The factoring method

That will change your life

Dr. Peyam Tabrizian

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The Culprit

$$y'' - 5y' + 6y = 0$$

STEP 1: Guess $y = e^{rt}$

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STEP 2: Plug into $y'' - 5y' + 6y = 0$

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STEP 6: $y = A\,e^{2t} + B\,e^{3t}$

STEP 1: Guess $y = e^{rt}$

STEP 2: Plug into $y'' - 5y' + 6y = 0$

STEP 3: $r = 2$ and $r = 3$

STEP 4: e^{2t} and e^{3t} are solutions

STEP 5: Wronskian $\neq 0$

STEP 6: $y = A\,e^{2t} + B\,e^{3t}$

Is there a more direct way?

Is there a more direct way?

YES

Dr Peyam

Dr Peyam

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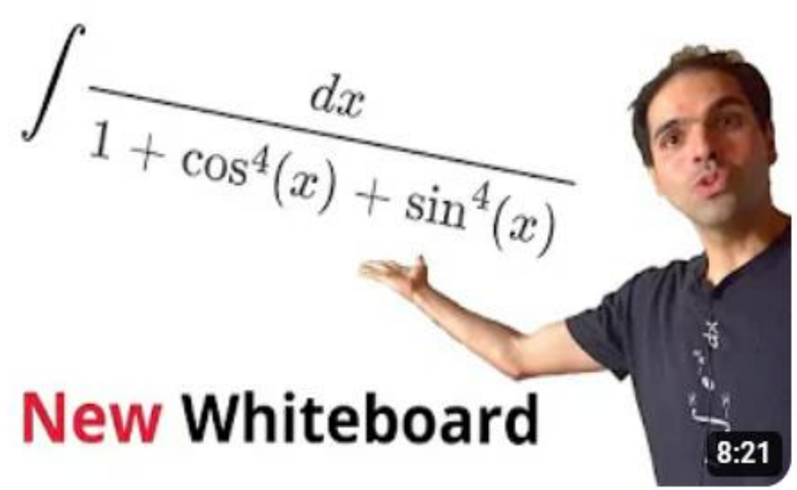

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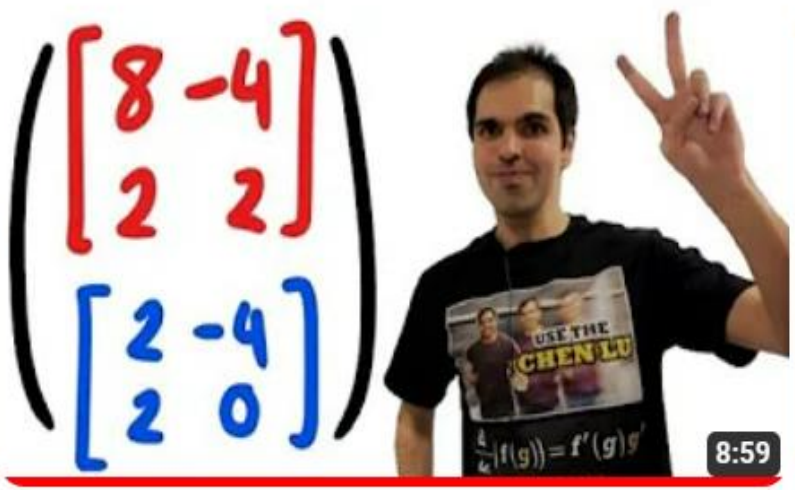

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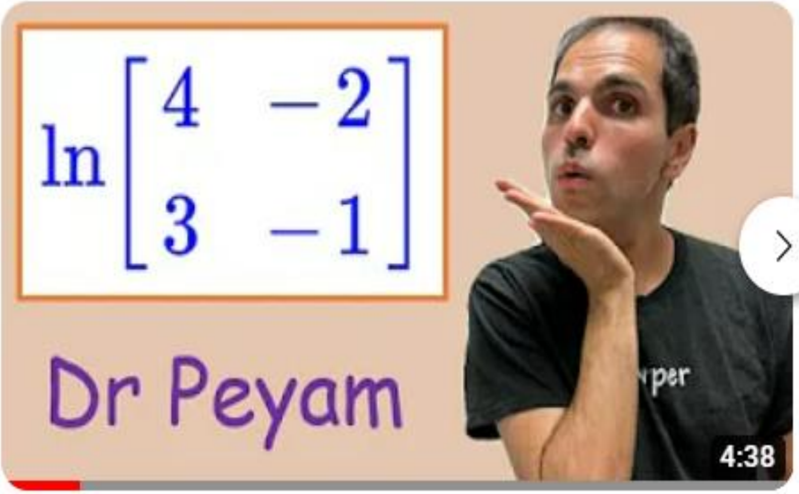

