Using **Maple** to Teach ODE-modeled Physics

A Paradigm Shift in Teaching Undergraduate Physics

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What I learned about my physics courses by teaching with a biologist and a chemist 7 times:

99% of the students in introductory undergraduate physics are *NOT* going to work towards a Ph.D. in physics

- Show students *why* physics is important to their interests
- *Emphasize* teaching applicable & transferable principles

Pedagogy for Physics:

1. Conceptual understanding of principles

2. Mathematical representation of principles

2. Hands-on exploration and measurement

2. Algebraic and numerical exploration

History: Adding computational problem-solving to introductory physics courses

Complexity

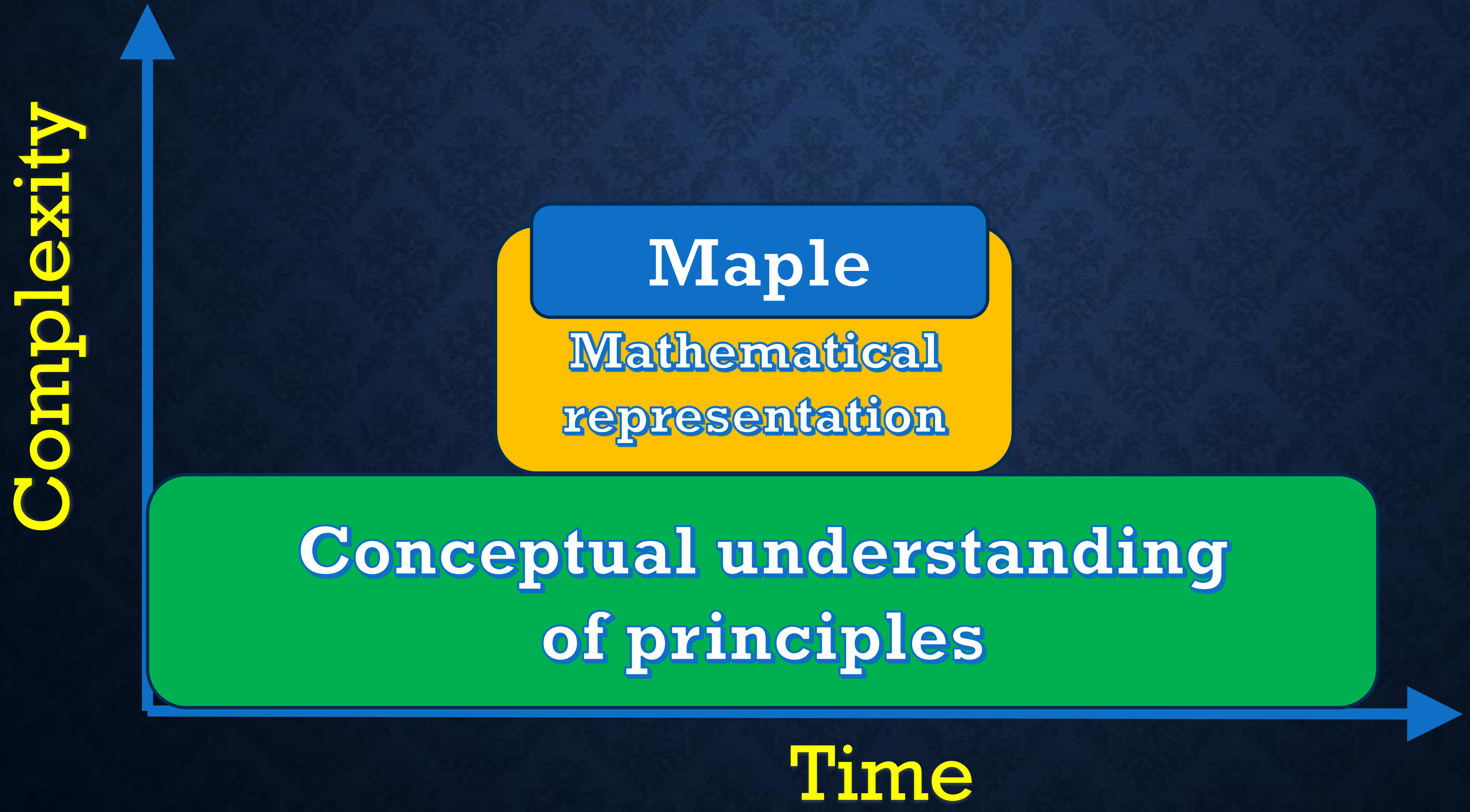
Coding

Mathematical representation, technique memorization, & tricks

Conceptual understanding

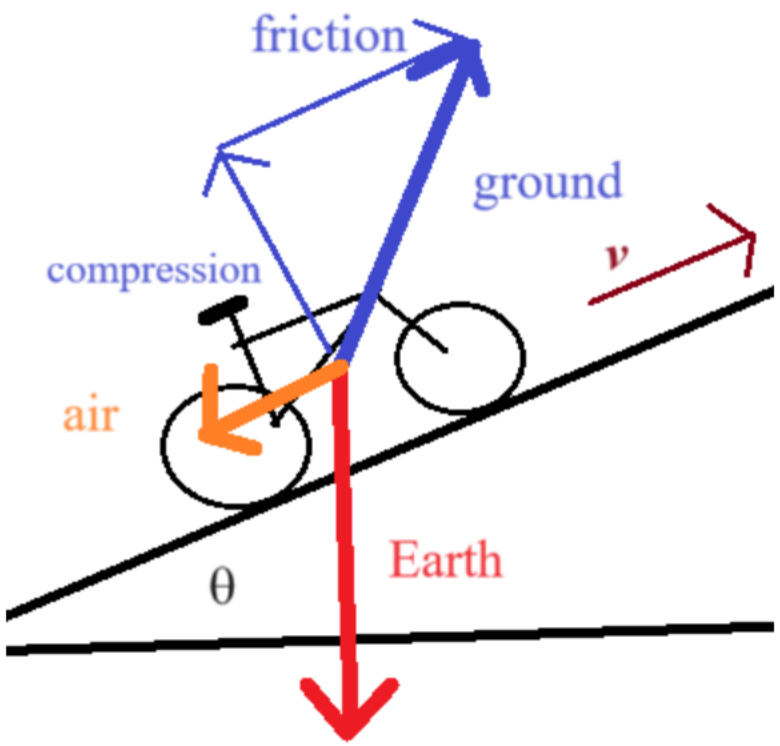
Time

Adding **Maple** to an introductory physics course



Physics: Top-down problem-solving

a) Riding a bike ***uphill*** at a constant velocity. Given that the speed, mass and power of the rider can be measured, derive the expression for the drag coefficient constant of the air. $|\vec{F}_{air}| = C\,v^2$.

