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Sustainability Modeling

Tandon School of Engineering's Sustainable Engineering Initiative

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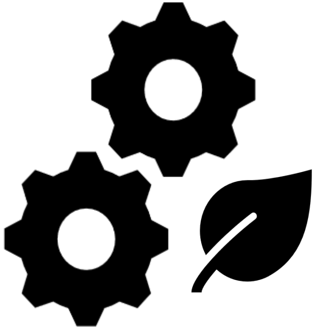
Today's Presentation



NYU's Sustainable Engineering Initiative

UN Definition of Sustainability

Meeting the needs of the present without compromising the ability of future generations to meet their own needs.

Modeling Sustainability with ODEs

What we are doing.

Summary of Our Results

Open Discussion & Suggestions

NYU Tandon's

Sustainable Engineering Initiative

01

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The Sustainable Engineering Initiative in Two Paragraphs

The Sustainable Engineering Initiative is a response to the reality that there are no formal requirements at NYU Tandon for our upcoming engineers to learn about sustainability and climate change in relation to their profession.

Design an educational framework so our engineers will understand:

1. The meaning of sustainability

2. The causes and impacts of climate change & environmental pollution

3. How to apply the principles of sustainable engineering to their work

Our Road Map

1. Integrate sustainable engineering concepts into discipline-specific courses

2. Introduce sustainable engineering principles through our core curriculum

3. Connect students to career opportunities in sustainable engineering

Current Courses Working with The Sustainability Initiative

- **Calculus 1** Worksheets & Projects comparing volcanic CO_2 emissions with those of humans.
- **Calculus 2** Worksheets First Order Linear DE models comparing various sequestration options in relation to a Carbon Budget,
- **Linear Algebra & Differential Equations** (Topic of this presentation)
- Physics
- Introduction to Engineering & Design
- The Advanced College Essay
- **Math Modeling Seminar**
- Proposed Elective

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Integrating Sustainable Engineering Concepts into Discipline Specific Courses.

Sustainability Modeling with ODEs

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Motivation:

- Build student engagement & skills
- Demonstrate that Mathematics is crucial for Engineering solutions.

“

Since Newton, humankind has come to realize that the laws of physics (and nature) are always expressed in the language of differential equations.

~Steven Strogatz

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Too Much to cover, too little time

Modeling Sustainability with ODEs In MA2034

Differential Equations with Linear Algebra

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Our engineering student have a very heavy schedule & ABET accreditation constraints

- One Semester, 4 credit Course
- Sophomore Engineering Students
- **Prerequisite for 20+ Courses**
- First Order DEs, Slope Fields,
- Higher Order Linear ODEs
- **Linear Algebra:** Gauss-Jordan Elimination, Matrix Algebra, Linear Independence, Vector Spaces, Linear Transformations, Bases,
- Eigenvalues and Vectors
- Non-Linear Systems:Predator-Prey
- Systems of First Order Linear DEs
- Laplace Transforms

Building Student Engagement & Skills In the Context of the Real World Models

DE Modeling Resources:

- Simiodes:
  Scenarios & Textbook
- Scudem Challenge
- Slopes App by Tim Lucas
- Blanchard, Devaney & Hall's *Differential Equations*

Considerations:

Not all faculty members are comfortable teaching material outside their field of expertise.

Adjunct workload & involvement

Manageable learning curve

Not favoring one discipline over others
(This is not a course on Climate Change or Sustainability)

Student engagement with topics of Climate Change & Sustainability is tricky: Topics must be specific & in context.

We wanted interesting specific modeling centered case studies that could convey the foundational mathematical concepts in the context of real world examples.

Topics We Chose:

- **Working with Parameters**
- **Nonlinearity & Parameter Sensitivity**
- **Bifurcation**
- **Hysteresis**
- **Unintended Consequences**

Semester Road Map

- First Order DE **autonomous** (Population, **Logistic, Mixing,** Newton's law of Heating and Cooling)
- **Worksheet (Project) Logistic with modifications, Constant & Proportional Harvesting, Introduce bifurcation, working with parameters,** stability of equilibrium Solutions, ***hysteresis***
- **Worksheet** Simple Harmonic Motion and Resonance
- Worksheet Linear Algebra
- **Worksheet Nonlinear Models, Predator Prey**
  Revisit with systems what we did with Logistic Model, interpreting and modifying the models. Changing parameters and equilibrium solutions.
- **Worksheet(Project) First Order Linear Systems**
  Revisit working with parameters, bifurcation, Simple Harmonic Motion and Resonance in the context of linear systems.