logarithm of a matrix

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$$y'' - 5y' + 6y = 0$$

The cool method not taught in calculus

Differential Operators

Differential Operators

$$Dy = y'$$

Differential Operators

$$Dy = y'$$

$$D^2y = y''$$

Differential Operators

$$Dy = y'$$

$$D^2y = y''$$

$$\mathrm{I}y = y$$

Dynamic Perspective

$$y'' - 5y' + 6y = 0$$

Dynamic Perspective

$$y'' - 5y' + 6y = 0$$

$$D^2y - 5Dy + 6Iy = 0$$

Dynamic Perspective

$$y'' - 5y' + 6y = 0$$

$$D^2y - 5Dy + 6Iy = 0$$

$$\left(D^2 - 5D + 6I\right)y = 0$$

Dynamic Perspective

$$y'' - 5y' + 6y = 0$$

$$D^2y - 5Dy + 6Iy = 0$$

$$\left(D^2 - 5D + 6I\right)y = 0$$

$$(D - 2I)(D - 3I)y = 0$$

Dynamic Perspective

$$y'' - 5y' + 6y = 0$$

$$D^2 y - 5Dy + 6Iy = 0$$

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Dynamic Perspective

$$y'' - 5y' + 6y = 0$$

$$D^2 y - 5Dy + 6Iy = 0$$

$$\left(D^2 - 5D + 6I\right)y = 0$$

$$(D - 2I)\underbrace{(D - 3I)y}_{z} = 0$$

First-Order ODE

$$(D - 2I)z = 0$$

First-Order ODE

$$(D - 2I)z = 0$$

$$z' - 2z = 0$$

First-Order ODE

$$(D - 2I)z = 0$$

$$z' - 2z = 0$$

$$z = A\,e^{2t}$$

First-Order ODE

$$(D - 2I)z = 0$$

$$z' - 2z = 0$$

$$z = A\, e^{2t}$$

$$(D - 3I)y = A\, e^{2t}$$

First-Order ODE

$$(D - 2I)z = 0$$

$$z' - 2z = 0$$

$$z = A\,e^{2t}$$

$$(D - 3I)y = A\,e^{2t}$$

$$y' - 3y = A\,e^{2t}$$

Integrating Factors

$$e^{-3t}y' - 3e^{-3t}y = A\,e^{-3t}e^{2t}$$

Integrating Factors

$$e^{-3t}y' - 3e^{-3t}y = A\,e^{-3t}e^{2t}$$

$$\left(e^{-3t}y\right)' = Ae^{-t}$$

Integrating Factors

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$$\left(e^{-3t}y\right)' = Ae^{-t}$$

$$e^{-3t}y = \int Ae^{-t}dt$$

Integrating Factors

$$e^{-3t}y' - 3e^{-3t}y = A\,e^{-3t}e^{2t}$$

$$\left(e^{-3t}y\right)' = Ae^{-t}$$

$$e^{-3t}y = \int Ae^{-t}dt$$

$$e^{-3t}y = -Ae^{-t} + B$$

Integrating Factors

$$e^{-3t}y' - 3e^{-3t}y = A\,e^{-3t}e^{2t}$$

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$$e^{-3t}y = \int Ae^{-t}dt$$

$$e^{-3t}y = -Ae^{-t} + B$$

$$y = e^{3t}(-Ae^{-t} + B)$$

Integrating Factors

$$e^{-3t}y' - 3e^{-3t}y = A\,e^{-3t}e^{2t}$$

$$\left(e^{-3t}y\right)' = Ae^{-t}$$

$$e^{-3t}y = \int Ae^{-t}dt$$

$$e^{-3t}y = -Ae^{-t} + B$$

$$y = e^{3t}(-Ae^{-t} + B)$$

$$y = -A\,e^{2t} + Be^{3t}$$

Solution

$$y'' - 5y' + 6y = 0$$

Solution

$$y'' - 5y' + 6y = 0$$

$$\downarrow$$

$$y = A\,e^{2t} + Be^{3t}$$

Pros

- Direct
- Elegant: No guessing whatsoever
- Dynamic point of view
- No “leap” between first order and second order ODE
- Uniqueness

