Suggested Topics and Timing

Differential Equations: A Toolbox for Modeling the World

available at
`https://qubeshub.org/community/groups/simiode/textbook`

Background

This topics and timing list is for a typical one-semester (14 instructional weeks) course that meets three days per week, perhaps with a one hour "recitation". This recitation could be used for students to work on one of the chapter-end projects. The general expectation is that students will do one of the projects at the end of each chapter, though these project sections are not explicitly listed as part of the topic list below. Supplementary resources for students and faculty are available at `https://qubeshub.org/community/groups/simiode/textbook`.

This list also assumes that boundary value problems and basic Fourier series are covered (two weeks) but if not that gives more time for the other topics.

Some sections are listed as "elective." Except as noted, each such section can be covered or omitted according to student and instructor interest without compromising the ability to cover any later material.

This time table assumes that the students have some familiarity with complex numbers and basic vector/matrix computations (notation, the mechanics of formulating and solving linear systems, and eigenvalues/eigenvectors). There are brief introductions to and reviews of these topics in Appendices A and B.

Week 1

Topics and Sections Introductory models: Hill-Keller (1.1), intracochlear drug delivery (1.2), fishery model and management (1.3). Solving ODEs by integration (1.4). Elective: Introduction to dimensional analysis (1.5).

Resources Introductory computer codes for solving ODEs in each of Maple, Mathematica, Matlab, and Sage.

Notes Sections 1.1 to 1.3 don't all have to be done in full detail in class; some portion can be given as reading or exercises. Section 1.5 is needed only for Section 4.5 (scaling and nondimensionalizing DEs), though the techniques are useful as sanity check when setting up models.

Week 2

Topics and Sections First-order linear equations (2.1) and separable equations (2.2), with application to some additional first-order models.

Resources Code and data for exercises 2.2.8 (yeast growth data) and 2.2.9 (position of a falling shuttlecock). Code and data for Project 2.5.1 (Money Matters 2) and Project 2.5.2 (Chemical Kinetics).

Notes Sections 2.1 and 2.2 are a bit longer, since they also develop some additional first-order models to which the analysis techniques can be applied. There is some emphasis on modeling using unspecified parameters.

Week 3

Topics and Sections Direction fields, qualitative analysis and phase line portraits for scalar ODEs; bifurcations (2.3). Existence-uniqueness theorem for scalar first-order ODEs (2.4). Euler's method for scalar ODEs (3.1).

Resources Computer codes for drawing direction fields; computer codes to support Euler's method, the improved Euler method, and the Runge-Kutta fourth-order method.

Notes Numerical methods for ODEs (including Section 3.1) could be omitted entirely if one is content to use built-in numerical solvers without knowing how they work.

Week 4

Topics and Sections Elective: The improved Euler method (3.2), modern numerical methods (3.3), and parameter estimation (3.4).

Resources Code and data for exercises 3.4.1-3.4.3 and 3.4.8-3.4.11 (parameter estimation), as well as all projects in Section 3.5.

Notes The material in Sections 3.2 and 3.3 isn't critical, but does build important background information on how modern numerical ODE solvers work and how the input arguments to these commands can affect the accuracy of the computed solution. The material in Section 3.4 is also elective, though parameter estimation appears occasionally later in the text and some exercises/projects.

Week 5

Topics and Sections	Motivational spring-mass models (Section 4.1: buildings in an earthquake, bike shock absorbers, vibration isolation tables) leading to the second-order harmonic oscillator/unforced spring-mass ODE. Solving the harmonic oscillator ODE, physical notions of under-,over-, and critically-damped system (4.2).
Resources	Introductory computer codes for solving ODEs (including second-order ODEs). Code to support Project 4.6.3 (Parameter Estimation for a Spring-Mass System).
Notes	The models of Section 4.1 need not all be covered in detail in class, as they are all variations of basic spring-mass systems.

Week 6

Topics and Sections	Forced/driven oscillators and the method of undetermined coefficients for solving forced oscillator ODEs (4.3); resonance and beats (4.4). Elective: scaling and nondimensionalizing ODEs (4.5).
Resources	Same as week 5.
Notes	In Chapter 4 it's worth emphasizing the value of linearity and the principle of superposition for constructing solutions to linear ODEs. There is plenty of opportunity to apply the material of Section 4.5 later in the text, but nowhere is it central or required.