Two Examples

Non-Linear

First Order

- Modified Logistic Constant Harvesting: Bifurcation-Fishery Collapse, Working with parameters, equilibrium solutions, hysteresis

First Order Linear Systems

- Predator-Prey
- Working with parameters
- Parameter sensitivity
- Modified Predator Prey
- Pesticide Problem
- Unintended Consequences

Linear DEs

Second Order Linear Nonhomogeneous

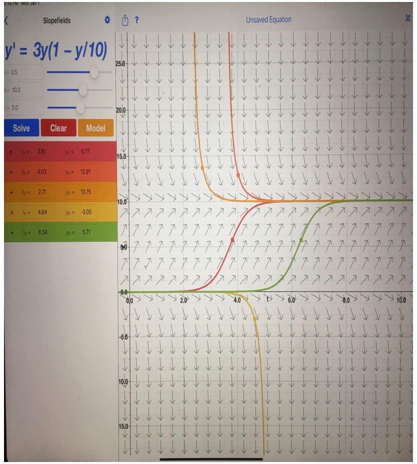
- *Simple Harmonic Motion, Resonance*

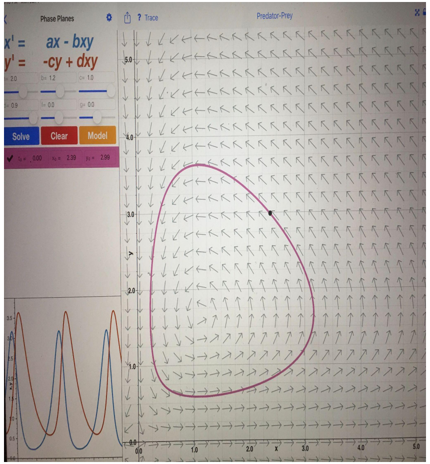
First Order Linear Systems

Carbon Budget & Simple Harmonic Motion

- Working with parameters
- Interpreting Solutions
- Bifurcation

Slopes: Powerful, Fun Popular App





SLOPES APP

by Tim Lucas

Predator-Prey Modeling

The traditional Lotka-Volterra predator-prey model is a fundamental ecological concept used to describe the interactions between two species in an ecosystem, typically a predator and its prey. Developed independently by Alfred Lotka and Vito Volterra in the early 20th century, this model provides insights into how the populations of predator and prey species can change over time in response to each other's presence. The Lotka-Volterra model has been applied to competition among pathogens, economic situations and competition in the corporate worlds, as well as social media.

The Lotka-Volterra model consists of two coupled **nonlinear** first-order differential equations that describe the dynamics of the predator and prey populations. These equations are as follows:

