Problem C: Dog Cannot Catch
A solution

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Outline

In this presentation, we will:

- Define the problem,
- Explain the assumptions,
- Create a model,
- Analyze the model,
- Explore limitations and future work.

Problem

- Make a model for Fritz's jaw movement,
- Track Fritz's jaw movement in the video to estimate constants,
- Model the errors in head orientation and path prediction,
- Estimate the effect of the initial throw, object size, and air resistance,
- Summarize the result and analyze the important factors.

Error Sources

Three main factors of failures:

- Parabola miscalculation,
- Mouth mistiming,
- Tricky shape/orientation.



- Took a different approach from many other teams
- Focused on mouth movement and observational error rather than body movement errors

Preliminary Assumptions

- The mouth is circular; the object is spherical.
- The mouth and the object are rigid.
- The dog moves to the correct left-right position and longitudinal orientation before the catch (2D systems).
- In the catching phase, the body is stationary.
- The dog only tracks motion with its eyes (not nose scent, etc.)
- The eyes accurately track the position/velocity of the object.
- The dog loses sight of the food once it opens its mouth.
- The dog has good sense of its mouth mechanics.
- The dog makes only one catching attempt.

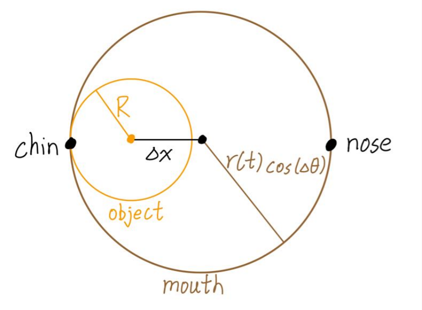
Model Framework

For the mouth to successfully catch the object,

$$R < r(t)\cos(\Delta\theta) - \Delta x,$$

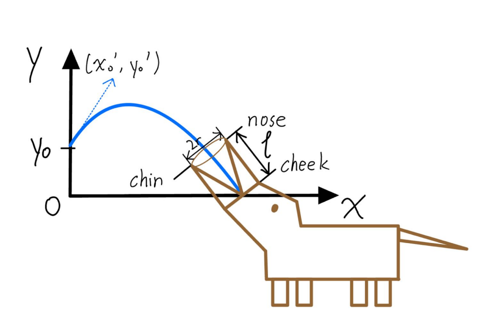
where

- R: object radius,
- $r(t)$: mouth radius at time t,
- $\Delta\theta$: orientation error,
- Δx: positional error.



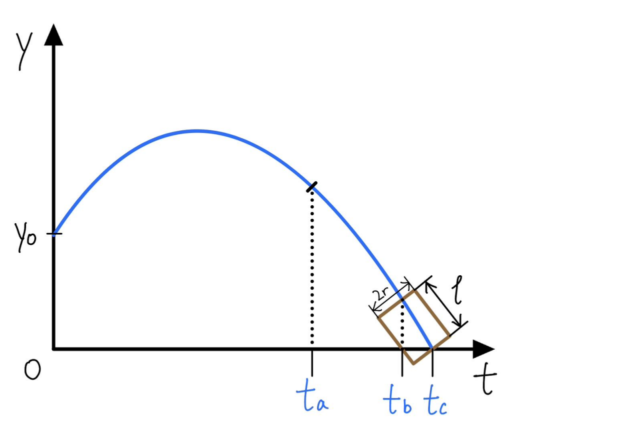
Coordinate System

- At the time of the catch, the dog's cheek is at $y = 0$.
- The object is thrown at position $(0, y_0)$ and velocity (x_0', y_0').
- The mouth opening diameter $(2r)$ is at the nose.
- The distance from nose/chin to cheek is $l = 6.5$ cm.



Time axis

- Three times,
- Need to solve the effective mouth radius $r(t_b)$.



