Manufacturing Models:

Helping New Modelers Model with a Clearly Defined Process

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Agenda

I teach a first-year modeling and ODE course to advanced students at USMA.

- Inconsistent experience modeling
- Mathematically inclined
- Not all STEM Majors (not a service course)

What I plan to cover

- What is strong modeling?
- How do we help students
- What is a modeling process?
- How to incorporate into a course
- Modeling and Science

Strong v. Weak modelers

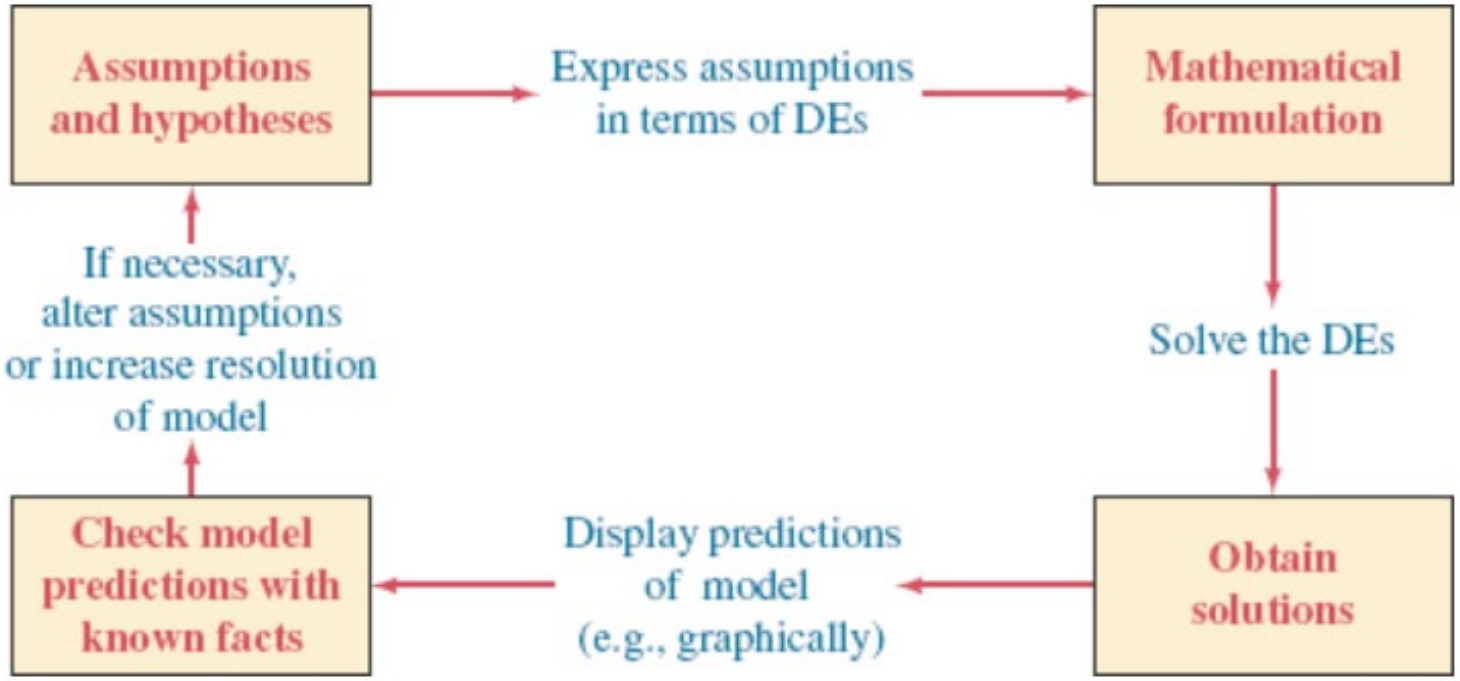
"Modeling problems can be divided, at least roughly, into "strong" and "weak" varieties. Strong modeling problems can be defined as those that are authentic (not contrived as a mere classroom exercise), admit many points of entry (e.g. allow different types of models based on different assumptions), and permit the development of simplified models initially and improved models iteratively. Weak problems have an 'obvious' way of being modeled that students tend to get forced into, which inhibits creativity."