$$\frac{dx}{dt} = ax - bxy$$

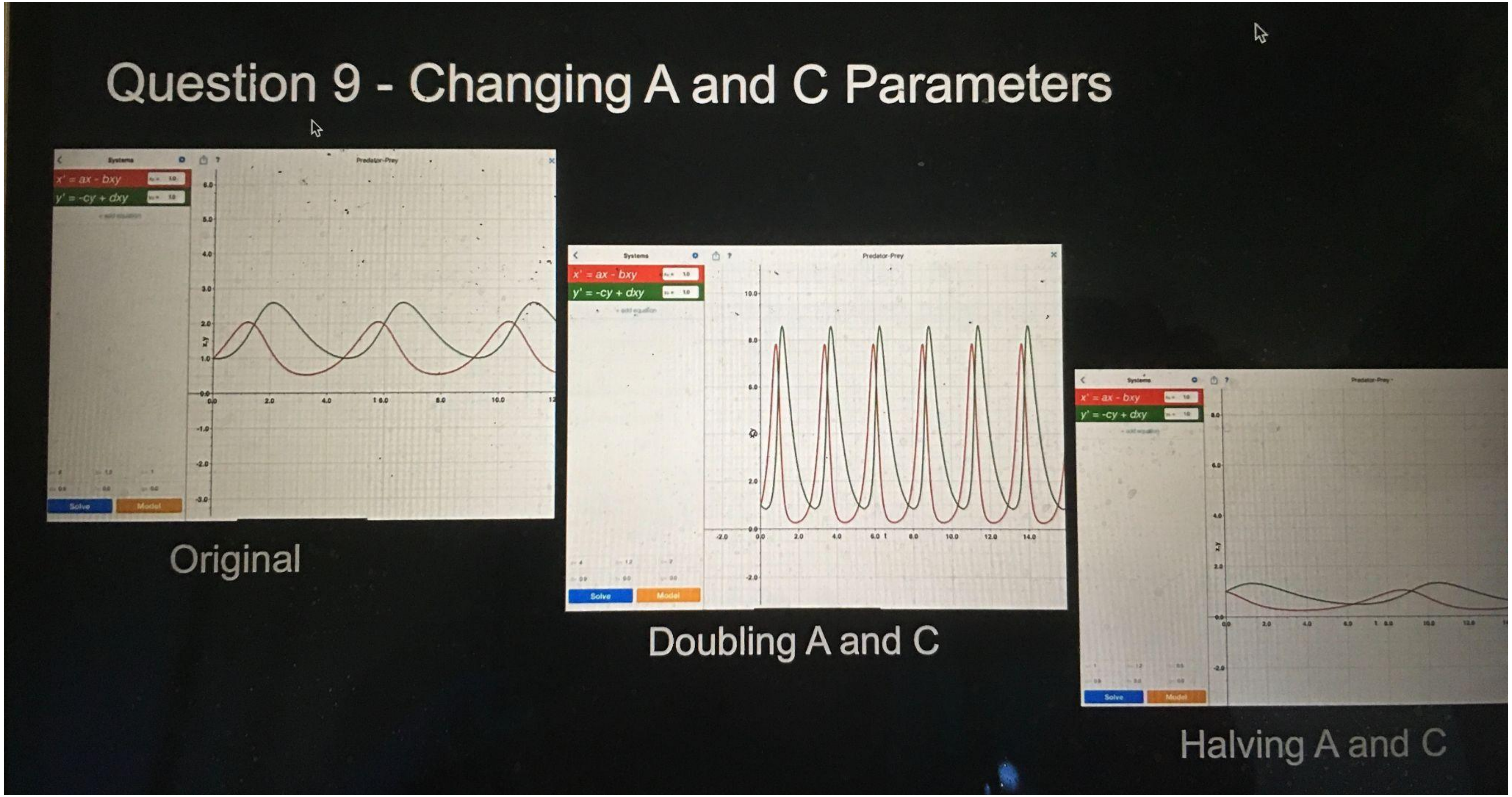
$$\frac{dy}{dt} = -cy + dxy$$

dx/dt represents the rate of change in the prey population x over time, and dy/dt represents the rate of change in the predator population y over time. and a, b, c, d are positive constants.

An entertaining and effective introduction to the Lotka-Volterra Predator-Prey system can be found in this video Predator Prey Numberphile

1. **PART I: Investigating Key Concepts Using Graphs.** Although the Lotka-Volterra system can be solved analytically, the solutions are beyond the scope of our class, so we will will explore key concepts such as *Cyclic Behavior, Parameter Sensitivity and Equilibrium Points* and other features of the Lotka-Volterra predator-prey model graphically by using the **Slopes App by Tim Lucas**.

   You can use any other app or program to graph the solutions of the predator-prey system, but the Slopes App is really convenient.


Question 9 - Changing A and C Parameters
x' = ax - bxy
y' = -cy + dxy
Solve
Model
Original
Doubling A and C
Halving A and C

