The modeling-first approach in an ODE classroom: utilizing fictional dragons as a practical modeling illustration

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Introduction

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Loyola Marymount University

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DE Models

Cancer Metastasis	HIV Prevention	Blood Clotting
HPC Vaccination	Population Crisis	Microbial Fermentation

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Students:

What do differential equations do?

Professors:

I don't use separable equations or integrating factors in my research.

I hate teaching ODE.

Common Questions & Concerns in a DE Class

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Key:

Modeling & Numerical Simulations

SIMIODE: Systemic Initiative for Modeling Investigations & Opportunities with Differential Equations

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The Modeling-First Approach

- **Model-Centered Learning**
- **Active Learning**
- **Contextual and Relevant Problems**
- **Development of Multiple Skills**
- **Iterative Learning**

Enhanced Engagement → Deeper Understanding → Skill Development

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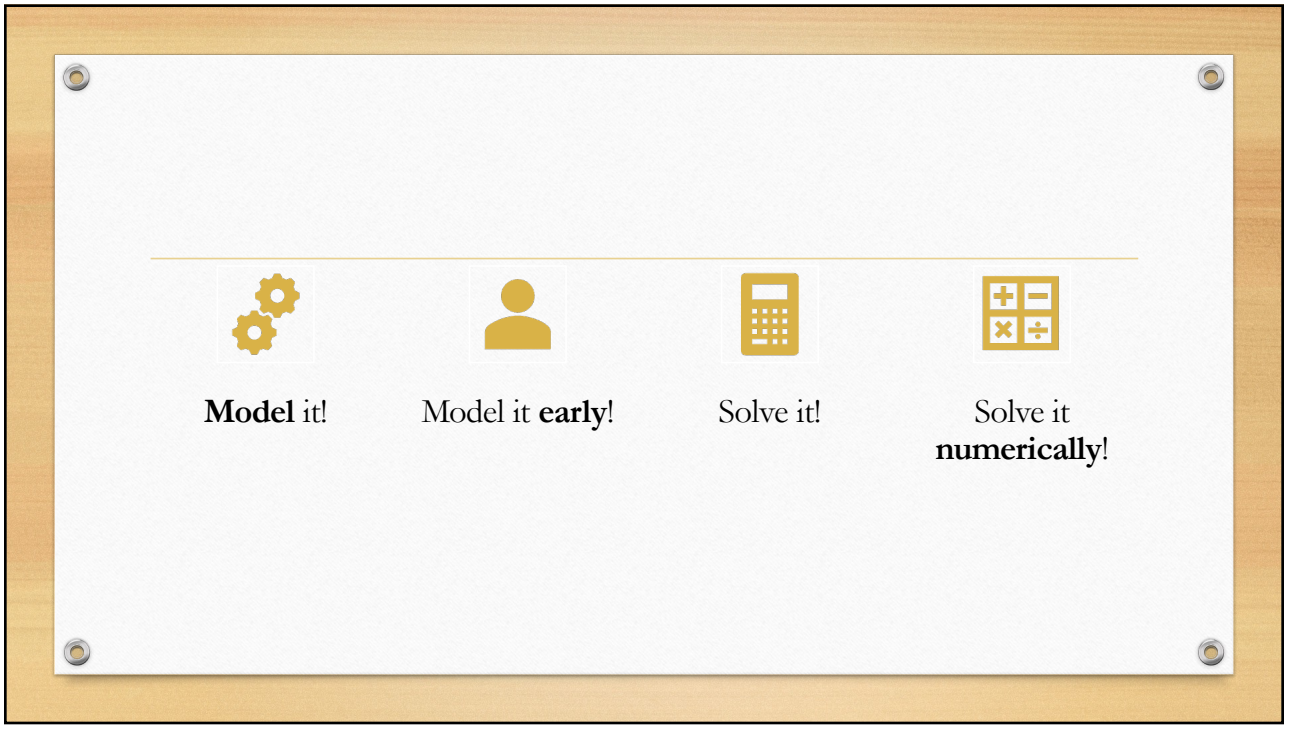

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Where to find recourses?

