SCUDEM: Teaching Fritz to Catch

Michael Hoefler, Alexander Seamark, Elizabeth Fetter

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Coach: MAJ Thomas Mussmann
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Location: West Point NY, United States
Problem Letter: C *Dog Cannot Catch*

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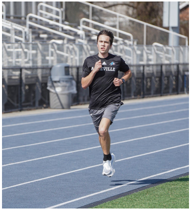


Figure: CDT Michael Hoefler

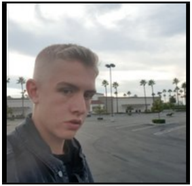


Figure: CDT Alexander Seamark

My name is Elizabeth Fetter. I am double majoring in International History and Mandarin.

My name is Michael. I am a freshman at West Point planning on studying Electrical Engineering.

My name is Alexander. I am a West Point freshman planning to major in Civil Engineering.

Why SCUDEM?

1. **How Did You Hear About SCUDEM?:** We heard of SCUDEM through Major Mussmann's MA153 class on differenital equations.
2. **Why Did You Participate In SCUDEM?:** We were interested in applying our knowledge of differential equations to a real world problem.

Figure: USMA at West Point

Problem Selection

Why Fritz?: We thought that a problem dealing with differential equations, physics, and dogs at the same time would be interesting!

How Did You Start?: We started by breaking down the problem into simpler and more understandable questions and then creating variables for equations to solve those equations. After creating our model and evaluating its shortcomings we looked into existing research on similar problems.

Figure: Fritz Trying His Best

Variables and Assumptions

Assumptions: No obstacles for the dog, dog can run fast, the dog will not jump, we can experimentally determine constants, the dog's velocity depends only on the position of the treat, the is no air resistance

symbol	explanation
t	time in seconds
x_0,y_0	initial position of the treat in m
v_x,v_y	x, y components of the initial velocity of the treat (m/s)
C	y position of the treat in m (constant for simplicity)
$x(t), y(t), z(t)$	position of the treat in m at time t seconds
$x_D(t), y_D(t)$	position of the dog in m at time t seconds

Motion of the treat

Motion of the Treat

Parametric equations for $x(t), y(t)$, and $z(t)$ of the treat.

$$x(t) = x_0 + v_x t$$
$$y(t) = C$$
$$z(t) = -\frac{1}{2}gt^2 + v_y t + y_0$$

A Very Basic Model

We start by considering the following linear system.

$$\frac{dx_D}{dt} = ax(t) + bz(t)$$
$$\frac{dy_D}{dt} = cy(t) + dz(t)$$

Where $a, b, c, d \in \mathrm{Z}$

Basic Model Solution

Clearly, since this first model is separable, we have the following.

$$x_D(t) = a \int x(t)dt + b \int z(t)dt$$
$$y_D(t) = c \int y(t)dt + d \int z(t)dt$$

When we pair this with the initial conditions $x_D(0) = y_D(0) = 0$ and $d(x_D(T), x(T)) < \varepsilon$ and $d(y_D(T), y(T)) < \varepsilon$ (the treat is caught from distance ε at time T)

Advantages: When we vary a, b, c, d we can get seemingly accurate catch paths. The animations look surprisingly accurate to real-life.

Disadvantages: It is tough to determine correct values for the constants. Lots of assumptions.

OAC Model

- The Optical Acceleration-Cancellation (OAC) model assumes that the dog runs a path where the toy will maintain constant optical velocity (Chapman, 1968).

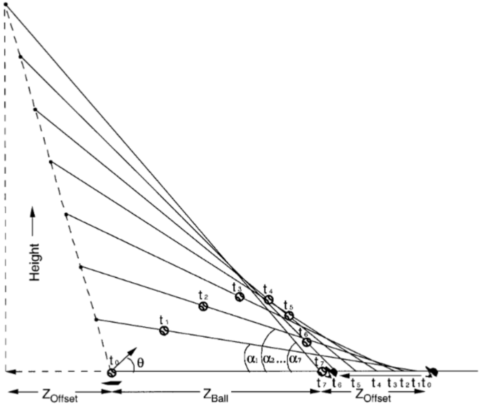
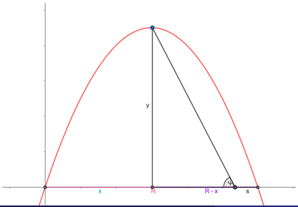


Figure: The OAC Model

OAC Model Explained

symbol	explanation
t	time in seconds
$x(t), y(t)$	x and y-positions in meters of the ball at time t seconds
R	distance from the dog to the thrower in meters
s	distance from the dog to the treat-landing spot
τ	delay in seconds for the dog to react to the treat
v	speed of the dog in m/s towards the treat



$$tan(\phi) = \frac{y}{R - x - s + v(t - \tau)}$$

Creator of the OAC Model:Chapman

- The Optical Acceleration-Cancellation (OAC) model was originally created by Chapman in 1968 in an effort to analyze the differences that lead a catcher to catch or not catch a baseball.

