Burn, Baby! Burn: Modelling Time to Ignition and the Critical Heat Flux of Solid Materials

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Mathematical preliminaries

Can your students solve the ODE

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Then let's explore a problem from fire engineering!

The experiment

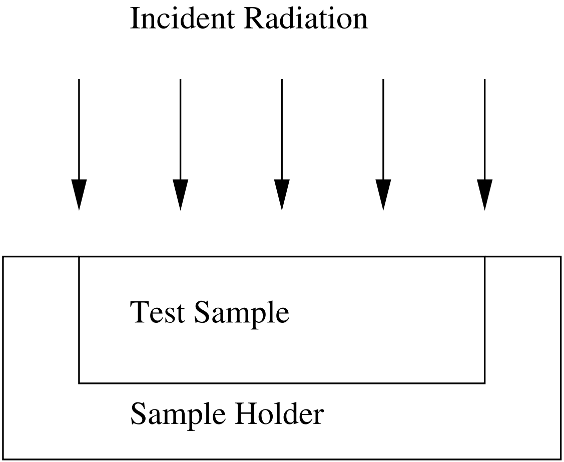

Figure 1: Common experimental configuration: horizontal heating.

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The model

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Most of my students rewrote the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -a\left(T - T_a\right) + b,$$

or

$$\frac{\mathrm{d}T}{\mathrm{d}t} = b - aT.$$

Solution

Solution. The solution of the differential equation is

$$T = T_a + \frac{\epsilon\mathcal{L}}{h} - \frac{\epsilon L}{h} \exp\left[-\frac{hS}{cm} \cdot t\right]. \tag{5}$$

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Show that

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Why do this?

Time-to-ignition and critical heat-flux

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Question 3.1 (Pre-calculus)

Use equation (6) to show there is a critical heat-flux, $\mathcal{L}_{cr}$:

1. *if $\mathcal{L} < \mathcal{L}_{cr}$ the sample does not ignite;*
2. *if $\mathcal{L} = \mathcal{L}_{cr}$ the sample ignites after an infinite amount of time;*
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$$\mathcal{L}_{\text{cr}} = \frac{h\left(T_{\text{ign}} - T_a\right)}{\epsilon}, \qquad t_{\text{ign}} = \frac{cm}{hS}\ln\left[\frac{\epsilon\mathcal{L}}{\epsilon\mathcal{L} - h\left(T_{\text{ign}} - T_a\right)}\right].$$

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$$t_{\mathrm{ign}} = \frac{cm}{hS} \ln \left[\frac{\epsilon \mathcal{L}}{\epsilon \mathcal{L} - h\left(T_{\mathrm{ign}} - T_a\right)} \right]. \tag{7}$$

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7. Show that...

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4. Data. . .

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5. Intuitive way to understand basic properties

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4. Increasing power-law.

   $$\mathcal{L}(t) = at^b, \qquad (b > 1).$$

   Linearly decreasing rate.

   $$\mathcal{L}(t) = \mathcal{L} - at.$$

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- $\dot{x} = a - bx, \quad x(0) = X_0$

References

B. Barnes and G.R. Fulford. (2015). *Mathematical Modelling with Case Studies Using Maple and Matlab*. CRC Press, Boca Rato London New York, third edition.

I used this book as the basis for my heat-transfer lecture notes. It teaches students how to derive heat-transfer models for *homogeneous* heat-transfer, when there is no temperature profile so the model is an ODE, and for *steady-state* heat conduction problems, so the model is a second-order ODE.

D. Drysdale. (1999). *An Introduction to Fire Dynamics*. John Wiley and Sons, 2nd edition.

Good introduction to Fire Dynamics. Has good sections on the ignition of thermally thin materials and thermally thick, which gives a PDE model.

R. Parot, J.I. Rivera, P. Reszka, J.L. Torero, and A. Fuentes. (2022). A simplified analytical model for radiation dominated ignition of solid fuels exposed to multiple non-steady heat fluxes. *Combustion and Flame*, 237:111866.

Comprehensive listing of functional forms for $\mathcal{L}(t)$ that have used in the literature. Uses them in extension of the ODE model I discussed to include radiative heat loss.

M.I. Nelson and P. Sebastianelli. (In preparation).

A souped up version of the talk including the missing details, more background and more discussion of our experiences using the model in the classroom. Ready to submit real soon.