- Mathematical Modeling Competitions
- Published Materials

Contests

Consortium for Mathematics and its Applications

IJMEST

International Journal of Mathematical Education Science and Technology

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SIMIODE OER

3-034-CarSuspension-ModelingScenario

We examine the spring-mass-dashpot that is part of a car suspension, how the ride is related to parameter values, and the effect of changing the angle of installation. We model a ``quarter car'', meaning a single wheel.

design | spring-mass-dashpot | spring constant | underdamping | car suspension | suspension tolerance | static equilibrium

6-022-CannibalismPredatorPrey-ModelingScenario

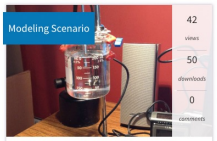
The Lotka-Volterra model tells us that the prey and predator exhibit a shifted cyclic behavior over time. In this module, we look at modifying this prey-predator model to consider the case when there is cannibalism in the predator species.

stability | predator-prey | equilibria | oscillations | cannibalism | phase planes | spiral sinks

1-132-DigoxinElimination-ModelingScenario

We model the concentration of digoxin eliminated from the human body at a rate proportional to the concentration. This is a ``first-order reaction'' in the language of pharmacokinetics -- the study of how drugs move in the body.

nullclines | pharmacokinetics | digoxin | drug elimination | two compartment



1-031-CoolIt-ModelingScenario

We offer data on the temperature of water in a beaker in a room of constant temperature and in a room of nonconstant temperature. Students consider both empirical and analytic modeling approaches. We offer additional data sets in Excel...

Spanish | Newton's Law of Cooling | cooling

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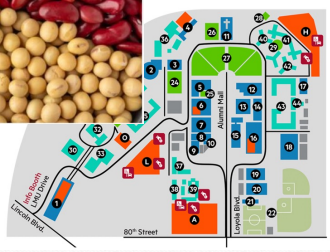
Hands-on activity studying the spread of Common Cold

- Modified from SIMIODE materials.
- Materials:
  - Cups
  - Beans (two colors) / M&Ms
  - Campus/ dorm Maps
  - Computer
- On the **first day of class**



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Recurring Love/ Relationship ODE model

- Published on IJMEST SIMIODE Special Issue (2024)
- We provide context for relating mathematics and cultural competence, by considering a five-part bundle of activities in which students draw on their own personal and/or cultural experiences with the concept of love. Students develop love scenarios, model them, identify limitations, and offer future model revisions.
- Valentine (2/14), White Valentine (3/14), and Black Valentine (4/14)

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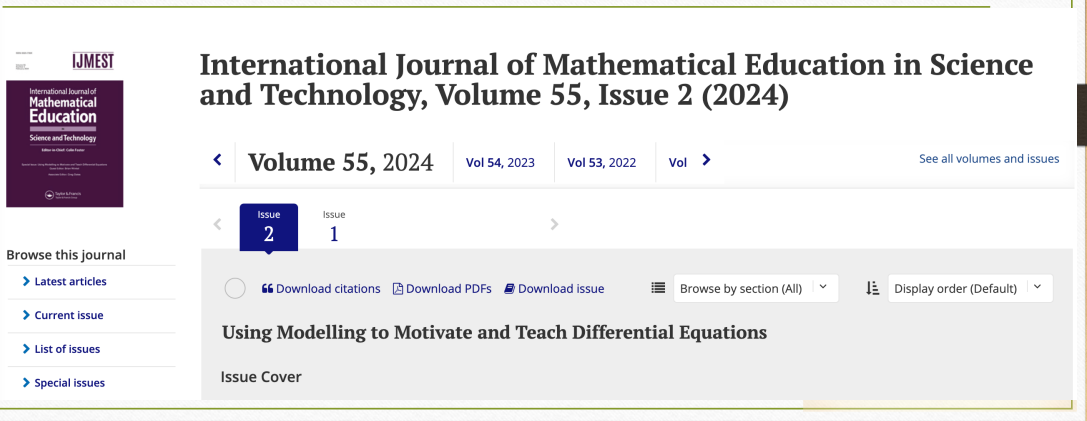
Recurring Love/ Relationship ODE model



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Bringing Imaginary Creatures Into the Class

Happy Lunar New Year~ The Year of Dragon~

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2019 MCM PROBLEM A: Game of Ecology

General question:

Examine what would happen if the fictional dragons from the television series **Game of Thrones** [Benioff and Weiss, 2019], based on the epic fantasy novel, **A Song of Ice and Fire** [Martin 2012], were living today.