-MAA Modeling Class Summary

My observations on Strong Modlers

- Modeling is not intuitive to many students.
- New students struggle to start when they encounter a strong modeling problem.
- We start eating an elephant one bite at a time.

Zill's Modeling Process

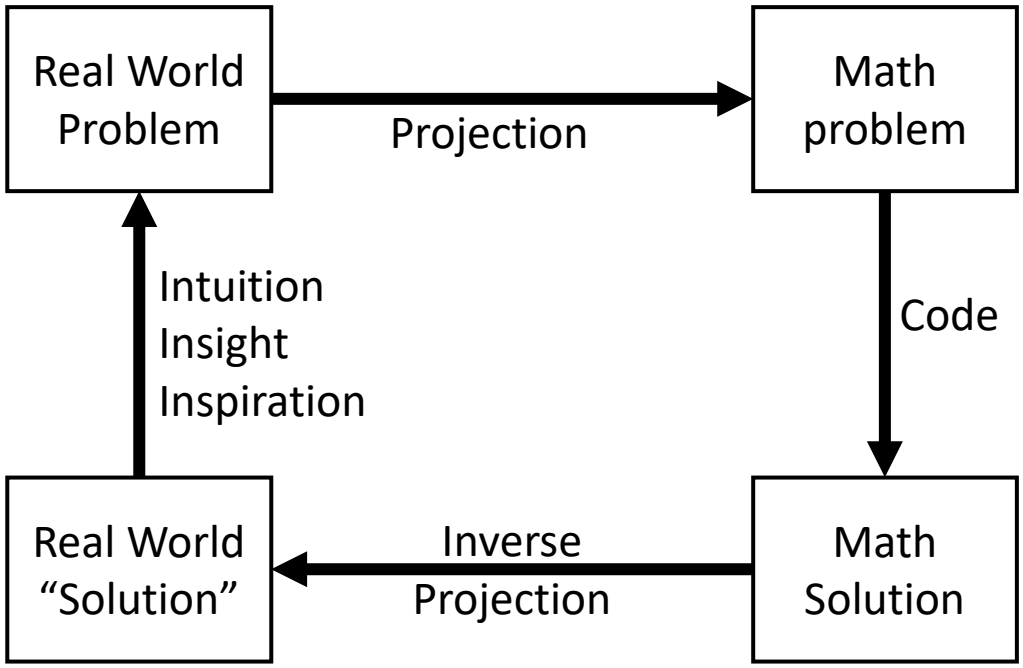


Boyce-DiPrima Modeling

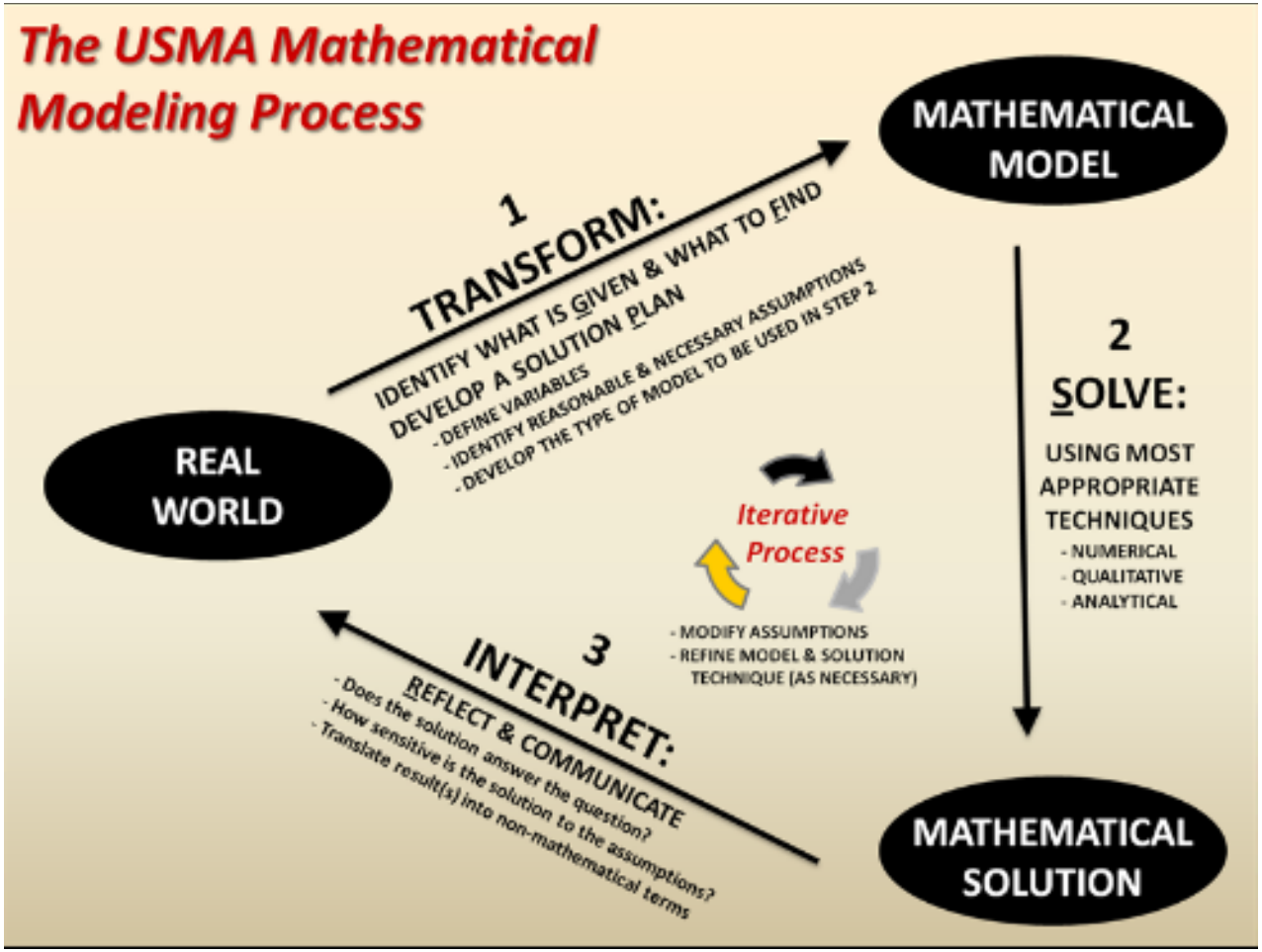
1. Identify Independent and Dependent Variables
2. Choose units of measure.
3. Articulate the basic principle that underlies the problem.
4. Express this principle in terms of your chosen variables.
5. Ensure that each term in your equation has the same units.
6. Note that sometimes more complex problems the mathematical model may become a system.

Summarized from Elementary ODEs, Boyce and DiPrima

Data Science Modeling Process



USMA Modeling Process



Breaking Down the Process

Points of Performance

1. **Transform.**
   (a) *Variables*: Define the Independent Variable (IV) and Dependent Variable(s) (DV), using either "triple equal" definition or another clear way, and include units.
   A *good practice* is to clearly state, in terms of your variables, what your problem is aiming to find.
   (b) *Assumptions*: State one or two key assumptions. Why do they matter, and what are the consequences if incorrect?
   (c) *Model*: What DE model governs (or approximates) this type of problem?
   Write down the model first, to help make assumptions and subsequent work clear.
   Remember, the *model* is an *Initial-Value Problem*, not just a differential equation!

2. **Solve**: Apply solution techniques from class to turn the relationship (i.e., a differential equation) into an explicit function that satisfies your DE.
   Use initial conditions to fully provide a solution to the IVP.

3. **Interpret**: Use a common-sense check back to the original problem to verify that your solution makes sense. Long-term behavior is an easy way to make sure you did not make a sign error.
   Use your solution to answer the original question posed, but remember you're just estimating with an imperfect model. Try to translate your solution from math back into words.