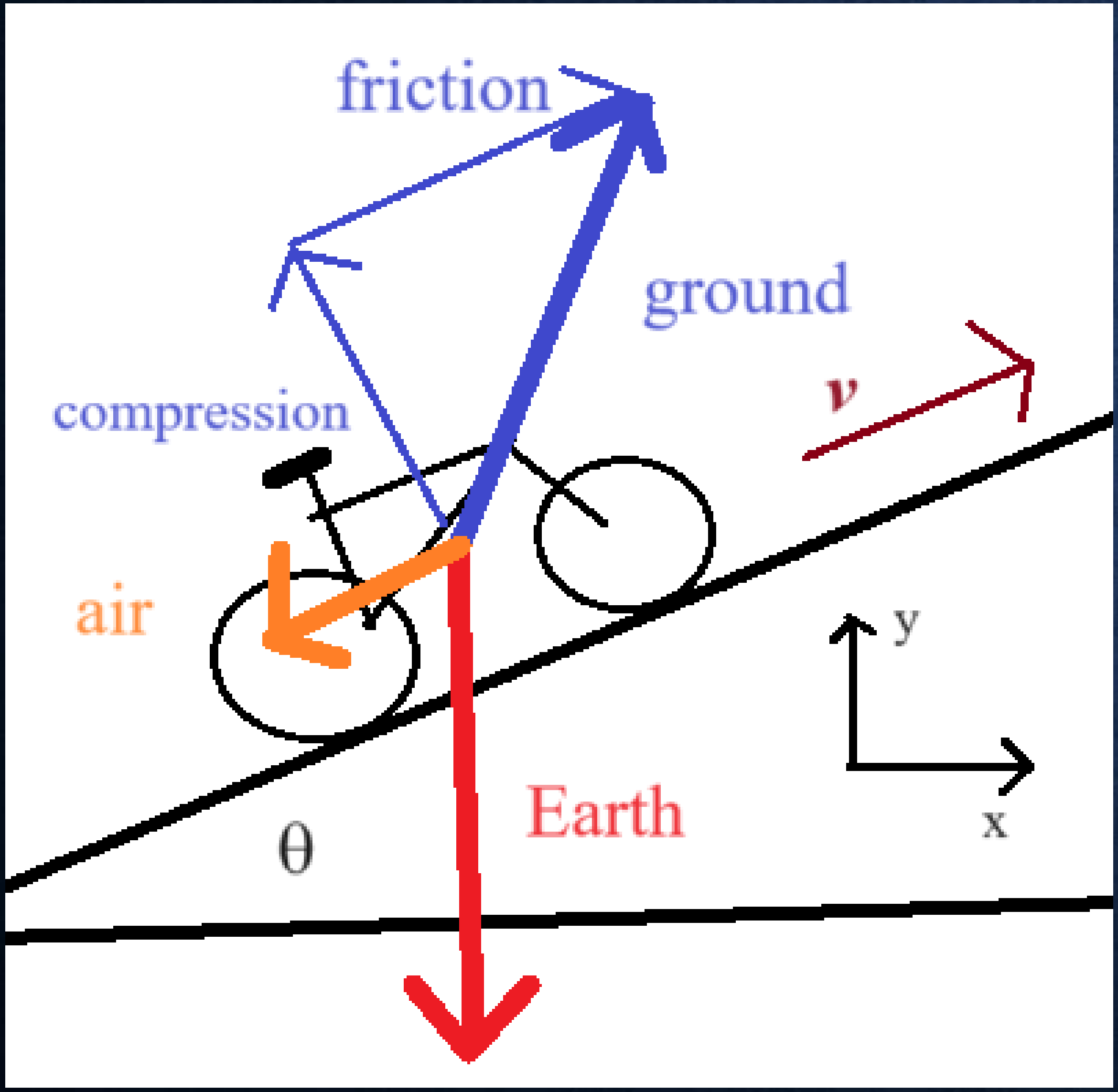


> $Force\_Equation := \vec{F}_{net} = m \cdot \vec{a}$:

$\vec{F}_{net} := \vec{F}_{Earth} + \vec{F}_{ground} + \vec{F}_{air}$:

← **Maple code**

Maple: Top-down problem-solving



$$> \vec{F}_{Earth} := \vec{v}_{2d}\left(m\cdot g, -\frac{\pi}{2}\right) :$$

$$\vec{F}_{ground} := \vec{F}_{g,\,friction} + \vec{F}_{g,\,compression} :$$

$$\vec{F}_{g,\,friction} := \vec{v}_{2d}\left(F_{scot}, \theta\right) : \quad F_{scot} := \frac{P}{v} :$$

$$\vec{F}_{g,\,compression} := \vec{v}_{2d}\left(F_{c}, \theta + \frac{\pi}{2}\right) :$$

$$\vec{F}_{air} := \vec{v}_{2d}\left(C\cdot v^2, \theta + \pi\right) :$$

$$> \vec{v}_{2d}(v, \theta) := \langle\, v\cdot\cos(\theta), v\cdot\sin(\theta)\,\rangle :$$

$$\vec{a} := \langle\, 0, 0\,\rangle :$$

Maple: Minimize mathematical minutia & coding

> *Force_Equation*

$$\begin{bmatrix} \frac{P\cos(\theta)}{v} - F_c \sin(\theta) - C v^2 \cos(\theta) \\ -m g + \frac{P\sin(\theta)}{v} + F_c \cos(\theta) - C v^2 \sin(\theta) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solve for the unknown variables, C, F_c