Mouth Mechanics Model

- Assumed to be a damped harmonic oscillator:

$$I\theta'' + C\theta' + K\theta = \tau.$$

- Convert to a force model:

$$mr'' + cr' + kr = F.$$

- e^{st} substitution for homogeneous solution:

$$ms^2 + cs + k = 0$$

$$s = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}.$$

Mouth Mechanics Model

- $c^2 > 4mk$ for overdamped mode.

$$r_h(t) = Ae^{s_+t} + Be^{s_-t}.$$

- Particular solution: $r_p(t) = \frac{F}{k}$.
- Total solution:

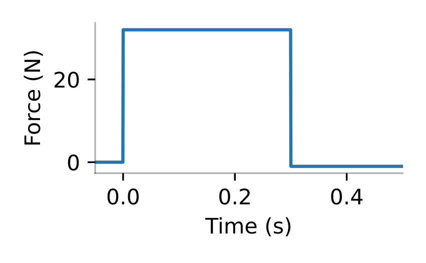
$$r(t) = \frac{F}{k} + Ae^{s_+t} + Be^{s_-t}.$$

- However, F(t) is actually a square bump.
- Can solve this ODE with opening and closing force values.

Mouth Mechanics Model

- Suppose the dog's input force switches from F_1 to F_2 at time T. $r(T)$ and $r'(T)$ for the F_1 solution can be used as initial conditions for the F_2 solution.

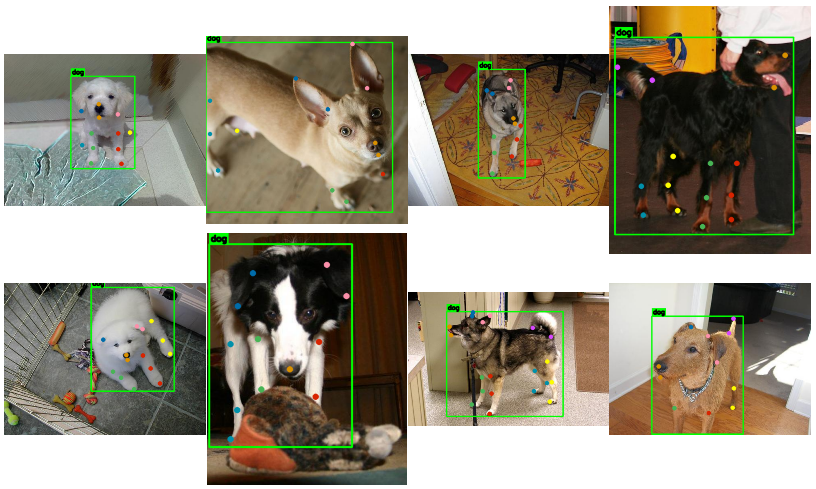
$$r(t) = \begin{cases} \frac{F_1}{k} + A_1 e^{S_+ t} + B_1 e^{s_- t} & \text{if } t_1 \leqslant t < t_1 + T \\ \frac{F_2}{k} + A_2 e^{S_+ t} + B_2 e^{s_- t} & \text{if } t \geqslant t_1 + T. \end{cases}$$



- F_1 initial condition: $r(0) = 0, r'(0) = 0$.
- Now, need to find values for the constants m, c, k, F_1, and F_2.

AI Tracker

AI Tracker



AI Tracker

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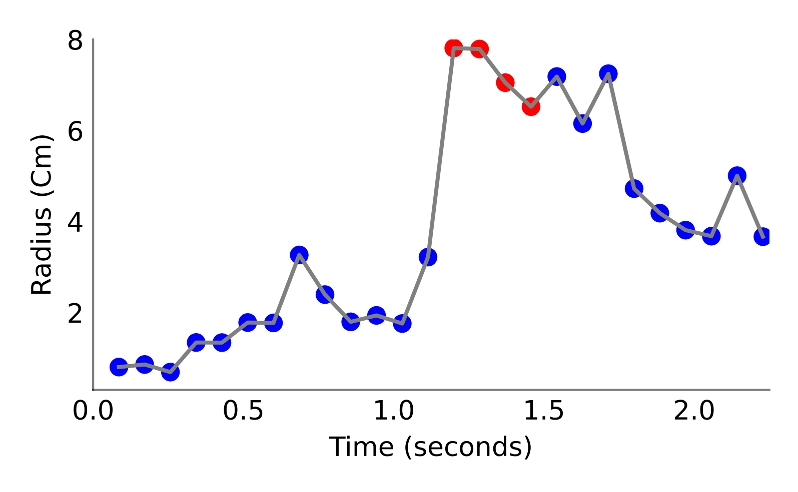
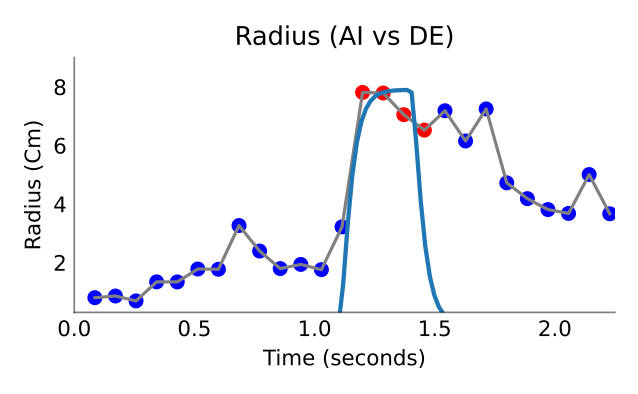