Cons

- No Wronskian
- No discussion of linear independence
- Hard to generalize to variable coefficients
- Complex roots case is awkward

JOKE

$$e^x$$

D

$$e^{x}$$

$$\frac{\partial}{\partial y}$$

$$e^{x}$$

Repeated Roots

$$y'' - 4y' + 4y = 0$$

Repeated Roots

$$y'' - 4y' + 4y = 0$$

$$\boxed{y = A\,e^{2t} + Bt\,e^{2t}}$$

Repeated Roots

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$$(D - 2I)^2 y = 0$$

Repeated Roots

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Repeated Roots

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$$D^2y - 4Dy + 4Iy = 0$$

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Repeated Roots

$$y'' - 4y' + 4y = 0$$

$$D^2y - 4Dy + 4Iy = 0$$

$$(D - 2I)^2y = 0$$

$$(D - 2I)\underbrace{(D - 2I)y}_{z} = 0$$

Repeated Roots

$$(D - 2I)z = 0$$

Repeated Roots

$$(D - 2I)z = 0$$

$$z' - 2z = 0$$

Repeated Roots

$$(D - 2I)z = 0$$

$$z' - 2z = 0$$

$$z = A\,e^{2t}$$

Repeated Roots

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$$z = A\,e^{2t}$$

$$(D - 2I)y = A\,e^{2t}$$

Repeated Roots

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$$y' - 2y = A\,e^{2t}$$

Repeated Roots

$$e^{-2t}y' - 2e^{-2t}y = A\,e^{-2t}e^{2t}$$

Repeated Roots

$$e^{-2t}y' - 2e^{-2t}y = A\,e^{-2t}e^{2t}$$

$$\left(e^{-2t}y\right)' = A$$

Repeated Roots

$$e^{-2t}y' - 2e^{-2t}y = A\,e^{-2t}e^{2t}$$

$$\left(e^{-2t}y\right)' = A$$

$$e^{-2t}y = \int A dt$$

Repeated Roots

$$e^{-2t}y' - 2e^{-2t}y = A\,e^{-2t}e^{2t}$$

$$\left(e^{-2t}y\right)' = A$$

$$e^{-2t}y = \int A dt$$

$$e^{-2t}y = At + B$$

Repeated Roots

$$e^{-2t}y' - 2e^{-2t}y = A\,e^{-2t}e^{2t}$$

$$\left(e^{-2t}y\right)' = A$$

$$e^{-2t}y = \int A dt$$

$$e^{-2t}y = At + B$$

$$y = (At + B)e^{2t}$$

Complex Roots

$$y'' + y = 0$$

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$$D^2 y + I y = 0$$

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$$(D + iI)(D - iI)y = 0$$

Complex Roots

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$$D^2 y + I y = 0$$

$$\left(D^2 + I\right) y = 0$$

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$$y = A\, e^{it} + B\, e^{-it}$$

Complex Roots

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Complex Roots

$$y = A\,e^{it} + B\,e^{-it}$$

$$y = A\,(\cos(t) + i\sin(t)) + B\,(\cos(-t) + i\sin(-t))$$

Complex Roots

$$y = A\,e^{it} + B\,e^{-it}$$

$$y = A\,(\cos(t) + i\sin(t)) + B\,(\cos(t) - i\sin(t))$$

Complex Roots

$$y = A\,e^{it} + B\,e^{-it}$$

$$y = A\,(\cos(t) + i\sin(t)) + B\,(\cos(t) - i\sin(t))$$

$$y = (A_1 + i\,B_1)(\cos(t) + i\sin(t)) + (A_2 + i\,B_2)(\cos(t) - i\sin(t))$$

Complex Roots

$$y = A\,e^{it} + B\,e^{-it}$$

$$y = A\,(\cos(t) + i\sin(t)) + B\,(\cos(t) - i\sin(t))$$

$$y = (A_1 + i\,B_1)(\cos(t) + i\sin(t)) + (A_2 + i\,B_2)(\cos(t) - i\sin(t))$$

Complex Roots

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$$y = (A_1 + i\,B_1)(\cos(t) + i\sin(t)) + (A_2 + i\,B_2)(\cos(t) - i\sin(t))$$