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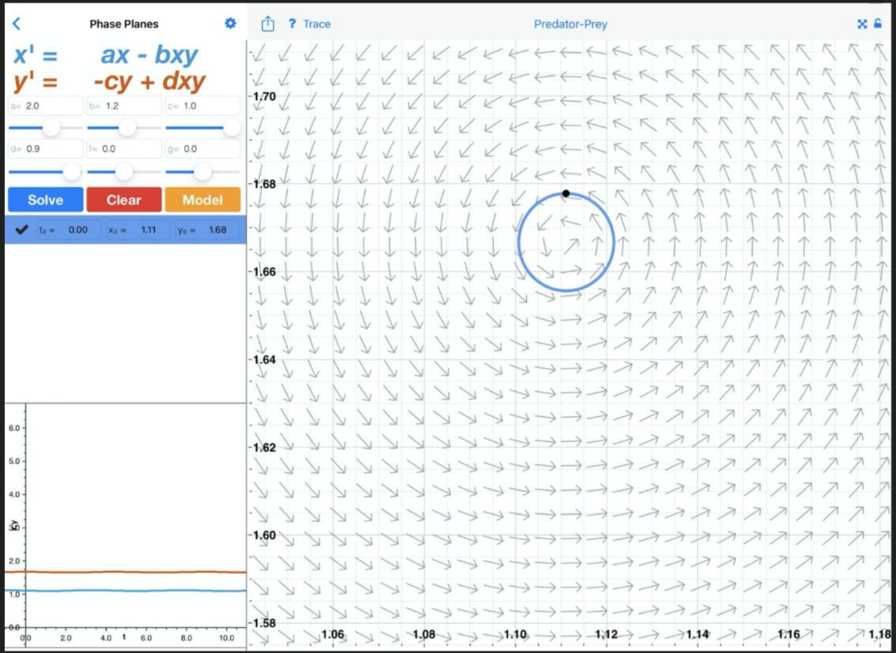
Question 11: Equilibrium Values



Predator-Prey Modeling

Pesticides Pesticides that kill all insect species are not only bad for the environment, they can also be inefficient at controlling the pest species and can have unintended consequences. Suppose a pest insect species in a particular field has a population $x(t)$ at a particular time t, and suppose its primary predator is another insect species with population $y(t)$ at time t. Suppose the population of these species are accurately modeled by the system

$$\frac{dx}{dt} = 2x - 1.2xy$$

$$\frac{dy}{dt} = -y + 0.9xy$$

Also suppose that at time $t = 0$ a pesticide is applied to the field that reduces both the pest and predator populations to a very small but non-zero numbers.

1. Use the Slopes app (or other similar app) to investigate what happens to this system. Choose 2 sets of initial conditions, **both near zero but whatever initial values you choose for your first set, the second set of values should be 1/100 of the first. Plot the solutions. Be sure to expand the scale if necessary, and look out to times 'far' in the future.** You will upload these solution curves to GS. You should also try other scenarios to get a feel for what can happen, but you only need to upload 2 graphs.

2. If the pest could be eliminated, would it be eliminated for all time? Explain your answer. Do you think that it is realistic to completely eliminate the pest?

3. If the pest could be eliminated, would it's predator be eliminated for all time too? Explain your answer.

4. Is the behavior of the pest that you found advantageous for the farmers? For the producers of the pesticides?

5. Write a short paragraph or two 'Letter to the Editor' to a Newspaper (or blog) warning about the implications and possible unintended consequences that pesticide applications can have on pest populations.

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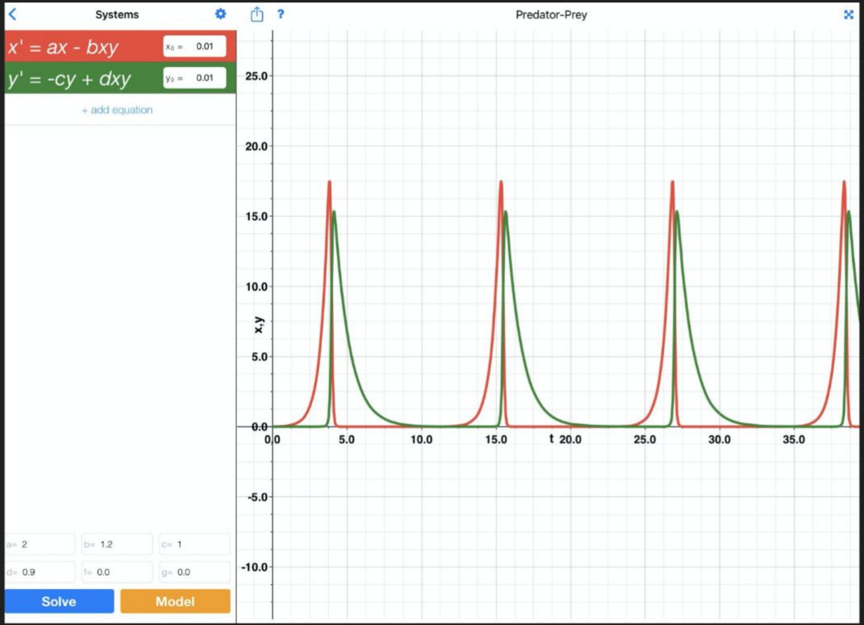
Question 2.1- Graph 1

Initial Condition: (0.01, 0.01)



Reintroduction of Wolves into Yellowstone National Park, Wyoming.