AI Tracker

- Constants: $m = 0.26\,\text{kg}$, $k = 400\,\text{N/m}$, $c = 21.5\,\text{kg/s}$.
- Force function: $F(t) = 32\,\text{N} \cdot H(t - t_1) - 33\,\text{N} \cdot H(t(t_1 + 0.3\,\text{s}))$.



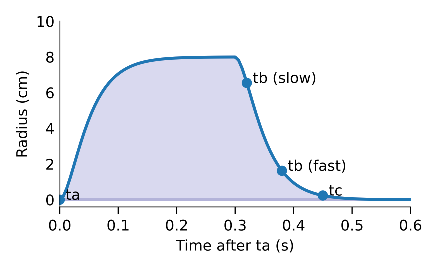
Timing Limit

Want mouth to close when the object reaches cheek.

- $t_c = t_a + T_{chomp}$,
- $t_b \approx t_c - \frac{l}{\sqrt{(x_c')^2 + (y_c')^2}}$.
- Let $T_{chomp} = 0.45$ s,
- $l = 6.5$ cm.

Assume the speeds at t_b and t_c are close.

Faster objects are harder to catch.



Tangent Line Approximation

- Lack of reference, straight line approximation,
- Observe the current y and y' to decide t_a,
- Opens $T_{chomp} = 0.45$ s before it thinks the object will reach its cheek ($y = 0$).
- Solve for t_a:

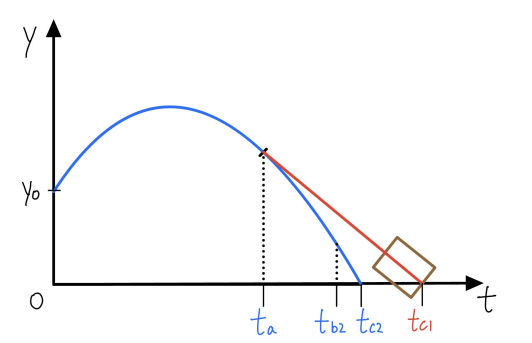
$$0 - y = y' \cdot T_{chomp}$$

$$-y_0 - y_0' t_a + \frac{1}{2} g t_a^2 = (y_0' - g t_a) T_{chomp}$$

$$t_a = \frac{(y_0' - gT_{chomp}) + \sqrt{(y_0' - gT_{chomp})^2 - 2g(y_0 + T_{chomp} y_0')}}{g}.$$

Tangent Line Approximation

- Fritz assumes the object will continue in a straight line beyond t_a,
- t_{c1}: Dog's time estimation,
- t_a, t_{b2}, t_{c2}: Actual time.



Positional Error

Calculate the variables of the two paths:

Dog's perception—linear drop:

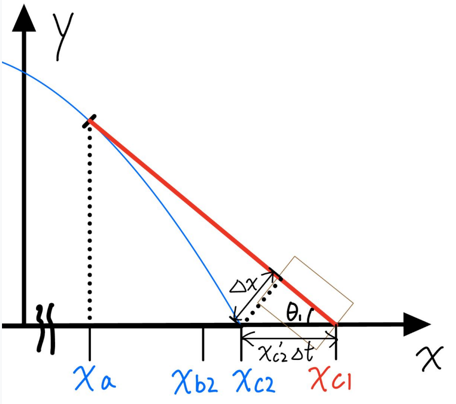
- $t_{c1} = t_a + T_{chomp} =$ linear drop time,
- $x'_{c1} = x'_0$ (ignore air resistance for now),
- $y'_{c1} = y'_a = y'_0 - gt_a$.