$$y = A_1\cos(t) - B_1\sin(t) + A_2\cos(t) + B_2\sin(t) + i\ \ldots$$

Complex Roots

$$y = A\,e^{it} + B\,e^{-it}$$

$$y = A\,(\cos(t) + i\sin(t)) + B\,(\cos(t) - i\sin(t))$$

$$y = (A_1 + i\,B_1)(\cos(t) + i\sin(t)) + (A_2 + i\,B_2)(\cos(t) - i\sin(t))$$

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$$y = A_1\cos(t) - B_1\sin(t) + A_2\cos(t) + B_2\sin(t) + i\ \ldots$$

$$y = (A_1 + A_2)\cos(t) + (-B_1 + B_2)\sin(t) + i\ \ldots$$

Complex Roots

$$y = A\,e^{it} + B\,e^{-it}$$

$$y = A\,(\cos(t) + i\sin(t)) + B\,(\cos(t) - i\sin(t))$$

$$y = (A_1 + i\,B_1)(\cos(t) + i\sin(t)) + (A_2 + i\,B_2)(\cos(t) - i\sin(t))$$

$$y = A_1\cos(t) - B_1\sin(t) + A_2\cos(t) + B_2\sin(t) + i\ \ldots$$

$$y = (A_1 + A_2)\cos(t) + (-B_1 + B_2)\sin(t) + i\ \ldots$$

$$y = C\cos(t) + D\sin(t) + i\ \ldots$$

Complex Roots

$$Re(y) = C\cos(t) + D\sin(t)$$

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Upshot: If y is real, then $Re(y) = y$