> $solutions := solve(Force\_Equation, \{C, F_c\})$

$$solutions := \left\{ C = \frac{-\sin(\theta)\, m g v + \cos(\theta)^2 P + \sin(\theta)^2 P}{v^3 \left(\cos(\theta)^2 + \sin(\theta)^2\right)}, F_c = \frac{\cos(\theta)\, m g}{\cos(\theta)^2 + \sin(\theta)^2} \right\}$$

> $C_{sol} := simplify(eval(C, solutions));$

$$C_{sol} := \frac{-\sin(\theta)\, m g v + P}{v^3}$$

Minimal # of procedures to learn

Calculate the value given a speed of 18 kph climbing at 2 degrees where the power is 180 W.

$$> C_{value} := eval\left(C_{sol}, \left\{m = 110, g = 9.8, P = 180, \theta = 0.02 \cdot \frac{\pi}{2.}, v = 5\right\}\right);$$

$$C_{value} := 0.08557$$

Use the value for C to calculate the maximimum angle given the maximum short-term power of the rider.

$$> \theta_{max} := solve\left(C_{sol} = C, \theta\right)$$

$$\theta_{max} := -\arcsin\left(\frac{C\,v^3 - P}{g\,m\,v}\right)$$

$$> \theta_{max\ value} := eval\left(\theta_{max}, \left\{C = C_{value}, m = 110, g = 9.8, P = 400, v = 1.7\right\}\right)$$