Actual—parabolic drop:

- $t_{c2} = \frac{y'_0 + \sqrt{{y'_0}^2 + 2gy_0}}{g} =$ parabolic drop time,
- $x'_{c2} = x'_0$,
- $y'_{c2} = y'_0 - gt_{c2}$.

Positional Error

$$\Delta x = x'_{b2} \Delta t \sin(\theta_1)$$
$$\approx x'_{c2}(t_{c1} - t_{c2}) \frac{y'_{c1}}{\sqrt{(x'_{c1})^2 + (y'_{c1})^2}}$$
$$= x'_{o}(t_{c1} - t_{c2}) \frac{y'_{a}}{\sqrt{(x'_{o})^2 + (y'_{a})^2}},$$

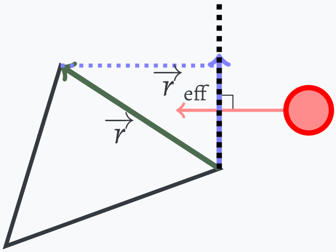


where θ_1 = perceived latitudinal angle.

Orientation Error

- Quantify how the dog's misestimation of head orientation affects catching outcome,
- Focus on latitudinal error $\Delta\theta$,
- Projection of Mouth width r onto normal of trajectory gives width food has to pass through.

$$r_{eff} = r\cos(\Delta\theta).$$



Orientation Error

- Affected by the parabola misestimation:

$$\begin{aligned}\Delta\theta &= \theta_2 - \theta_1 \\ &= \arctan\left(\frac{y'_{b2}}{x'_{b2}}\right) - \arctan\left(\frac{y'_{b1}}{x'_{b1}}\right) \\ &\approx \arctan\left(\frac{y'_{c2}}{x'_{c2}}\right) - \arctan\left(\frac{y'_{c1}}{x'_{c1}}\right),\end{aligned}$$

where

θ_1 = latitudinal angle in dog's perception,

θ_2 = actual latitudinal angle.

Model Summary

Under all errors, is the effective mouth opening big enough to catch the object?

$$R_{max} = r(t_{b2})\cos(\Delta\theta) - a_x \Delta x,$$

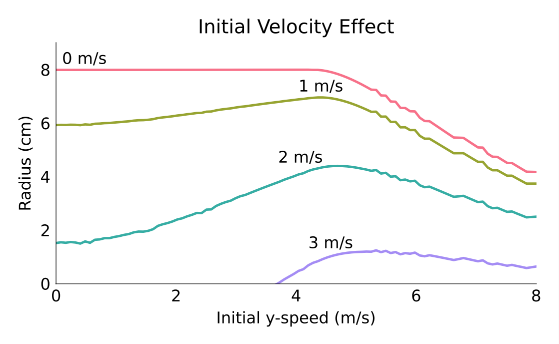
where a_x is a learning factor.

- Variables: $y_\circ, y'_\circ, x'_\circ, a_x$,
- Analyze R_{max} as a function of the variables,
- Default configuration:
  $y_\circ = 1.2\,\text{m}, y'_\circ = 2\,\text{m/s}, x'_\circ = 2\,\text{m/s}, a_x = 0.25$,
- Object is spherical, no air resistance.

Initial Velocity

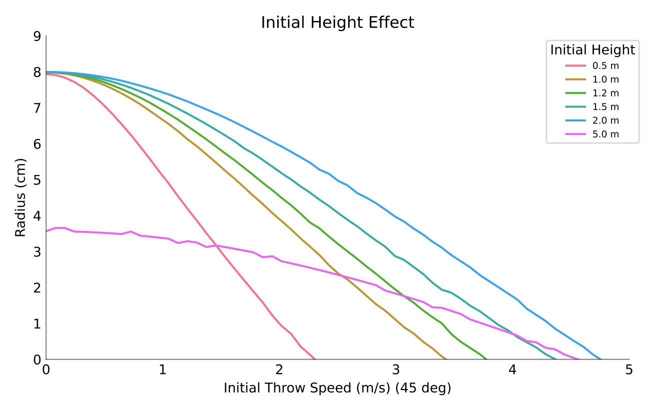
To make Fritz look bad:

- High x speed (parabola misestimation),
- Low y speed (parabola misestimation),
- High y speed (mouth mechanics).

