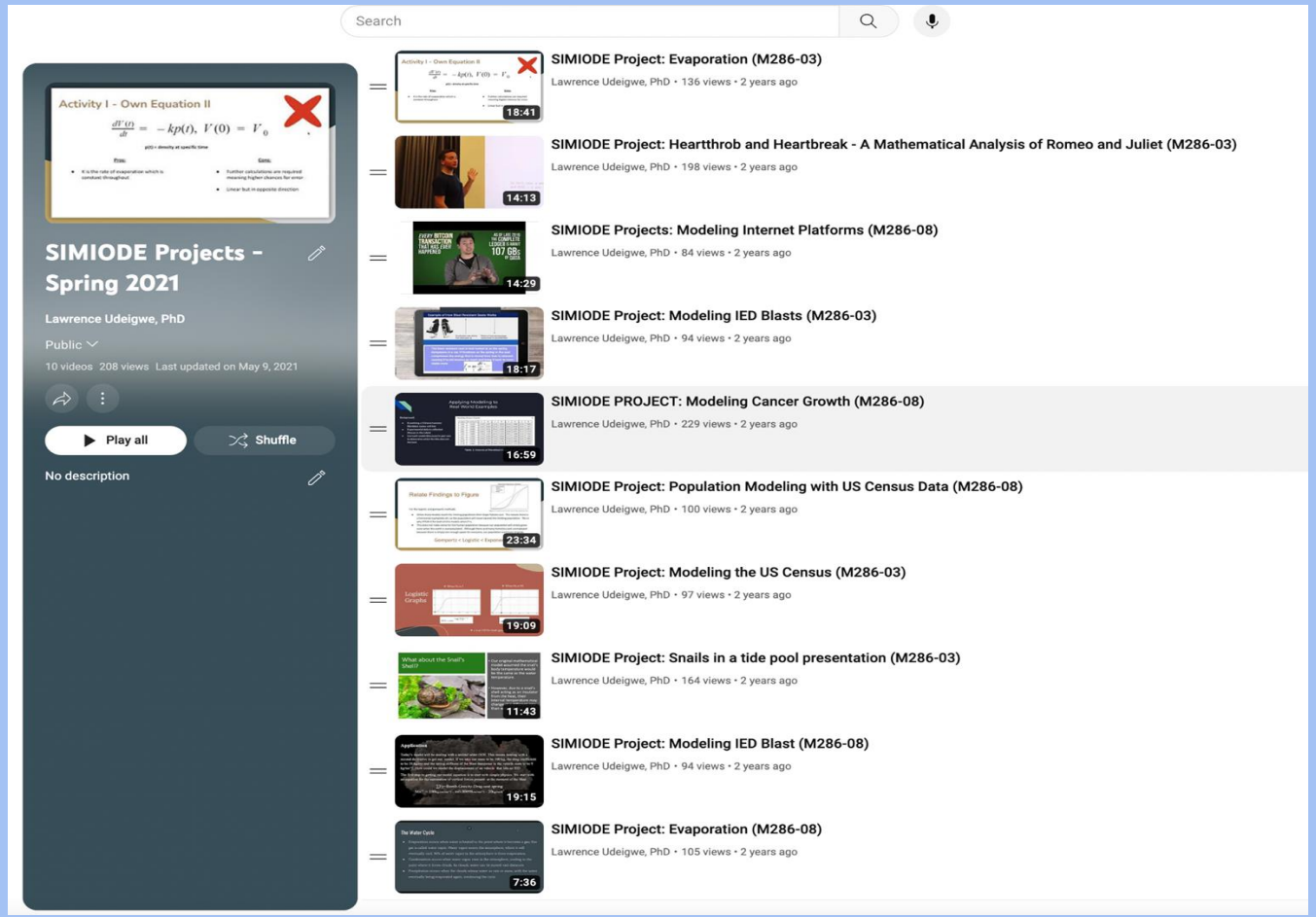
Parts:

- Introduction and Background
- Mathematical Exploration
- Conclusion

Major Rule: Your target audience is a high school student. Your job is to teach this student without intimidating them.

Main Product: A 20-minute video recording of your presentation uploaded on YouTube

Playlist of Final Project



Audience Feedback

Evaporation
Presented by the Calc-Aholics. Adapted from Brian Winkel (2015), "1-026-S-Evaporation," https://www.simiode.org/resources/797.

@janinegayanelo4885 2 years ago
I thought I understood evaporation in simple biological terms, but looking at it mathematically and modeling it's rate was super interesting. I would definitely like to learn more about it and the different ways to apply this. Even, relating back to biology, for example, something like the evaporation of sweat and how it cools us down.