2. **Solve.**

$$\frac{dA}{A} = \frac{-dt}{20}$$
$$\int \frac{dA}{A} = \int \frac{-dt}{20}$$
$$\ln|A| = \frac{-t}{20} + C_1$$
$$\underline{A(t) = \underbrace{C_2 e^{-t/20}}_{\text{general solution}}}$$

$A(0) = 10^7 = C_2 e^{0/20} \Longrightarrow C_2 = 10^7$, so

$$\underline{\underline{A(t) = 10^7 e^{-t/20}.}}$$

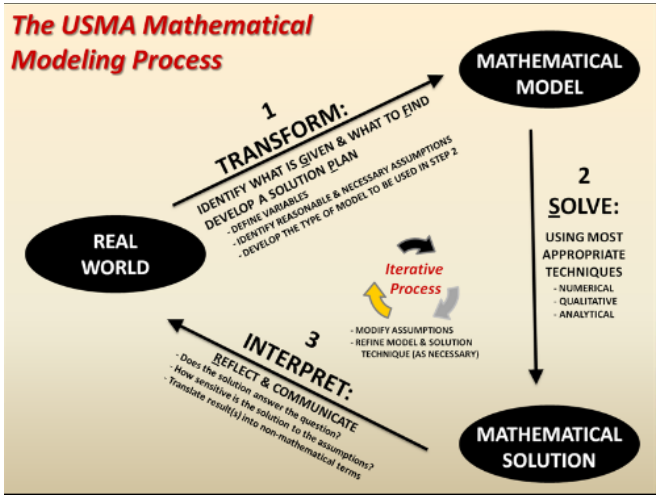
3. **Interpret.**
   (a) Long-term behavior. In this case, as time goes on, the system should go to zero. Makes sense.
   (b) When will $\frac{A(t)}{400} \leq 4\frac{mg}{L}$?

$$1600 = 10^7 \, e^{-t/20}$$
$$\ln\left(\frac{16}{10^5}\right) = \frac{-t}{20}$$
$$t = -20 \ln\left(\frac{16}{10^5}\right)$$
$$t \approx 174.807$$

The water should be safe to drink after 174 minutes, or $\underline{\underline{\text{about 3 hours}_{ANS}}}$.

Does Application Matter

Do we need to differentiate between Biological and Sociological models versus Physics and Chemistry models?