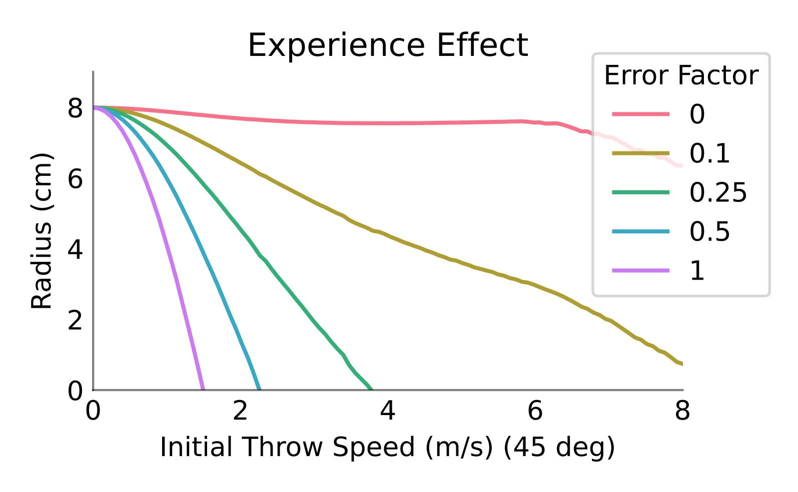


Initial Height

- Translates to y_0'.
- Graph is with a 45-degree throw angle.



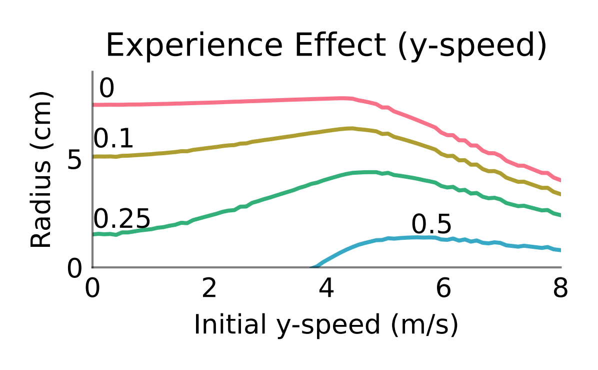
Experience


Experience Effect
Error Factor
0
0.1
0.25
0.5
1
Radius (cm)
0
2
4
6
8
Initial Throw Speed (m/s) (45 deg)

Experience

Fix x'_0 at 2 m/s.

- Parabola misestimation (low y-speed),
- Mouth mechanics issue (high y-speed).



Air Resistance

$$F_{drag} = \frac{1}{2}\rho A c_d v^2.$$

Solve differential equations:

$$\begin{cases} mx'' = -cx'|x'| \\ my'' = -mg - cy'|y'|, \end{cases}$$

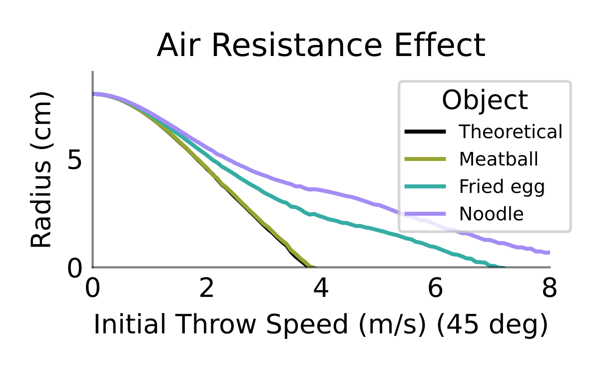
where $c = \frac{1}{2}\rho A c_d =$

- 2.15×10^{-4} kg/m for meatball,
- 5.94×10^{-3} kg/m for fried egg,
- 0.01×10^{-2} kg/m for noodle.

Air Resistance

Beneficial!

- y-axis: terminal velocity and linear approximation,
- x-axis: slight error.