Modeling Internet Platforms

Presented by The Average Joe's. Adapted from Victoria Rayskin (2018), "6-065-S-InternetPlatformUsers," https://www.simiode.org/resources/5540.

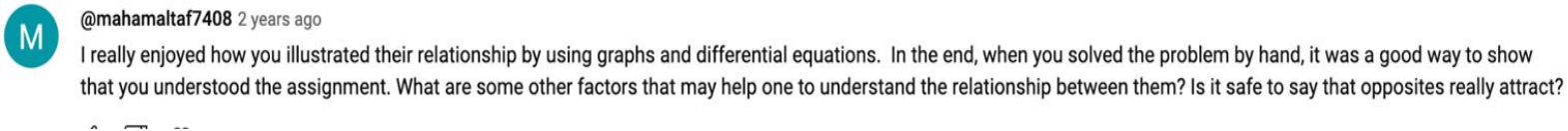
@mariamiosepaishvili5850 2 years ago
This was a very captivating topic for me since I've always wanted to be introduced to cryptocurrency. I liked how you explained bitcoin in detail to introduce the topic. You did a good job explaining how modeling the differential equations can be used to observe internet platforms, especially I liked the "odd system" where you got the eigenvalues to be complex numbers and fixed points became spiral and then you connected it to the outcome for buyers and sellers. The phase portrait looked fascinating. Overall, the presentation was organized and easy to follow. I am excited to learn more about this.

Reply

Audience Feedback

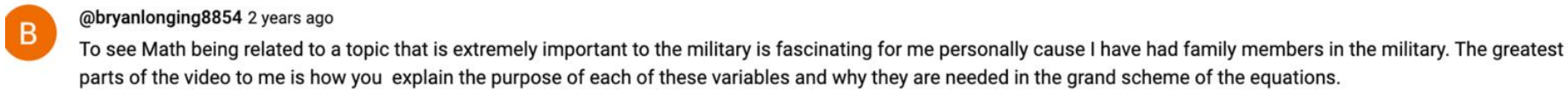
Heartthrob and Heartbreak - A Mathematical Analysis of Romeo and Juliet

Presented by The Integreats. Adapted from Lawrence C Udeigwe (2019), "5-002-S-PhasePortraitForRelationshipDynamics," https://www.simiode.org/resources/6263.



Modeling IED Blasts

Presented by The Lambda's. Adapted from Jonathan Paynter; George Hughbanks (2016), "4-020-S-AnIEDBlast," https://www.simiode.org/resources/1514.



Audience Feedback

Modeling Cancer Growth

Presented by The Manhattan Integrals. Adapted from Jue Wang (2018), "1-102-S-CancerGrowth," https://www.simiode.org/resources/4846.

@salvatoreschillace2410 2 years ago

I appreciate how you gave background information about what cancer actually is. Also it is helpful how you showed 5 different model types and explained why only 3 of them worked for cancer growth. This presentation was excellent, I really enjoyed how you took real data and used it to explain which models were the most accurate. 👍🏽

@evasanz9572 3 months ago

Hi! I'm doing a paper on tumor growth formulas and this has been the best resource I've found so far about it. Thanks so much for your video, it was extremely helpful!

Snails in a tide pool presentation

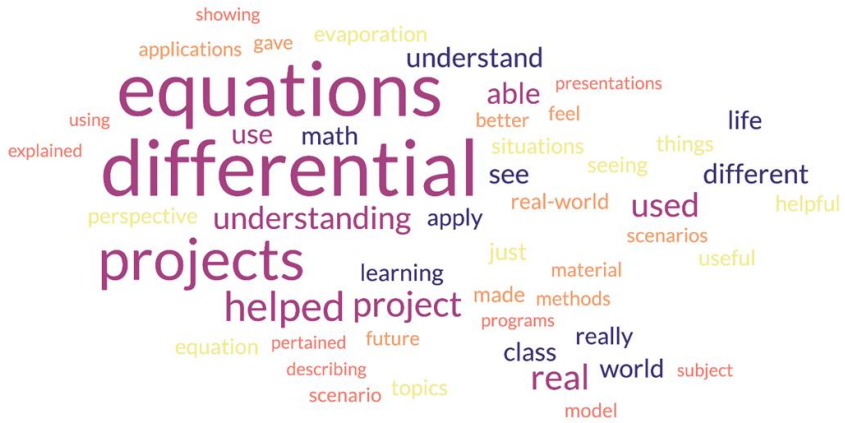
Presented by the The Snail Squad. Adapted from Lisa Driskell; Audrey Malagon (2016), "1-110-S-SnailsInaTidePool," https://www.simiode.org/resources/2924.