UNITED STATES MILITARY ACADEMY
WEST POINT.
Boyce Diprima

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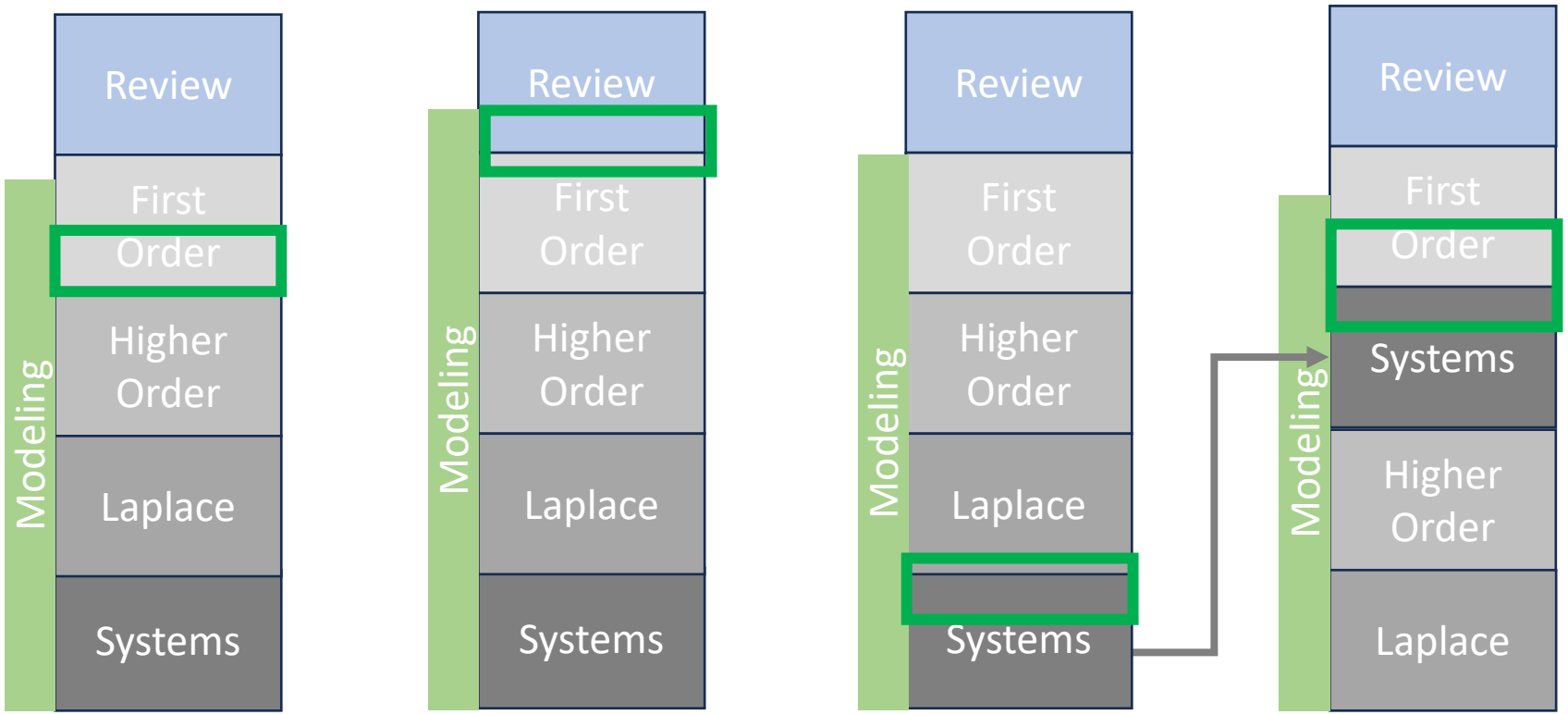
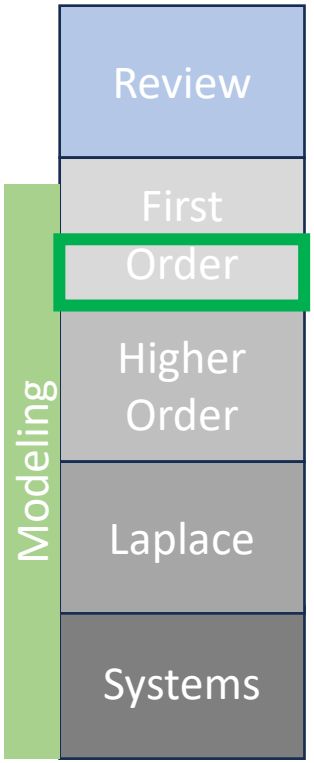
Options for When to Teach a Process

Modeling Process



First Order Before ODE Systems Hybrid

Modeling Process with First Order

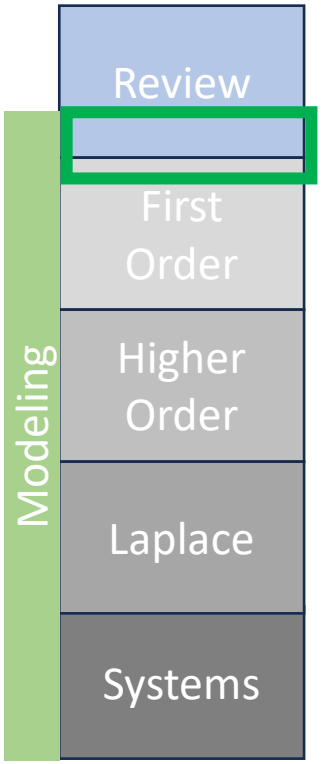


First Order

Concept:

- Teach one or more of the traditional models (mixing tank, heat transfer, draining tank, drag, population, disease).
- Teach students to build on the models by adding terms (evaporation, temperature, time of year).

