Finding Microplastics in the Ocean

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Microplastics and Algal Biofouling in the Ocean

Exposure to microplastics adversely affects ecosystems. Empirical studies of plastic particles show that microplastics are lost in deeper regions of the ocean. It is important to understand the trajectories of the microplastics in the ocean and to attempt to find their locations. Microplastics bonds with algae.

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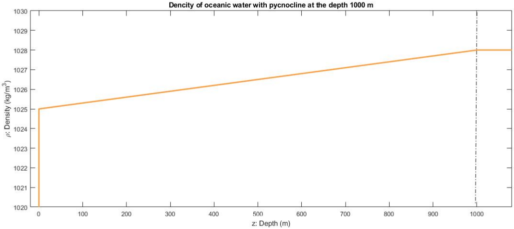
- Usually, density of microplastics (low-density polyethylene, high density polyethylene, polypropylene) are $<$ than density of water.
- Usually, density of algal cells are $\geq$ density of water.
- Water density is increasing from $\sim 1025\frac{kg}{m^3}$ at the surface to $\sim 1028\frac{kg}{m^3}$ at the pycnocline depth ($\sim 1000m$).



Occillatory Behavior??

A buoyant microplastic particle submerged at the ocean's surface is exposed to algae which attach and grow on the particles' surface as a biofilm. The size of this biofilm is determined by both the rate of attachment and the relative growth and death rates of the biofilm population. Algal density typically exceeds that of seawater, so once a biofilm is sufficiently large, combined plastic particle and algal aggregates will begin to sink. Once the particle is submerged below the euphotic zone, algal growth from photosynthesis is not possible due to limited light exposure. Algal population in the biofilm reduces due to death and fauna grazing, implying that the biofouled particle can become rising toward the ocean's surface. Traveling back above the euphotic zone and experiencing an increase in light intensity allows the algae to photosynthesize and grow once more.

Model Development

We want to construct a 3-dimensional dynamical system's model that describes the trajectory of the microplastics's with variable amount of algae. We study how changes of depth of microplastics with algal biofouling depend on the amount of algal cells.

- We can describe the changes in the number of algal cells N as a population dynamics process
- We can describe the depth z of the aggregate with the help of 2nd Law of Newton (and hydrodynamic friction – Stokes formula).

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- $\frac{dN}{dt} = \text{growth rate} - \text{death rate} + \text{attachment of new cells rate}$

- $\frac{dz}{dt} = v$
  $m \cdot \frac{dv}{dt} = m \cdot g - \text{mass of displaced water} \cdot g - \text{friction}$

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- $$\frac{dN}{dt} = \text{growth rate}(z) - \text{death rate} + \text{attachment of new cells rate}(z)$$

- $$\frac{dz}{dt} = v$$
  $$m(N) \cdot \frac{dv}{dt} = m(N) \cdot g - \text{mass of displaced water}(N) \cdot g - \text{friction}(N)$$

A Model

- $$\frac{dN}{dt} = g(z) \cdot N - r_d \cdot N + A(z)$$
  $$\frac{dz}{dt} = v$$
  $$\frac{dv}{dt} = g - g\frac{\rho_w(z)(V_p + N \cdot V_A)}{V_p\rho_p + N \cdot V_A\rho_A} - C\frac{\rho_w(z) \cdot R(z) \cdot v}{V_p\rho_p + N \cdot V_A\rho_A}.$$
- Constants that we can adjust/bifurcate are red.
  r_d rate of death of algal cells
  V_p volume of microplastic particle
  V_A volume of algal cell
  ρ_p density of microplastic
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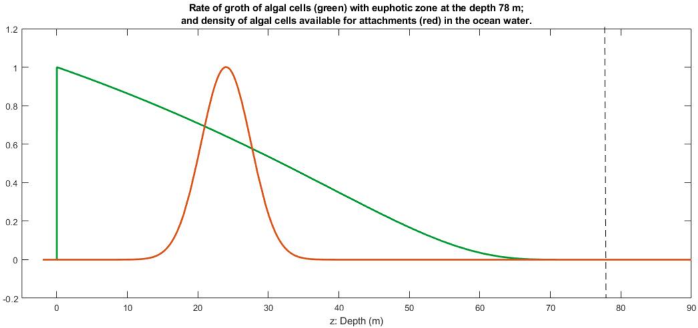
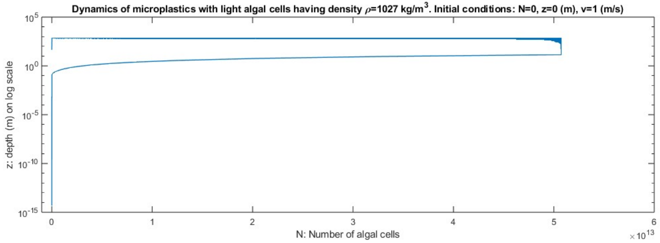


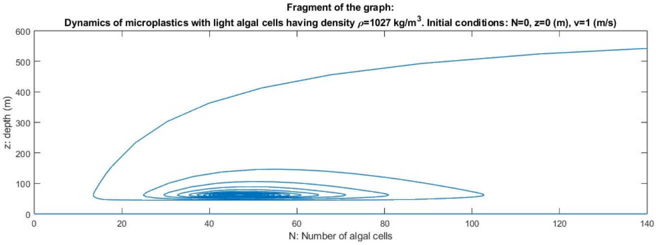
Figure: $g(z) = g_{max} \cdot$ "green graph", and $A(z) = A_{max} \cdot$ "brown graph".