The reintroduction of wolves to Yellowstone National Park is a fascinating case study in ecosystem management and conservation biology. Here's a brief overview of key aspects of this case:

In 1995, wolves were reintroduced to Yellowstone National Park after being eliminated in the 1920s. The primary goal was to restore a vital part of the park's ecosystem that had been missing for nearly 70 years.

The initial reintroduction involved 31 wolves from Canada. These wolves were released in several groups, and their populations grew steadily, benefiting from the abundant prey and lack of human interference.

The reintroduction of wolves had profound effects on the Yellowstone ecosystem which are nicely summarized in this interesting (and somewhat controversial) video How Wolves Change Rivers

While the Lotka-Volterra model simplifies complex ecological interactions, it provides a useful framework for understanding predator-prey relationships in nature. It's important to note that ecological data can be complex and context-dependent. The interaction between wolves and elk in Yellowstone is influenced by many factors beyond just simple population counts, including environmental changes, human activities, and the presence of other predators and prey species and many modified models have been applied to this case.

Expanding the Model

One modification of the predator-prey model would be to assume that, in the absence of predators, the prey population obeys a logistic rather than an exponential growth model. One such model is provided by the system of differential equations given by

$$\frac{dx}{dt} = 2x\left(1 - \frac{x}{2}\right) - 1.2xy$$
$$\frac{dy}{dt} = -y + 0.9xy$$

1. Find the equilibrium points for the system shown above. Explain the significance of these points in terms of the predator and prey populations.
2. How would you modify the above equations to include the effect of hunting on the prey at a rate of α units per unit of time (matching the units in the above equation, whatever they are chosen to be). In the case of elk and wolf example, the elk are hunted in addition to being prey for the wolves.
3. How would you modify the above equations to include the effect of hunting on the predators at a rate proportional to the number of predators?

Project PART I. Our Carbon Budget

A major driver of global climate change is the emissions of greenhouse gases such as CO_2 into the atmosphere from human activities such as the burning of fossil fuels for transportation, electric power generation, manufacturing, agriculture, home heating, and so forth. Global average temperatures are now slightly higher than 1° C above their pre-Industrial Revolution levels. Damaging sea level rise and extreme weather events (such as the recent wildfires in Canada that led to the unhealthy air in NYC last summer) will become an increasingly major problem around the world if global average temperature rises more than 1.5° C. Scientists estimate that the total additional amount of CO_2 that can be put into the atmosphere without reaching the 1.5° C level is about 51 parts per million (ppm) or approximately 400 Gigatons (Gt). Let's call 51 ppm our carbon budget. In this project, we want to use linear systems of differential equations to analyze the predictions of various model scenarios regarding carbon dioxide emissions and investigate when we might exceed our carbon budget and how changes in the parameters affect the solutions.

Creating a linear system of first-order differential equations to model atmospheric CO_2 and CO_2 emissions involves setting up a system where each equation represents one of these two variables, and their interaction is characterized by four parameters. The system can be solved using the method of eigenvalues and eigenvectors. This is a highly simplified model of the complex interactions between CO_2 and the environment.

Consider the linear first-order differential equation system with four parameters to model atmospheric CO_2 and CO_2 emissions. Let $x_1(t)$ represent the concentration of atmospheric CO_2 at time t in ppm, and $x_2(t)$ represents the CO_2 emissions from fossil fuel and other human sources at time t in ppm. The system of differential equations can be expressed as:

$$\frac{dx_1}{dt} = ax_1 + bx_2$$
$$\frac{dx_2}{dt} = cx_1 + dx_2$$

Here, $a, b, c,$ and d are the parameters of the system, representing various factors:

1 General Questions

1. What does $x_2(t)$ represent in the system?
   (a) Concentration of atmospheric CO_2 at time t
   (b) Natural absorption rate of CO_2
   (c) Impact of CO_2 on fossil fuel emissions
   (d) CO_2 emissions at time t

1

Solving the First Order System

The system can also be written in matrix form as:

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

where the parameters are defined as follows:

- $a = -0.004$
- $b = 1.4$
- $c = 0$
- $d = 0.01$

The initial conditions are assumed to be:

- $x_1(0) = 424$, ppm starting from atmospheric CO_2 concentration in 2023 baseline.
- $x_2(0) = 7$ ppm, representing the initial global emission concentration.