Complex Roots

$$Re(y) = C\cos(t) + D\sin(t)$$

Upshot: If y is real, then $Re(y) = y$

$$\boxed{y = C\cos(t) + D\sin(t)}$$

Complex Roots

$$y'' + y = 0$$

Complex Roots

$$y'' + y = 0$$

$$\downarrow$$

$$\boxed{y = A\cos(t) + B\sin(t)}$$

Inhomogeneous Equations

$$y'' - 3y' - 4y = 5e^{4t}$$

Inhomogeneous Equations

$$y'' - 3y' - 4y = 5e^{4t}$$

$$(D^2 - 3D - 4I)y = 5e^{4t}$$

Inhomogeneous Equations

$$y'' - 3y' - 4y = 5e^{4t}$$

$$\left(D^2 - 3D - 4I\right)y = 5e^{4t}$$

$$(D + I)(D - 4I)y = 5e^{4t}$$

Inhomogeneous Equations

$$y'' - 3y' - 4y = 5e^{4t}$$

$$\left(D^2 - 3D - 4I\right)y = 5e^{4t}$$

$$(D + I)(D - 4I)y = 5e^{4t}$$

Inhomogeneous Equations

$$y'' - 3y' - 4y = 5e^{4t}$$

$$\left(D^2 - 3D - 4I\right)y = 5e^{4t}$$

$$(D + I)\underbrace{(D - 4I)y}_{z} = 5e^{4t}$$

Inhomogeneous Equations

$$(D + I)z = 5e^{4t}$$

Inhomogeneous Equations

$$(D + I)z = 5e^{4t}$$

$$z' + z = 5e^{4t}$$

Inhomogeneous Equations

$$(D + I)z = 5e^{4t}$$

$$z' + z = 5e^{4t}$$

$$e^{t}z' + e^{t}z = 5\, e^{t}\, e^{4t}$$

Inhomogeneous Equations

$$(D + I)z = 5e^{4t}$$

$$z' + z = 5e^{4t}$$

$$e^t z' + e^t z = 5\, e^t\, e^{4t}$$

$$(e^t z)' = 5\, e^{5t}$$

Inhomogeneous Equations

$$(D + I)z = 5e^{4t}$$

$$z' + z = 5e^{4t}$$

$$e^t z' + e^t z = 5\, e^t\, e^{4t}$$

$$(e^t z)' = 5\, e^{5t}$$

$$e^t z = e^{5t} + A$$

Inhomogeneous Equations

$$(D + I)z = 5e^{4t}$$

$$z' + z = 5e^{4t}$$

$$e^t z' + e^t z = 5\, e^t\, e^{4t}$$

$$(e^t z)' = 5\, e^{5t}$$

$$e^t z = e^{5t} + A$$

$$z = e^{4t} + Ae^{-t}$$

Inhomogeneous Equations

$$y' - 4y = e^{4t} + A\,e^{-t}$$

Inhomogeneous Equations

$$y' - 4y = e^{4t} + A\,e^{-t}$$

$$e^{-4t}y' - 4\,e^{-4t}y = e^{-4t}(e^{4t} + A\,e^{-t})$$

Inhomogeneous Equations

$$y' - 4y = e^{4t} + A\,e^{-t}$$

$$e^{-4t}y' - 4\,e^{-4t}y = e^{-4t}(e^{4t} + A\,e^{-t})$$

$$(e^{-4t}y)' = 1 + Ae^{-5t}$$

Inhomogeneous Equations

$$y' - 4y = e^{4t} + A\,e^{-t}$$

$$e^{-4t}y' - 4\,e^{-4t}y = e^{-4t}(e^{4t} + A\,e^{-t})$$

$$(e^{-4t}y)' = 1 + Ae^{-5t}$$

$$e^{-4t}y = t - \left(\frac{A}{5}\right)e^{-5t} + B$$

Inhomogeneous Equations

$$y' - 4y = e^{4t} + A\,e^{-t}$$

$$e^{-4t}y' - 4\,e^{-4t}y = e^{-4t}(e^{4t} + A\,e^{-t})$$

$$(e^{-4t}y)' = 1 + Ae^{-5t}$$

$$e^{-4t}y = t - \left(\frac{A}{5}\right)e^{-5t} + B$$

$$y = t\,e^{4t} - \left(\frac{A}{5}\right)e^{t} + B\,e^{4t}$$

Inhomogeneous Equations

$$y'' - 3y' - 4y = 5e^{4t}$$

Inhomogeneous Equations

$$y'' - 3y' - 4y = 5e^{4t}$$

$$y = A\,e^{t} + B\,e^{4t} + t\,e^{4t}$$

Systems

$$\mathbf{x}'' - 5A\mathbf{x}' + 6A^2\mathbf{x} = \mathbf{0}$$

Systems

$$\mathbf{x}'' - 5A\mathbf{x}' + 6A^2\mathbf{x} = \mathbf{0}$$

$$D^2\mathbf{x} - 5A\,D\mathbf{x} + 6A^2\mathbf{x} = 0$$

Systems

$$\mathbf{x}'' - 5A\mathbf{x}' + 6A^2\mathbf{x} = \mathbf{0}$$

$$D^2\mathbf{x} - 5A\,D\mathbf{x} + 6A^2\mathbf{x} = 0$$

$$\left(D^2 - 5AD + 6A^2\right)\mathbf{x} = 0$$

Systems

$$\mathbf{x}'' - 5A\mathbf{x}' + 6A^2\mathbf{x} = \mathbf{0}$$