Mathematical Contest in Modeling (MCM)

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2019 MCM PROBLEM A: Game of Ecology

Expectations:

- Model and analyze the dragons' characteristics, behavior, habits, diet, and interactions with their environment
- Discuss the energy expenditures of the dragons, the ecological impacts of the dragons on the environment, and how climate or community assistance might change the system
- Write a two-page letter to guide the author, G. Martin, about how to maintain ecological realism in the fictional story

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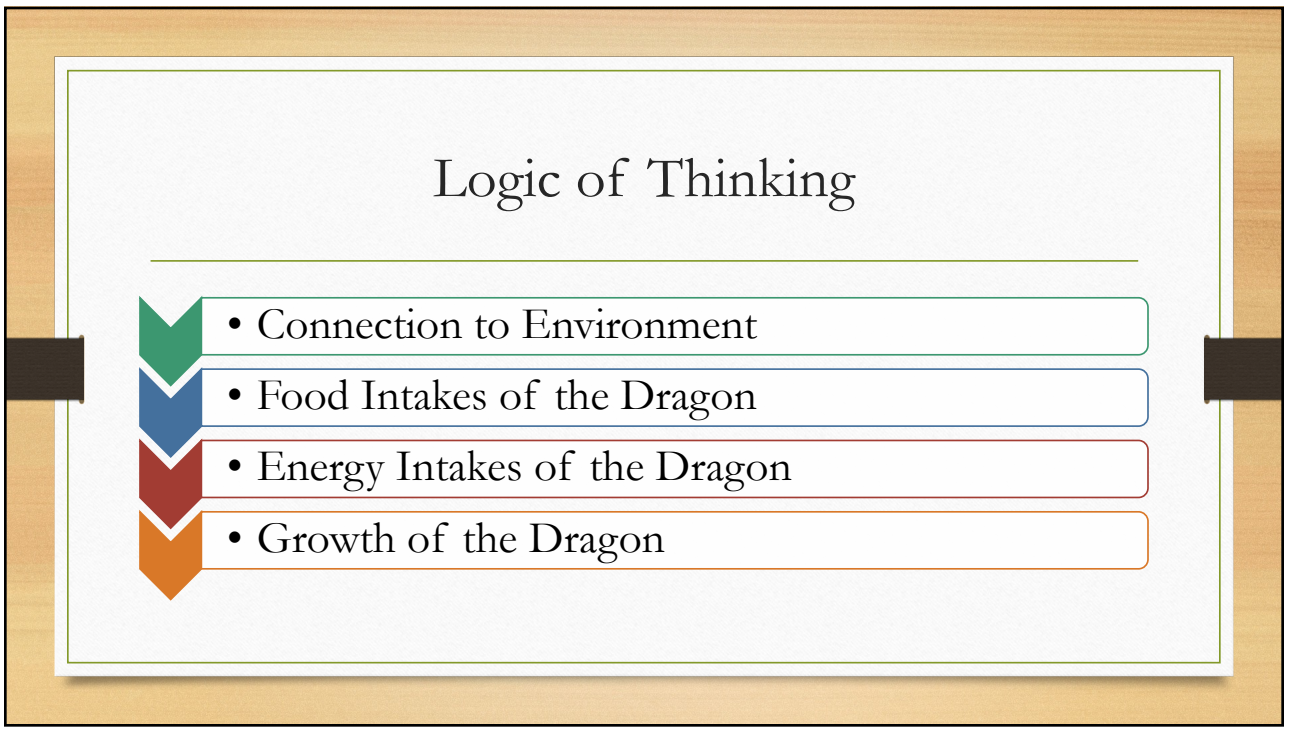
Logic of Thinking
• Connection to Environment
• Food Intakes of the Dragon
• Energy Intakes of the Dragon
• Growth of the Dragon