@santiagosantos9992 2 years ago

That intro scene was a great. I like how all the info was presented, everything was well organized and pleasant to look at. And it is interesting how math could tell us how the temperature effects snails.

Student Feedback

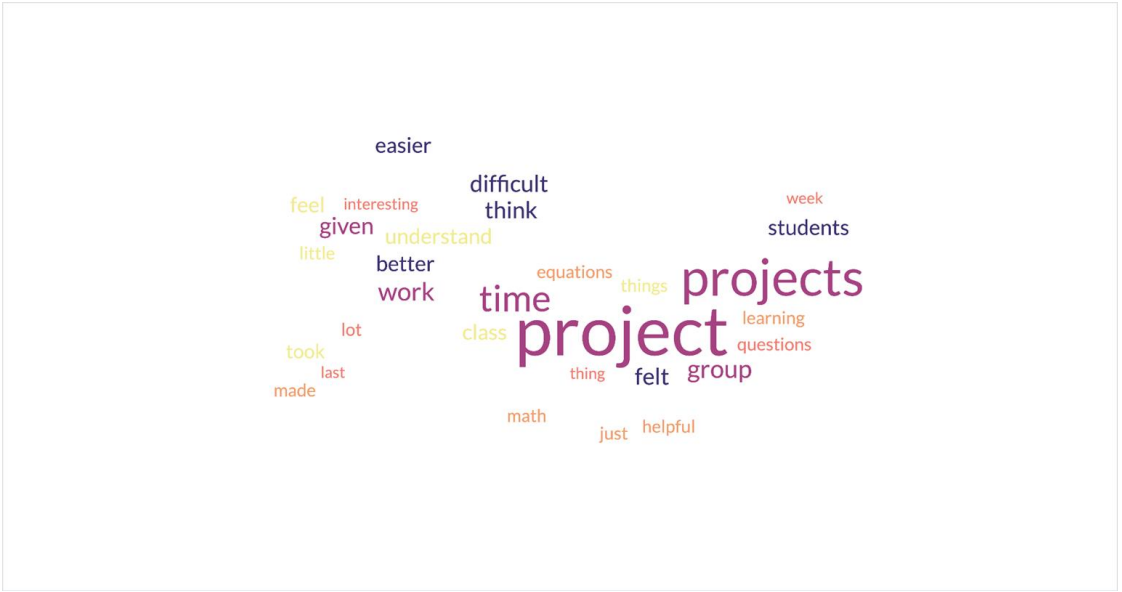
How did the projects help your understanding of differential equations?



The projects helped my understanding of differential equations by showing me how differential equations can be used in all parts of life. This was something that was explained to us but I never really considered how these differential equations would be used in the real world. Watching the projects and seeing how the topics were so different and differential equations were still useful in them, showed me that even in areas where you wouldn't expect a differential equation to be useful, they are still useful, for example snails in a tidepool.

Student Feedback

What do you not like about the projects?



The only thing that I didn't like about the projects was the short amount of time we were given to approach this task and explore it. I wish we had more time to work on it to grasp and analyze it better. Also, to be creative and incorporate our imagination and knowledge of mathematics into these projects.

Student Feedback

Are there any helpful experiences you gained from working on the final project?

Overall the presentation aspect was really helpful because it has you put yourself out there, the feedback from other people on how you did was motivating, and seeing everyone else's work to compare how we could do better in the future was thought provoking. It's good practice because we're going to have to be making a lot of presentations in our college and work career, and this is just a step towards getting better.

My Takeaway

Students show ability to synthesize the multiple facets of mathematics that they learned.

Students showed a great deal of interest in the non-math aspect of projects.

Non-math majors showed a lot of interest in the videos.

Next time

- will give more time
- mid-project submission and feedback from me

Framing Ethical Reasoning – Four Questions

Question 1: Does project tackle at least one learning outcome/objective of course? Does the project pose an ethical reasoning challenge?

Question 2: Does project adhere to the ethical standards of any related fields?

Question 3: Does project effective and sensitively address the interest/position of each stakeholder?

Question 4: Are enough resources devoted to addressing follow up questions from the general public?

DISCUSSION

Acknowledgement

My Spring 2021 Differential Equations students at Manhattan College

My Manhattan College Colleagues

Brian Winkel
- SIMIODE
- Mentorship

Thank you :-)