2 Questions

Parameter Set I

1. (a) Use the above parameters and solve the system. Show all of your work and write the complete solution to the IVP (initial value problem) showing the eigenvalues and vectors and the values of the constants that your obtained.
   (b) Use a numerical solver to graph the solution of the system.
   (c) Using your graph approximate the atmospheric CO_2 concentration 30 years from today and 100 years from today.
   (d) Upload a graph of your solution to GS.

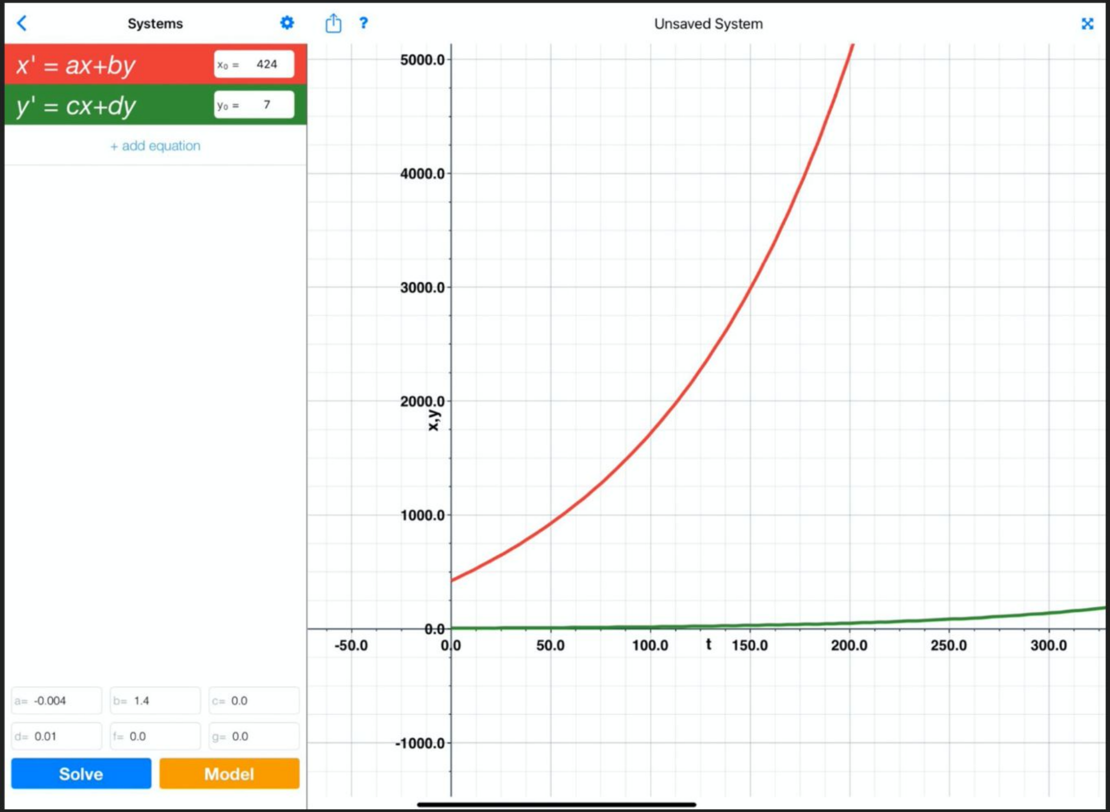
Parameter Set II- Changing b

2. Answer the questions that follow using the parameters shown below.
   - $a = -0.004$
   - $b = 0.95$
   - $c = 0$
   - $d = 0.01$