What can we study?

What subjects of differential equations can be studied with this model?

Simulations





Stationary Points

- A simplified system of ODEs:

$$\frac{dN}{dt} = g(z)\cdot N - r_d \cdot N + \cancel{A(z)}$$
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- Stationary points and their physical meaning:

  $v^* = 0$ – obvious.

  1. $z^* \sim g^{-1}(r_d)$ and $N^* \sim \frac{\rho_w(z^*)-\rho_p}{\rho_a-\rho_w(z^*)}\cdot\frac{V_p}{V_A}$
  2. If $N^* = 0$, then $\rho_w(z^*) = \rho_p$. But plastic is lighter than water.

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Let us vary the parameters

- Many parameters are imprecise or have a large range of possible values.
- Let us vary ρ_A
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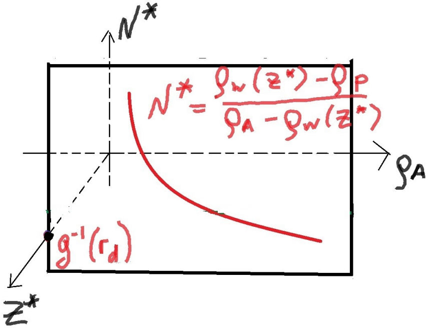

Stability Analysis

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Linearization at $(N^*, z^*, 0)$:

$$\begin{bmatrix} 0 & g'(z^*)N^* & 0 \\ 0 & 0 & 1 \\ gV_pV_A\frac{\rho_A-\rho_p}{\rho_w(z^*)(V_p+NV_A)^2} & -g\rho_w'(z^*)\frac{(V_p+N^*\cdot V_A)}{V_p\rho_p+N^*\cdot V_A\rho_A} & -C\frac{\rho_w(z^*)(V_p+N^*\cdot V_A)^{1/3}}{V_p\rho_p+N\cdot V_A\rho_A} \end{bmatrix}$$

- Recall that usually $\rho_A > \rho_p$, $g'(z) < 0$ and $\rho_w'(z) > 0$.

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- Recall that usually $\rho_A > \rho_p$, $g'(z) < 0$ and $\rho_w'(z) > 0$.
- Then, the characteristic equation $\lambda^3 + a\lambda^2 + b\lambda + c = 0$ has positive coefficients a, b, c.

  $\Longrightarrow$ all roots have negative real parts (Descartes' rule of signs).

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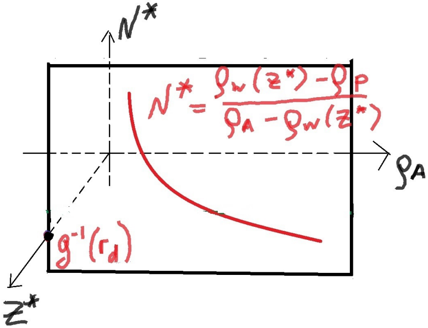

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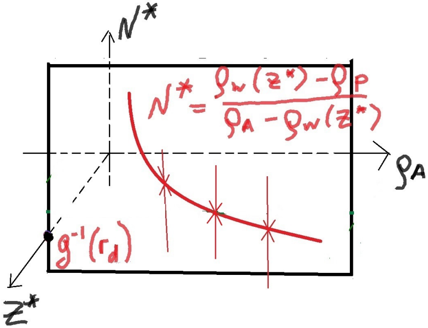

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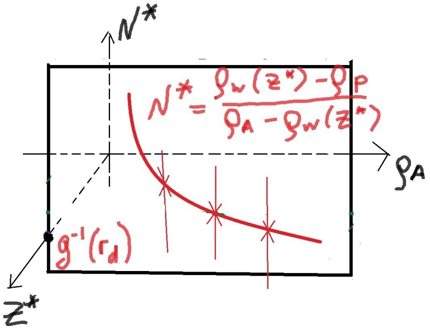

- What if we add attachment function $A(z)$???

References

H. Kreczak, A.J. Willmott, and A.W. Baggaley. (2021) Subsurface dynamics of buoyant microplastics subject to algal biofouling. Limnology and Oceanography **66.9**, 3287–3299. doi: 10.1002/lno.11879.

V. Rayskin, Convergence of the trajectory of buoyant microplastics with algal biofouling towards underwater equilibrium. ResearchGate.

M. Kooi, E. H. v. Nes, M. Scheffer, and A. A. Koelmans. (2017). Ups and downs in the ocean: Effects of biofouling on vertical transport of microplastics. Environ. Sci. Technol. **51**: 7963–7971. doi:10.1021/acs.est.6b04702

G. Bratbak, I. Dundas. (1984). Bacterial dry matter content and biomass estimations. Appl. Environ. Microbiol., **48(4)**:755–757. doi:10.1128/AEM.48.4.755-757.1984.

J.H. Simpson, J. Sharples. 2012. *Introduction to the physical and biological oceanography of shelf seas*. Cambridge University Press.

J. Uitz, H. Claustre, A. Morel, and S. B. Hooker. (2006). Vertical distribution of phytoplankton communities in open ocean: An assessment based on surface chlorophyll. J. Geophys. Res.: Oceans, **111(C8)**. doi:10.1029/2005JC003207.

S.F. Hoerner, 1965. Fluid Dynamic Drag: Practical Information on Aerodynamic Drag and Hydrodynamic Resistance, 400 pp. John G. Webster, 2000. Mechanical variables measurement: solid, fluid and thermal, CRC Press, Boca Raton, London, New York, Washington D.C.