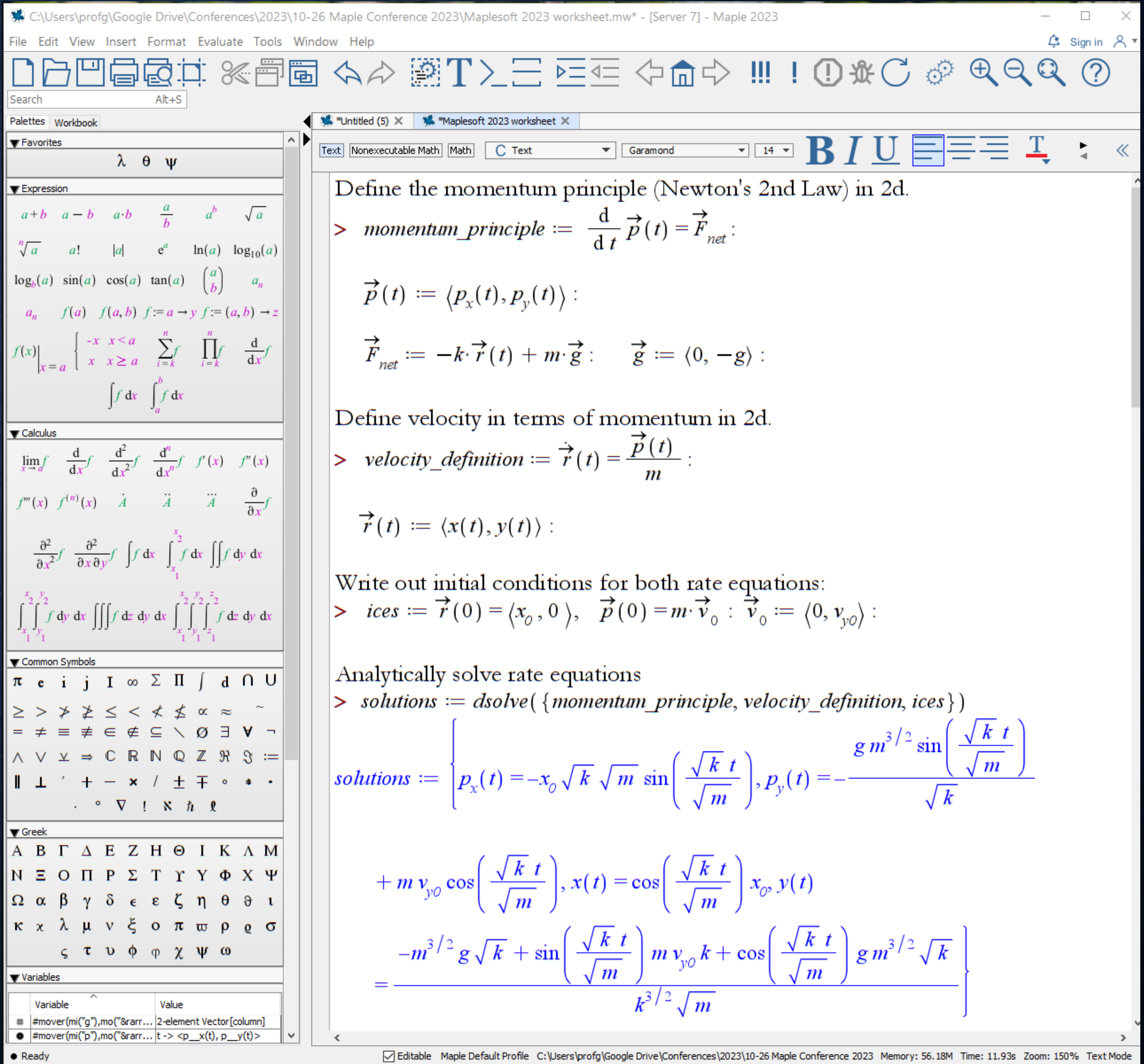
$$\theta_{max\ value} := 0.21981$$

$$> \theta_{degrees} := \theta_{max\ value} \cdot \left(\frac{2}{\pi}\right) \cdot 100$$

$$\theta_{degrees} := 13.99325$$

Student buy-in? Yes!

- Low barrier to generate content
- Maple math = written math
- Symbolic calculations
- Minimal coding → *solve, eval, plot*
- Immediate feedback
- Not a black box sim.
- Linked ODEs in intro



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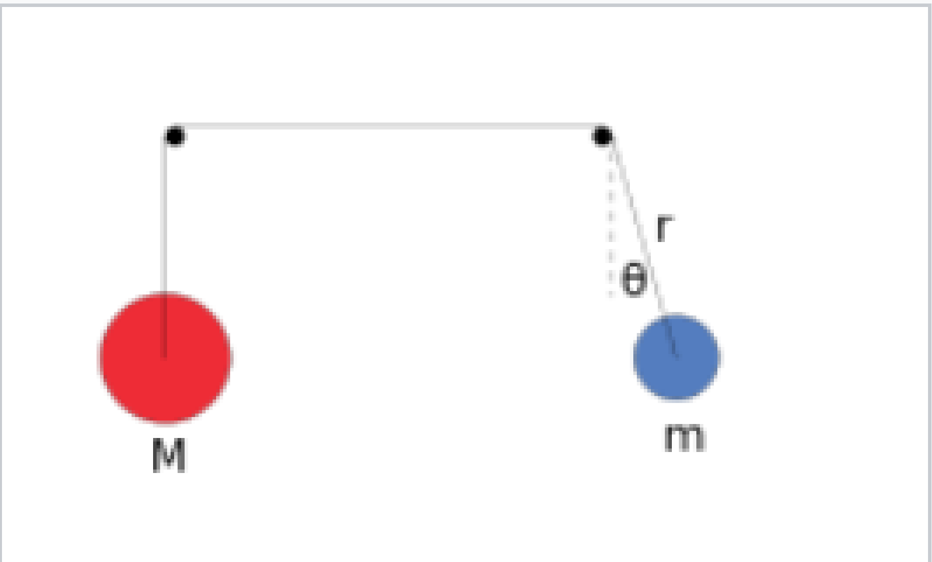
“*What if*” example



Derive the Lagrangian and initial condition equations

> $odes := \quad (M+m)\cdot\ddot{r}(t) = m\cdot r(t)\cdot\dot{\theta}(t)^2 - M\cdot g + m\cdot\cos(\theta(t)),$
>
> $\qquad r(t)^2\cdot\ddot{\theta}(t) + 2\cdot r(t)\cdot\dot{r}(t)\cdot\dot{\theta}(t) = -g\cdot r(t)\cdot\sin(\theta(t)) :$
>
> $ices := \quad r(0) = r_0, \quad \dot{r}(0) = 0, \quad \theta(0) = \theta_0, \quad \dot{\theta}(0) = 0 :$

Numerical problem: all constants need a value unless on-the-fly parameters.