Modeling Process Without ODEs

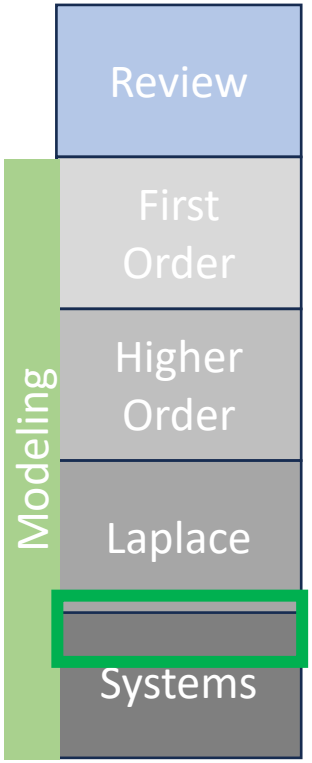


Before ODE

Examples:

- How much will it cost to go on a vacation to Hawaii with your family?
- How much water is in the Hudson River?
- The English introduced a male and female rabbit to Australia. How long until there are 1,000 rabbits? (Fibonacci)

Modeling Process with Systems

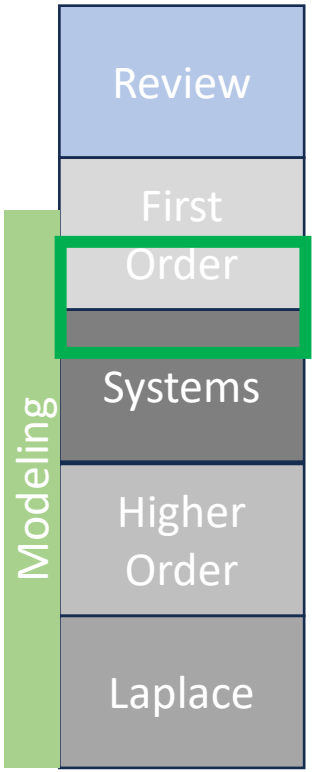


Systems

Concept:

- Start modeling in first order but do more “weak” examples.
- Introduce a process to help students tackle hard (strong) problems later.

Modeling Process Hybrid

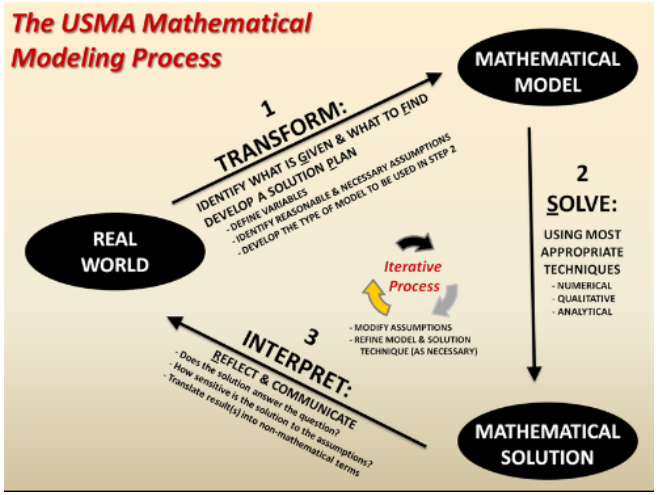


Hybrid

Level 1: Coffee, Air

Level 2: Coffee, Mug, Air

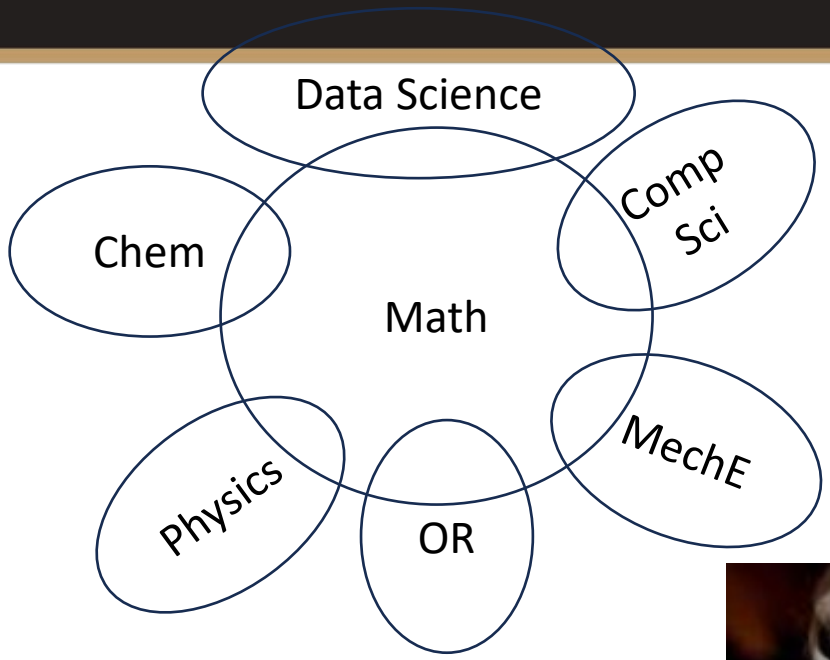
Level 3: Coffee inside, Coffee outside, Mug, Air