Week 7

Topics and Sections	Introductory models (pharmacokinetic), to introduce discontinuous or impulsive forcing, to motivate the need for the Laplace transform (5.1). Computing and inverting Laplace transforms, and solving ODEs with Laplace transforms (5.2). Section 5.2.7 (the initial and final value theorems) could be elective, though it is used briefly in Section 5.6 on PID control theory.
Resources	There are allied exercises in Section 5.1 to introduce other models (monetary, thermal, mechanical) that have discontinuous or impulsive forcing. There is code to support computing, inverting, and using Laplace transforms to solve ODEs.

Notes It's worth emphasizing the big idea that many operations in the time domain have counterparts in the Laplace s-domain, and that it is often easier to work in the s-domain.

Week 8

Topics and Sections The Heaviside (unit step) function and its use in ODE models (5.3). The Dirac delta function and its use in ODE models (5.4). Elective: Input-output relations and transfer functions (5.5). Elective: the application of Laplace transforms to control theory (5.6).

Resources There is code to support Laplace transform computations and ODE solutions involving Heaviside and Dirac functions, and the Section 5.6 computations (including the projects in Section 5.7).

Notes The discontinuous and impulsive forcing models in Sections 5.3 and 5.4 are used as a primary motivation for developing the Laplace transform in Section 5.1.

Week 9

Topics and Sections Linear systems of ODEs, motivated by a pharmacokinetic model (6.1). Analysis and solution of homogeneous linear systems of ODEs (6.2), and application to other models.

Resources Computer code to support the eigenvalue/eigenvector analysis of matrices and linear systems of ODEs.

Notes This schedule assumes that student are familiar with basic matrix algebra and the notion of eigenvalues/eigenvectors.

Week 10

Topics and Sections Solving non-homogeneous systems of ODEs via undetermined coefficients or Laplace transforms (6.3). Elective: The matrix exponential (6.4) as a solution technique.

Competing species and epidemics to motivate nonlinear systems of ODEs; direction fields (7.1).

Resources Code for solving linear systems via Laplace transforms, and support the pharmacokinetic project in Section 6.5. Also code to illustrate direction fields.

Notes The matrix exponential assumes students are familiar with diagonalizing matrices; this topics is discussed in Appendix B.

Week 11

Topics and Sections Eigenvalues for linear systems and their relation to the direction fields for the system (7.2). Sketching phase portraits and performing stability analysis for autonomous nonlinear systems (7.3).

Resources Code to support sketching and analyzing two-dimensional direction fields for autonomous systems.

Notes The focus is on planar systems, though there is some discussion of higher-dimensional systems.

Week 12

Topics and Sections Elective: Direction fields and qualitative analysis for systems with unspecified parameters and using trace-determinant analysis (7.4). Elective: numerical methods for systems of ODEs and stiff systems (7.5). Elective: further techniques for the analysis of systems, including conserved quantities/first integrals, Lyapunov functions, and the Routh-Hurwitz theorem (7.6).

Resources Code to illustrate numerical methods for systems, and support the project "Parameter Estimation for Competing Species."

Notes Again, the focus is on planar systems, though there is some discussion of higher-dimensional systems.

Week 13

Topics and Sections One-space-dimensional time-dependent models, conservation modeling, the continuity equation, and the heat equation (8.1). Introduction to Fourier series (8.2).

Resources Code for computing and graphing Fourier series.

Notes The text focuses first on Fourier sine and cosine series, which are most immediately useful as tools for solving the heat and/or wave equations. There are exercises that explore Fourier sine/cosine series (exercise 8.2.6) and Fourier series with complex exponentials (exercise 8.2.10), as well as other basis functions.

Week 14

Topics and Sections Solving the heat and general diffusion equation with Fourier series (8.3). The advection and wave equations (8.4).

Resources Code for the project "Frequency Analysis of Signals" discusses using Fourier techniques to analyze real data.

Notes The focus is on the homogeneous heat and wave equations, but there is a discussion of the nonhomogeneous heat equation at the end of Section 8.3. Also, the D'Alembert solution to the wave equation on the real line is discussed at the end of Section 8.4. Both are optional.