> $M := \alpha\cdot m : \quad m := 1.0 : \quad r_0 := 1.0 : \quad g := 9.8 :$

Solve differential equations numerically and extract solutions:

> $solutions := dsolve(\ \{odes, ices\},\ numeric,$
>
> $s = [\alpha, \theta_0],\ output = listprocedure) :$
>
> $= eval(\theta(t), solutions) :$

arameters, 2) plots trajectory of blue ball.

$]) :$

$\cdot\cos(\theta(t)),\ t = 0..10],$

view = [... 1.3 ..1.3], *title* = "Trajectory of Small Mass");

end proc:

Plot trajectory of blue ball

Parameters varied on demand

* relative mass of red ball &

* initial angle

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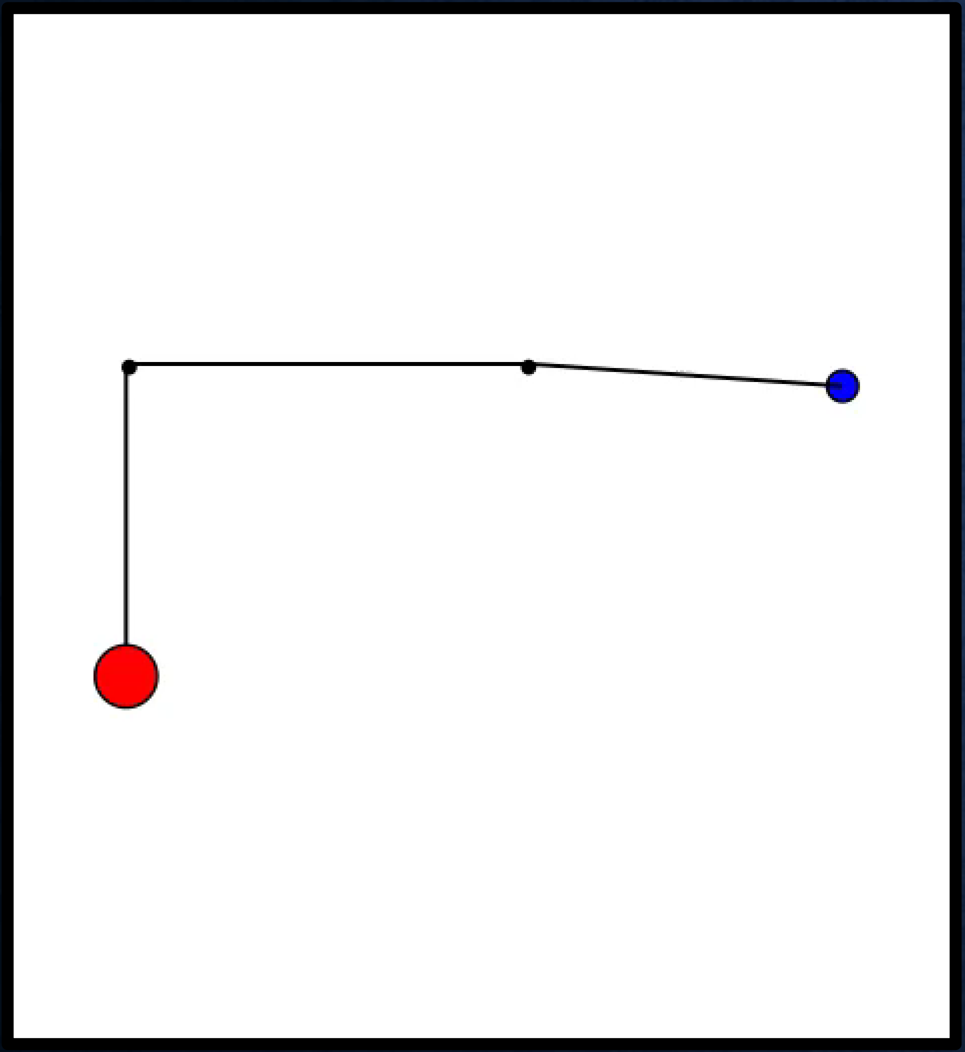
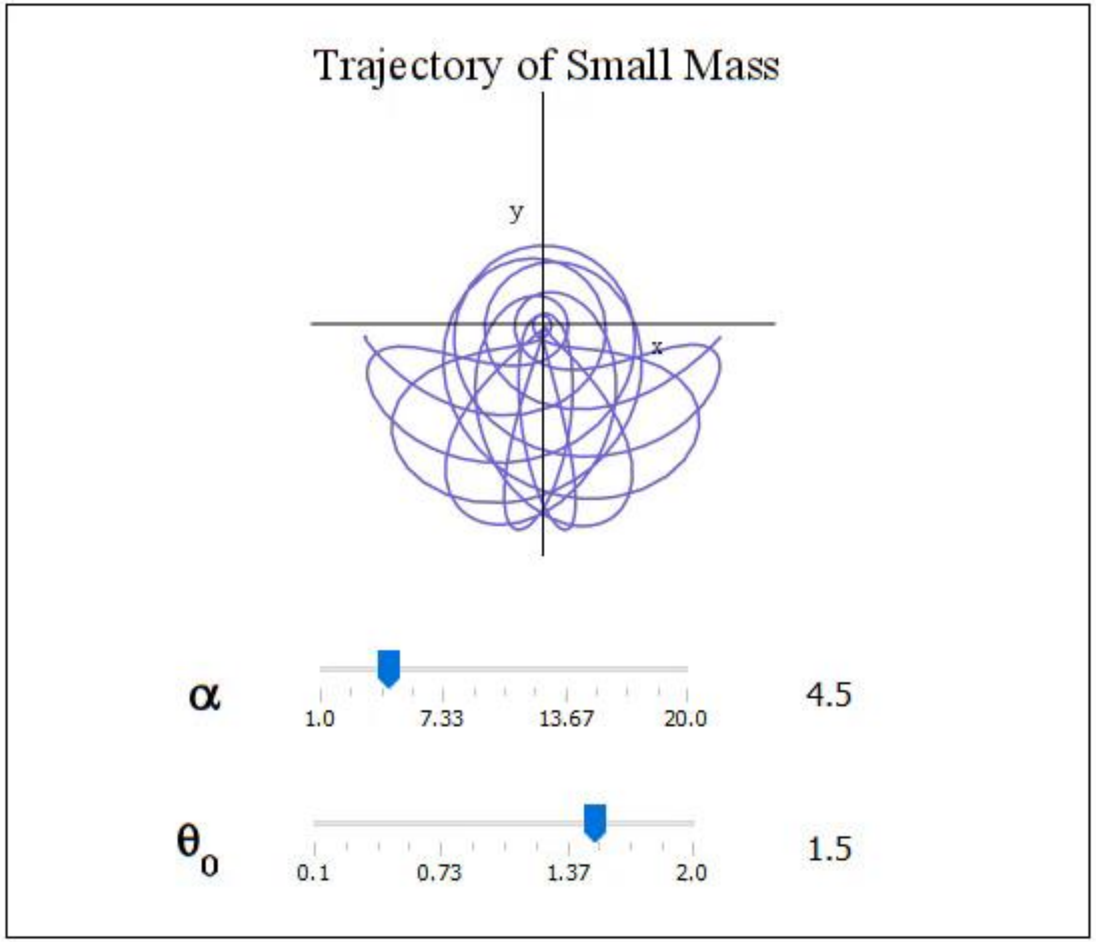


Explore the procedure with parameters on sliders:

$$> \; Explore\left(Trajectory\left(\alpha, \theta_0\right),\; parameters = \left[\alpha = 1 \,..20.0,\; \theta_0 = 0.1 \,..2.0\right],\; initialvalues = \left[\alpha = 4.5,\; \theta_0 = 1.5\right]\right);$$



Maple Immersion:

- Use Maple from day 1. In class, walk through calculations, line by line, building confidence.