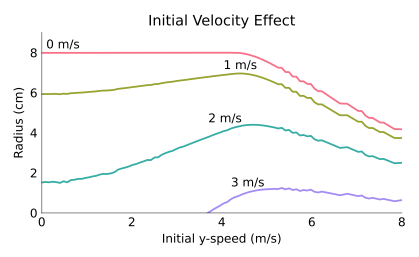
Non-spherical Shapes

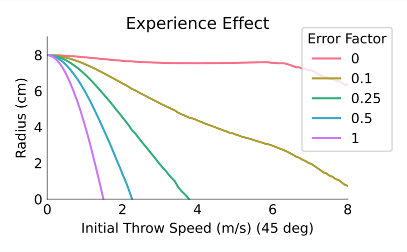
- If the object is non-spherical, the dog must predict its rotation and twist its head.
- If the object is rigid, it has to be caught by the center of mass.
- If the object is deformable, the dog can catch the edge and hold on.

Critial Factors

Tricky throw + clumsy Fritz,

- Initial throw speed (especially horizontal),
- Object size,
- The dog's experience.





Summary

$$R < r(t)\cos(\Delta\theta) - \Delta x$$

- Success (Is the mouth wide enough to fit the object?)
  - Mouth timing
    - Initial throw
    - Mouth mechanics limitation
  - Position error
    - Initial throw
    - Experience
    - Air resistance
  - Angle alignment
    - Object size
    - Object shape
    - Object rotation

Model Limitations

In the model:

- The dog can move to the intended x-position-Did not account for the body dynamics.
- Position and velocity are correctly perceived when the dog can see the food.
- Object is spherical, so rotation in flight wasn't modeled.
- The food is rigid enough that if it hits the dog's lips, the catch will fail due to bouncing.

Reflection on Feedback

- Needs more context and specify the problem more clearly;
- Order of topics;
- Data presentation.

Unused Complexities

- We had many ideas for how to model this situation
- Had to use a framework simple enough to be developed fully in reasonable time
- Narrowed the scope of the model to fit in the video and our schedules

Visual Limitation

- Fritz estimates distance based on object's length in its eyes l.
- Let y be the actual distance.

$$l = \frac{A}{y}$$
$$dl = -\frac{A}{y^2}dy$$
$$dy = -\frac{1}{A}dly^2$$
$$dy \propto y^2$$

- The farther the distance, the greater the error.

Inverse ODE for Jaw Movement

- The DHO model of the dog's mouth does not take into account the anatomical limit to how wide the jaw can open.
- This can be amended with an ODE with an inverse:

$$mr'' = \frac{k}{(r - r_{max})^n}$$

- With this ODE, the resisting force could be very low until the mouth reaches its maximum width r_{max}
- Possibly more accurate than DHO model across many driving forces
- However, solving this ODE analytically took too much time
- DHO solution matched dog's mouth movement well enough

Data Collection

- We used an AI to gather data
- The AI struggled to gather data
- We should have just used Tracker Video Analysis

Future Work

- Determine what factors still cause error once the dog learns to predict parabolas correctly,
- Account for odd shapes by modeling rotation and the need to catch the center of mass
- Introduce stochastic errors into the ODE to give the catching probability given different throws and learning levels
- This would simulate random errors inherent in any living organism

Alternative approaches from other teams

- Simulate the dog's body dynamics;
- AI-trained dog to model learning;
- Calculate the optimal standing point given a random initial

Works Cited I

- Sizing instructions for dog collars.

  DTDogCollars Website.

  https://www.dtdogcollars.com/Sizing-Instructions-for-Muzzles-Harness-Collars-Leashes-s/192.htm.

- Fritz Dog.

  Fritz learns to catch compilation 1, 2015.

- Fritz Dog.

  Fritz learns to catch compilation 2, 2015.

Works Cited II

- Aditya Khosla, Nityananda Jayadevaprakash, Bangpeng Yao, and Li Fei-Fei.

  ImageNet Dogs.

  http://vision.stanford.edu/aditya86/ImageNetDogs/, 2011.

- P. C. Sereno.

  Taxonomy, morphology, masticatory function and phylogeny of heterodontosaurid dinosaurs.

  ZooKeys, 226:1–225, 2012.