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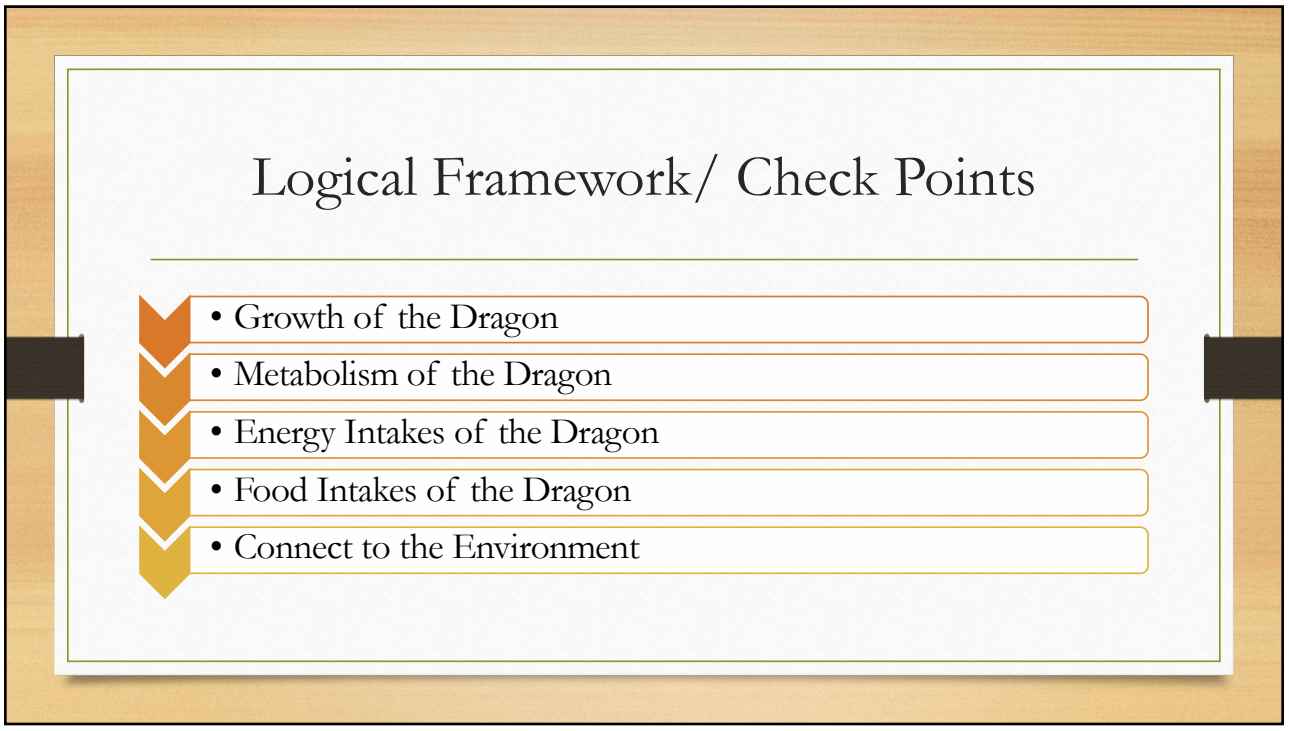
Logical Framework/ Check Points
• Growth of the Dragon
• Metabolism of the Dragon
• Energy Intakes of the Dragon
• Food Intakes of the Dragon
• Connect to the Environment

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Dragon Growth

- Von Bertalanffy Growth Function (VBGF)

$$\frac{dW}{dt} = k \cdot (W_\infty - W)$$

- Gompertz Function:

$$\frac{dW}{dt} = k \cdot W \cdot \ln\left(\frac{W_\infty}{W}\right)$$

- Logistic Growth Equation:

$$\frac{dW}{dt} = k \cdot W \cdot \left(1 - \frac{W}{W_\infty}\right)$$

(k: growth rate, W_∞: asymptote weight, W: weight, t: time)

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Dragon Metabolism

- Kleiber's law for Basic Metabolic Rate (BMR) [Thommen et al., Science, 2009]

$$B = B_0 * W^{4/3}$$

- Incorporate body temperature [Kolokotrones et al., Nature, 2010]

$$lg\, B = \beta_0 + \beta_1\, lg\, W + \beta_2 (lg\, W)^2 + \frac{\beta_3}{T}$$

- Incorporate activity type by Harris Benedict Formula [Harris & Benedict, PNAS, 1918]. For example, if the dragon needs to fly and breath fire, the Metabolic each day maybe twice to three times of the BMR.