3 Random Quotes

Why major in Mathematics?

To help people use Math,
we must be good modelers



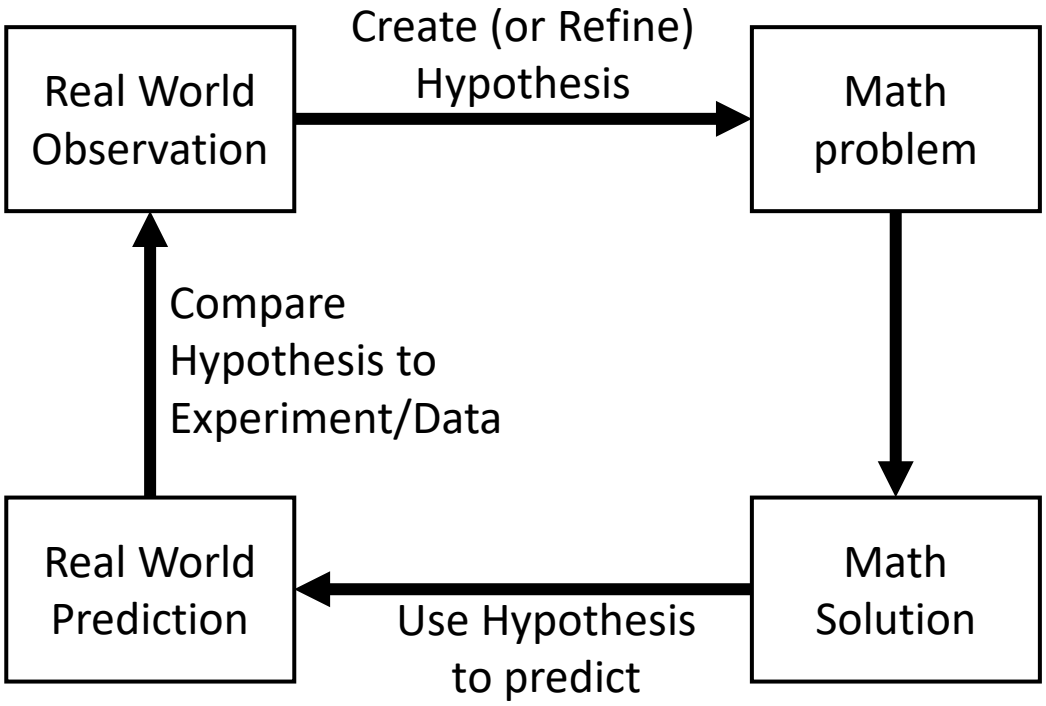
What are modelers, but bad Physicists?

Yes, but as we intended

"..two fields that never should have been separated in the first place: Literature and History" -Susan Wise Bauer

Perhaps the best modeling is to mix our Science and Math

Scientific Method



Should We Use a Process?

- A process gives students a problem solving framework.
- There are lots of potential processes and integration timelines based on audience.
- A process can make them better scientists, engineers, and mathematicians.
- Our process helped USMA build strong modelers (5/17 outstanding at SCUDEM).

I know I am quite abstract in this talk, to learn more about how we implemented what we did, please attend Brittany Oletti's talk

Aknowledgements

- My fellow Math Modeling with ODE teachers here at USMA for all their discussions and opinions: Devon Zillmer, Jonathan Paynter, Fr. Gabriel Costa, Brittany Oletti.

- Dr. Brian Winkel for starting me down the modeling process.

- SIMIODE Expo staff for putting on this great conference

References

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