$$D^2\mathbf{x} - 5A\,D\mathbf{x} + 6A^2\mathbf{x} = 0$$

$$\left(D^2 - 5AD + 6A^2\right)\mathbf{x} = 0$$

$$(D - 2A)(D - 3A)\mathbf{x} = 0$$

Systems

$$\mathbf{x}'' - 5A\mathbf{x}' + 6A^2\mathbf{x} = \mathbf{0}$$

$$D^2\mathbf{x} - 5A\,D\mathbf{x} + 6A^2\mathbf{x} = 0$$

$$\left(D^2 - 5AD + 6A^2\right)\mathbf{x} = 0$$

$$(D - 2A)\underbrace{(D - 3A)\mathbf{x}}_{\mathbf{z}} = 0$$

Systems

$$(D - 2A)\mathbf{z} = \mathbf{0}$$

Systems

$$(D - 2A)\mathbf{z} = \mathbf{0}$$

$$\mathbf{z}' - 2A\mathbf{z} = \mathbf{0}$$

Systems

$$(D - 2A)\mathbf{z} = \mathbf{0}$$

$$\mathbf{z}' - 2A\mathbf{z} = \mathbf{0}$$

$$\mathbf{z} = e^{2At}\mathbf{C}$$

Systems

$$(D - 2A)\mathbf{z} = \mathbf{0}$$

$$\mathbf{z}' - 2A\mathbf{z} = \mathbf{0}$$

$$\mathbf{z} = e^{2At}\mathbf{C}$$

$$(D - 3A)\mathbf{x} = e^{2At}\mathbf{C}$$

Systems

$$(D - 2A)\mathbf{z} = \mathbf{0}$$

$$\mathbf{z}' - 2A\mathbf{z} = \mathbf{0}$$

$$\mathbf{z} = e^{2At}\boldsymbol{C}$$

$$(D - 3A)\mathbf{x} = e^{2At}\boldsymbol{C}$$

$$\mathbf{x}' - 3A\mathbf{x} = e^{2At}\boldsymbol{C}$$

Systems

$$e^{-3At}\mathbf{x}' - 3e^{-3At}\mathbf{x} = e^{-3At}e^{2At}\boldsymbol{C}$$

Systems

$$e^{-3At}\mathbf{x}' - 3e^{-3At}\mathbf{x} = e^{-3At}e^{2At}\boldsymbol{C}$$

$$\left(e^{-3At}\mathbf{x}\right)' = e^{-At}\boldsymbol{C}$$

Systems

$$e^{-3At}\mathbf{x}' - 3e^{-3At}\mathbf{x} = e^{-3At}e^{2At}\boldsymbol{C}$$

$$\left(e^{-3At}\mathbf{x}\right)' = e^{-At}\boldsymbol{C}$$

$$e^{-3At}\mathbf{x} = \int e^{-At}\boldsymbol{C}\,dt$$

Systems

$$e^{-3At}\mathbf{x}' - 3e^{-3At}\mathbf{x} = e^{-3At}e^{2At}\boldsymbol{C}$$

$$\left(e^{-3At}\mathbf{x}\right)' = e^{-At}\boldsymbol{C}$$

$$e^{-3At}\mathbf{x} = \int e^{-At}\boldsymbol{C}\,dt$$

$$e^{-3At}\mathbf{x} = -A^{-1}e^{-At}\boldsymbol{C} + \boldsymbol{D}$$

Systems

$$e^{-3At}\mathbf{x}' - 3e^{-3At}\mathbf{x} = e^{-3At}e^{2At}\boldsymbol{C}$$

$$\left(e^{-3At}\mathbf{x}\right)' = e^{-At}\boldsymbol{C}$$

$$e^{-3At}\mathbf{x} = \int e^{-At}\boldsymbol{C}\,dt$$

$$e^{-3At}\mathbf{x} = -A^{-1}e^{-At}\boldsymbol{C} + \boldsymbol{D}$$

$$\mathbf{x} = e^{3At}\left(-A^{-1}e^{-At}\boldsymbol{C} + \boldsymbol{D}\right)$$

Systems

$$e^{-3At}\mathbf{x}' - 3e^{-3At}\mathbf{x} = e^{-3At}e^{2At}\boldsymbol{C}$$

$$\left(e^{-3At}\mathbf{x}\right)' = e^{-At}\boldsymbol{C}$$

$$e^{-3At}\mathbf{x} = \int e^{-At}\boldsymbol{C}\, dt$$

$$e^{-3At}\mathbf{x} = -A^{-1}e^{-At}\boldsymbol{C} + \boldsymbol{D}$$

$$\mathbf{x} = e^{3At}(-A^{-1}e^{-At}\boldsymbol{C} + \boldsymbol{D})$$

$$\mathbf{x} = e^{2At}(-A^{-1}\boldsymbol{C}) + e^{3At}\boldsymbol{D}$$

Systems

$$\mathbf{x}'' - 5A\mathbf{x}' + 6A^2\mathbf{x} = \mathbf{0}$$

Systems

$$\mathbf{x}'' - 5A\mathbf{x}' + 6A^2\mathbf{x} = \mathbf{0}$$

$$\downarrow$$

$$\boxed{\mathbf{x} = e^{2At}\boldsymbol{C} + e^{3At}\boldsymbol{D}}$$

Remarks

- If A is not invertible, use Jordan form
- What if you had:

$$(D - 3A)(D - 2B)\mathbf{x} = \mathbf{0}$$

Tricky because matrix exponentials don't necessarily commute

Matrix cosine

$$\mathbf{x}'' + A^2\mathbf{x} = \mathbf{0}$$

Matrix cosine

$$\mathbf{x}'' + A^2\mathbf{x} = \mathbf{0}$$

$$\mathbf{x} = \cos(At)\,\mathbf{C} + \sin(At)\,\boldsymbol{D}$$

PDE

$$u_{tt} = c^2 u_{xx}$$

PDE

$$u_{tt} = c^2 u_{xx}$$

$$\frac{\partial}{\partial t} u = u_t$$

PDE

$$u_{tt} = c^2 u_{xx}$$

$$\frac{\partial}{\partial t} u = u_t$$

$$\left(\frac{\partial}{\partial t}\right)^2 u = u_{tt}$$

PDE

$$u_{tt} = c^2 u_{xx}$$

$$\frac{\partial}{\partial t} u = u_t \qquad \frac{\partial}{\partial x} u = u_x$$

$$\left(\frac{\partial}{\partial t}\right)^2 u = u_{tt} \qquad \left(\frac{\partial}{\partial x}\right)^2 u = u_{xx}$$

PDE

$$u_{tt} - c^2 u_{xx} = 0$$

PDE

$$u_{tt} - c^2 u_{xx} = 0$$

$$\left[\left(\frac{\partial}{\partial t}\right)^2 - c^2\left(\frac{\partial}{\partial x}\right)^2\right]u = 0$$

PDE

$$u_{tt} - c^2 u_{xx} = 0$$

$$\left[\left(\frac{\partial}{\partial t}\right)^2 - c^2\left(\frac{\partial}{\partial x}\right)^2\right]u = 0$$

$$\left[\left(\frac{\partial}{\partial t}\right) - c\left(\frac{\partial}{\partial x}\right)\right]\left[\left(\frac{\partial}{\partial t}\right) + c\left(\frac{\partial}{\partial x}\right)\right]u = 0$$

PDE

$$u_{tt} - c^2 u_{xx} = 0$$

$$\left[\left(\frac{\partial}{\partial t}\right)^2 - c^2\left(\frac{\partial}{\partial x}\right)^2\right]u = 0$$

$$\left[\left(\frac{\partial}{\partial t}\right) - c\left(\frac{\partial}{\partial x}\right)\right]\underbrace{\left[\left(\frac{\partial}{\partial t}\right) + c\left(\frac{\partial}{\partial x}\right)\right]u}_{v} = 0$$

PDE

$$\left[\left(\frac{\partial}{\partial t}\right) - c\left(\frac{\partial}{\partial x}\right)\right] v = 0$$

PDE

$$\left[\left(\frac{\partial}{\partial t}\right) - c\left(\frac{\partial}{\partial x}\right)\right] v = 0$$

$$v_t - c\, v_x = 0$$

PDE

$$\left[\left(\frac{\partial}{\partial t}\right) - c\left(\frac{\partial}{\partial x}\right)\right]v = 0$$

$$v_t - c\,v_x = 0$$

$$v(x,t) = f(x + ct)$$

PDE

$$\left[\left(\frac{\partial}{\partial t}\right) - c\left(\frac{\partial}{\partial x}\right)\right] v = 0$$

$$v_t - c\, v_x = 0$$

$$v(x,t) = f(x + ct)$$

$$\left[\left(\frac{\partial}{\partial t}\right) + c\left(\frac{\partial}{\partial x}\right)\right] u = f(x + ct)$$