- Present all calculations/complex derivations in Maple.

- Assign problems that cannot be solved by hand.

- Emphasize creating graphics & "What-if apps."

- **Homework & exam submissions: Maple worksheet *ONLY***

- Rely on the **Learning Maple Textbook / Video series** to teach and remind students how to use Maple.

Learning Maple: Max Productivity-Min Coding

Maple Instructional Videos/Documents for Science and Engineering

https://**gould.prof** *or* https://**YouTube/@MapleProf**

- Each video is limited to**12 minutes**
- Minimal number of procedures to learn
- Practice problems from the physics undergraduate curriculum
- Embedded Maple coding instruction where appropriate
- Documents: additional problems & Troubleshooting

Learning Maple: Max Productivity-Min Coding

Maple Fundamentals:

- 1: **Setting Up Maple and Finding Help** (document)
- 2: **Maple as a Calculator** (document)
- 3: **Writing Symbolic Expressions** (document)
- 4: **Solving Symbolic Equations** (document)
- 5: **Solving Numeric Equations** (document)
- 6: **User-generated Functions** (document)
- 7: **2d Plotting** (document)
- 8: **Document Enhancement** (document)

Mathematics:

- **Complex Numbers 1**: Fundamentals (document)

- **Vectors 1**: Cartesian Coordinates (document)
- **Vectors 2**: Vector Products (document)

- **Statistics 1**: Descriptive Statistics (document)
- **Statistics 2**: Curve Fitting (document)

- **Calculus 1**: Limits & Differentiation (document)
- **Calculus 2**: Integration (document)
- **Calculus 3**: Summation & Series (document)

Advanced Mathematics:

- **Ordinary Differential Equations 1:** Symbolic (document)
- **Ordinary Differential Equations 2:** Numeric (document)
- **Ordinary Differential Equations 3:** Systems of ODEs (document)
- **Ordinary Differential Equations Topic:** Boundary Value Problems (document)

- **Linear Algebra 1:** Matrices Arithmetic (document)
- **Linear Algebra 2:** Eigenvalues & Eigenvectors (document)
- **Linear Algebra Topic:** Linear Transformations (document)

- **Advanced Mathematics Topic:** Fourier Series (document)
- **Advanced Mathematics Topic:** Transformations (document)
- **Advanced Mathematics Topic:** Dirac delta function (document)

- **Partial Differential Equations 1:** Basics (document)
- **Partial Differential Equations Topic:** Heat Equation (document)

- **Vector Calculus 1:** Div, Grad, Curl (document)
- **Vector Calculus 2:** Integrals (document)
- **Vector Calculus 3:** Fundamental Theorems (document)

Useful Maple Procedures:

- **Evaluate expressions - *eval*:** (document)
- **Sequence generator - *seq*:** (document)
- **Conditional procedures - *ifelse, piecewise, Heaviside* :** (document)

- **Random numbers - *rand, randomize*:** (document)
- **Extrema - *minimize, maximize*:** (document)

- **Animation - *plots:-Animate* :** (document) (worksheet)
- **Exploration application generator - *Explore*:**

Advantages of teaching Physics using Maple – concepts, concepts, concepts

- Principles: emphasized through mathematics, not black box
- Top-down problem-solving approach – like physics
- Reduction of math minutia and teaching tricks
- Interpretative interface → near-immediate feedback.
- Solutions are symbolic & beyond the spherical cow
- Transferable principles and skills ($149 – student version)

Final recommendations:

- Take a STEM course
- Question everything you teach

Learning Maple

Navigate to https://**gould.prof** *or*

YouTube/@MapleProf