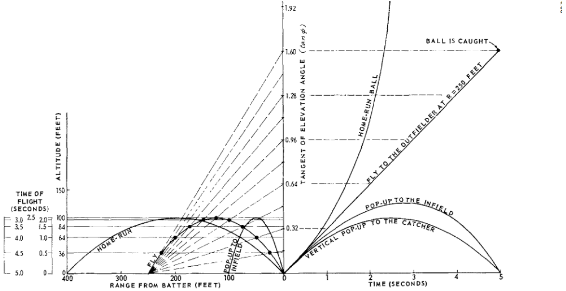


Figure: Chapman's Model of Catching a Baseball

- The elevation angle ϕ of the treat as seen by a dog standing a distance s from where the treat comes down at a distance R from the origin (Chapman,1968).

Result

After several algebraic manipulations we get the following equation which tells us that if the dog runs at the proper speed so as to maintain a constant rate increase of tanψ he will catch the treat (Chapman,1968).

$$tan(\phi) = \frac{gt}{2(V cos\theta - v)} = (constant)t$$

LOT Model

- The Linear Optical Trajectory (LOT) model assumes that that the dog runs so as to keep the apparent trajectory of the toy linear (from the dogs point of view), such that it appears to move in a constant direction γ relative to the horizontal.

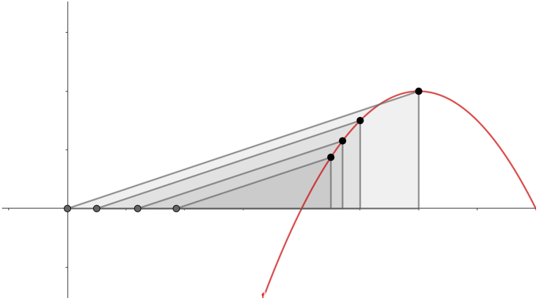


Figure: LOT Model Similar Triangles Diagram

Creating the Model

We start by using the relationship between the sides of the similar triangles.

$$tan(\gamma) = \frac{y(t)}{x(t) - x_D(t)}$$

Solving for x_D yields the following model.

The LOT Model in 2-Dimensions

The motion of the dog in the x-axis will be governed by the following equation, where γ is the ratio between the treat's maximum height and the dog's starting position.

$$x_D(t) = x(t) - \frac{y(t)}{\tan(\gamma)}$$

LOT Model in 3D

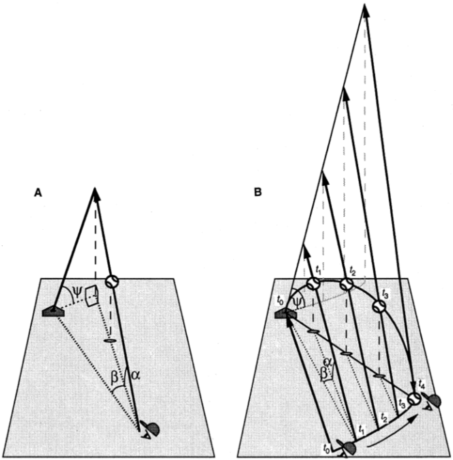


Figure: LOT Model

Conclusions

- The models we studied showed that there are lots of complex patterns that the brain needs to recognize in order to catch a treat in midair.
- Fritz should practice more in order to better recognize the acceleration and/or angular patterns of projectile motion.
- The owner could make it harder to catch by throwing the treat at an oblique (i.e. not head-on) angle and/or adding spin.

Overcoming Difficulties

1. Most Strategies didn't actually use differential Equations

2. Deciding on which approach was the best approach

3. Our differential equations contained lots of parameters but we had no data to solve for the parameters.

4. The problem is very subjective/open-ended

Future Ideas

1. Make the model truly 3D
2. What would happen if Fritz was perhaps on a leash?
3. Accounting for air resistance
4. Accounting for the shape of the treat

Thank You!

Thank you for taking the time to view our presentation today. Have a great day!

Works Cited

- Rhettallain, May 2021, Using Optical Acceleration Cancelation Method to Catch a Basetreat, Accessed 8th November 2023, https://youtu.be/ZmgHaUfIYHk?si=XaII1MQg9UCEncbP https://www.researchgate.net/publication/23707007$_Catching_fly_b$