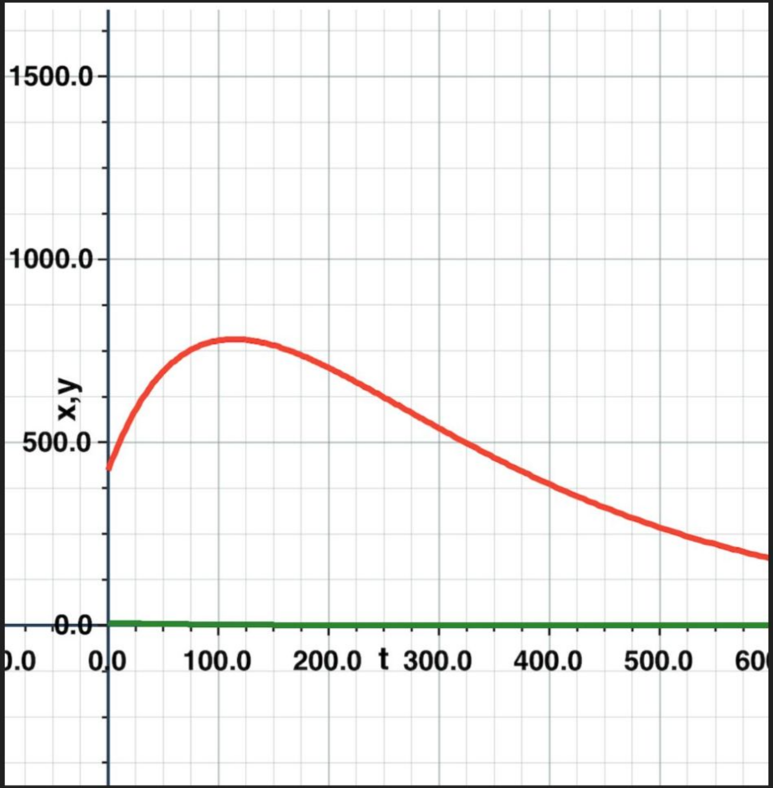
   The initial conditions are assumed to be:

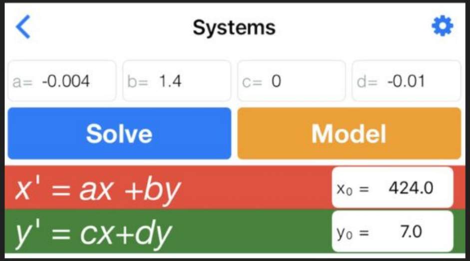
   - $x_1(0) = 424$ppm,
   - $x_2(0) = 7$ ppm.

   (a) Use a numerical solver to graph the solution of the system.
   (b) Using your graph approximate the atmospheric CO_2 concentration 30 years from today and 100 years from today.
   (c) What does the constant b represent in the model?
   (d) Did changing b have any effect on the solution? If so how would you describe the effect?
   (e) Upload a graph of your solution to GS.



Systems
Unsaved System
x' = ax+by
x₀ = 424
y' = cx+dy
y₀ = 7
+ add equation
a= -0.004
b= 1.4
c= 0.0
d= 0.01
f= 0.0
g= 0.0
Solve
Model
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
-1000.0
x,y
-50.0
0.0
50.0
100.0
t
150.0
200.0
250.0
300.0


1500.0
1000.0
500.0
0.0
x,y
0.0
100.0
200.0
t
300.0
400.0
500.0


Systems
a= -0.004
b= 1.4
c= 0
d= -0.01
Solve
Model
x' = ax +by
x0 = 424.0
y' = cx+dy
y0 = 7.0

Summary

Questions & Discussion

Suggestions

Part Three

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Summary of Results

- Increased student & faculty engagement,
  Students & faculty find the Slopes App fun and easy to use.
- Deepened student understanding of course material making connections between topics by using different learning modalities.
- Created a framework for working with sustainability topics.
- Required A LOT of extra support such as extra Q & A sessions, Video tutorials, **Totally worth it!**
- The Slopes App does not have much documentation (that I am aware of).
- Created a community of practice & interest among faculty,
  Other faculty are incorporating these topics into their courses.

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Big Thanks To:

NYU Sustainability Initiative Team

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And to Brian Winkel for all his work with Simiode, Scudem and his consistent & kind & patient support over the years.

Questions? Suggestions?

Interested in Collaborating?

vwagenen@nyu.edu

Math Modeling Seminar
Students looking at SLOPES App on screen, telling the instructor in real time how to modify Newton's Law of Cooling for various scenarios.

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