(B: Metabolic rate, W: weight, T: body temperature, B_0, β_0, β_1, β_2, and β_3 are regression coefficients)

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Dragon Metabolism

Activity types	None	Minimum	Medium	Active	Professional	Extreme
Metabolism	$1.2\times B_0$	$1.375\times B_0$	$1.55\times B_0$	$1.725\times B_0$	$1.90\times B_0$	$2.90\times B_0$

$$\lg B = \beta_0 + \beta_1 \lg W + \beta_2 (\lg W)^2 + \frac{\beta_3}{T}$$

- Incorporate activity type by Harris Benedict Formula [Harris & Benedict, PNAS, 1918]. For example, if the dragon needs to fly and breath fire, the Metabolic each day maybe twice to three times of the BMR.

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Dragon's Total Caloric Intake

Basic Metabolic + Temperature + Life Style

Extra Calories for Growth Before Certain Age

- Body composition for different types of animals [water, protein, fat, carbohydrates, minerals, etc]
- Calory needed for growth of unit weight $\Delta\text{Energy} = \sum \alpha_i \cdot p_i \cdot \Delta W$

(α_i: Energy required to increase 1 unit weight of protein, fat, or carbohydrates, etc; p_i: propotion of each component in Dragon; ΔW: weight change.)

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Connect to Predation

- If the dragon is like carnivorous reptiles
  - Calories needed -> Food needed (weight of meat) -> # of sheep needed
- If the dragon is like herbivorous reptiles or omnivorous reptiles
  - Calories needed -> Food needed (weight of meat/grass) -> # of sheep needed / grass

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Modified Predator-Prey Model

- Grass Growth Model

$$\frac{dG}{dt} = g_1 G\left(1 - \frac{G}{K}\right) - d_1 G - c_1 S$$

- Sheep Population Model

$$\frac{dS}{dt} = g_2 GS - d_2 S - h_1 S W_f - c_2 W_f$$

- Wolf Population Model

$$\frac{dW_f}{dt} = g_3 S W_f - d_3 W_f$$

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Skelton of the Project

- Introduction
- Model Development
- Assumptions and Justifications
- Mathematical Modeling and Analysis
- Sensitivity Analysis
- Results and Interpretations
- Letter to Authors

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Iteration and Touch Back

First Week of Class	Introduction
ODE Classifications and Euler's Method	Various ways to model growth
First-Order ODE	Dragon Growth Metabolism
Second-Order Linear ODE	Food Intake Sensitivity Analysis
Systems of ODEs	Connect to Predation Numerical Solutions
Laplace Transform	Report preparation

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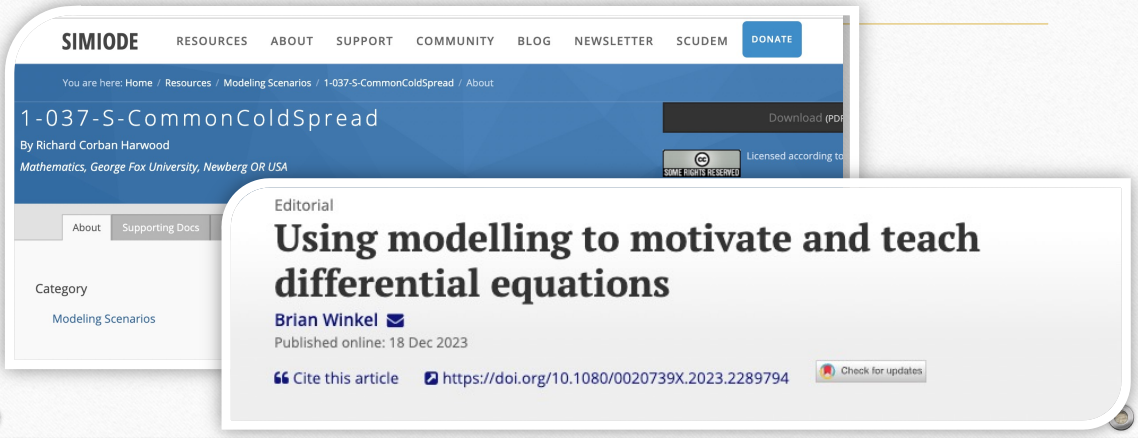
Main Reference

References



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Questions
Suggestions
Comments

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