PDE

$$\left[\left(\frac{\partial}{\partial t}\right) - c\left(\frac{\partial}{\partial x}\right)\right] v = 0$$

$$v_t - c\, v_x = 0$$

$$v(x,t) = f(x + ct)$$

$$\left[\left(\frac{\partial}{\partial t}\right) + c\left(\frac{\partial}{\partial x}\right)\right] u = f(x + ct)$$

$$u(x,t) = F(x + ct) + G(x - ct)$$

PDE

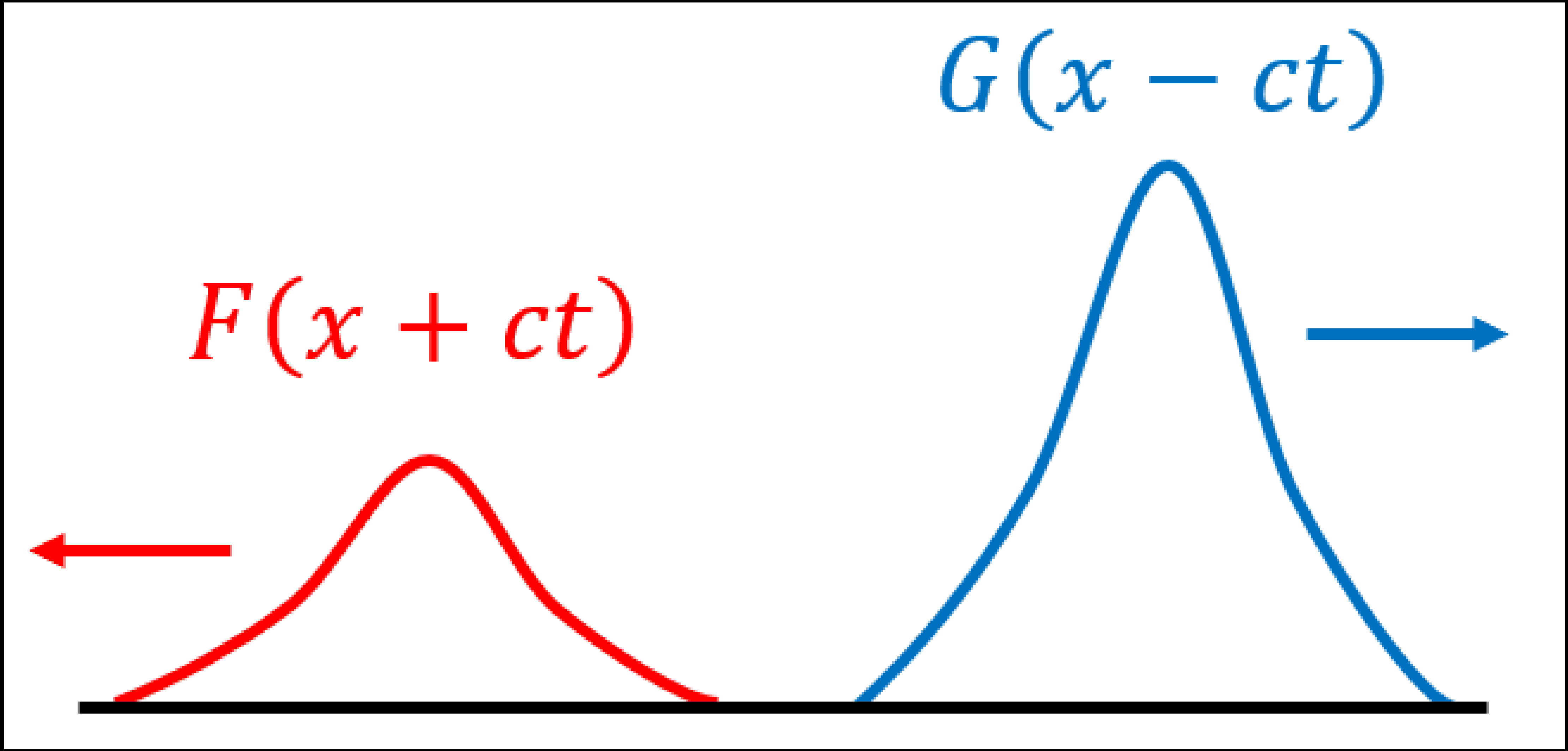
$$u_{tt} = c^2 u_{xx}$$

PDE

$$u_{tt} = c^2 u_{xx}$$

$$\downarrow$$

$$\boxed{u(x,t) = F(x+ct) + G(x-ct)}$$



$G(x - ct)$
$F(x + ct)$

Dr Peyam

Thank you!!!