Stress-Strain Curves and Young's Modulus

# Introduction

Solid materials are often categorized by their mechanical behavior. One such category is tensile materials, which operate by resisting being pulled upon. Four common types of tensile materials are found in living organisms: silk, collagen, cellulose, and chitin. Silk and collagen are both composed of proteins, while cellulose and chitin are composed of polysaccharides (sugars). The properties of tensile materials are often investigated using stress-strain tests, which involve pulling on a sample from each end. Spider webs, which function in prey capture for many species, are made of silk, a well-studied example of a tensile material.

# Importance

Spider webs must be able to withstand destruction from a variety of forces. Wind affects the strands of the web and the substrate(s) to which the web is anchored (leaves, branches, blades of grass, etc.). Additionally, insects flying into the web exert force not only upon impact, but as they struggle to free themselves.

# Questions

How does the web of a spider balance the conflicting requirements of being strong enough to trap prey, fine enough to resist wind disturbance, and flexible enough to resist deformation from struggling insects and movement of anchoring substrate?

# Variables

|  |  |
| --- | --- |
| s | stress (MPa) |
| e | strain (dimensionless) |
| E | Young's modulus of elasticity (MPa) |

# Methods

A strain on a material can be defined as any change in the materialís dimension, and any force acting on a material produces a stress. With tensile materials, strain () is the same as stretch, and is simply the ratio of the change in size to some basic (or original) size (often given as a percentage;  = 0.1 indicates that each unit of length has extended by 10%). The unit for stress () is the pascal (Pa) or megapascal (MPa), which is the force per unit area (a newton or meganewton, respectively, per square meter). Young's modulus of elasticity (*E*), also known as the elastic modulus, is the ratio between stress and strain:

|  |  |
| --- | --- |
| $$E={σ}/{ε}$$ | LaTeX Code: \[ E = \sigma / \varepsilon \] |

and has the same units as stress. *E* is the slope of the stress-strain graph: the steeper the slope, the stiffer the material. The maximum height of the stress-strain curve is called the tensile strength (also given in MPa), which is a measure of the amount of stress a material can take before tearing apart. The extensibility, or breaking strain, is the furthest horizontal extent of the stress-strain curve, and like strain is dimensionless.

The basic orb web is composed of several parts (the mooring threads, frame, radii, hub, and the sticky catching spiral). The mooring threads attach the web to its substrate while the frame, radii, and hub provide structural support. None of these types are sticky. The radii support the catching spiral, which is made out of a sticky type of silk that entangles prey. Köhler and Vollrath (1995) examined the thread biomechanics of the capture spiral for the orb-weaving spider *Araneus diadematus* (data estimated and stress-strain curve redrawn from Fig. 5b in Köhler and Vollrath, 1995), and from these data we can calculate Young's modulus (*E* = */*):

|  |  |  |  |
| --- | --- | --- | --- |
| Stress (MPa) | Strain (%) | * /* | Young's modulus (MPa) |
| 0 | 0 | -- | -- |
| 20 | 50 | 20/0.5 | 40.0 |
| 55 | 100 | 55/1.0 | 55.0 |
| 130 | 150 | 130/1.5 | 86.7 |
| 249 | 200 | 249/2.0 | 124.5 |
| 430 | 250 | 430/2.5 | 172.0 |
| 631 | 300 | 631/3.0 | 210.3 |
| 831 | 350 | 831/3.5 | 237.4 |
| 1031 | 400 | 1031/4.0 | 257.8 |
| 1338 | 476 | 1338/4.76 | 281.1 |

We can graph Young's modulus to see how stiffness changes as strain increases:



We can also graph the standard stress-strain curve:



# Interpretation

Young's modulus of elasticity can be thought of as a measure of how well a substance stands up to tension. The capture spiral silk's ability to withstand increasing strain improves quickly at lower levels of strain, but past a certain point this improvement increases more slowly until the thread breaks.

From the stress-strain graph we can see that the spiral's mean extensibility, which is the maximum strain (or stretch) before breaking, was 476%, as compared to the radii's mean extensibility of 39.4% (data from Köhler & Vollrath not shown). The tensile strength of the capture spiral is 1,338 MPa, while the tensile strength of the radial thread is 1,154 MPa. For comparison, the tensile strength of "mild" steel is 400 MPa (in Vogel 1988, p. 185). The capture spiral must absorb most of the kinetic energy from an insect's initial impact, while the radial threads serve primarily as scaffolding for the spiral.

# Conclusion

In order for a flying insect to be trapped by a web, its motion must be stopped. The force required to stop its motion is inversely proportional to the distance over which the motion must be stopped. In other words, the greater the distance over which the insect is slowed down the smaller the force necessary to stop it. The capture spiral's high extensibility enables spiders to trap insects with a fairly minimal amount of force and reduces the potential for damage to the web. The extensibility and tensile strength of spider silk in general, combined with its light weight, enable it to resist damage from wind and from being pulled by anchoring points of the web.

# Additional Questions

1. How do strength (as in tensile strength) and stiffness (as in Young's modulus of elasticity) differ conceptually? Under what conditions would it be desirable to maximize one over the other?

2. What type(s) of equation(s) might fit the line of the stress-strain curve?

# Source

Foelix, R. F. 1996. *Biology of Spiders, 2nd edition*. Oxford University Press, New York, NY.

Köhler, T. and F. Vollrath. 1995. Thread biomechanics in the two orb-weaving spiders Araneus diadematus (Araneae, Araneidae) and Uloboris walckenaerius (Araneae, Uloboridae). *Journal of Experimental Zoology 271*:1-17.

Vogel, S. 1988. *Life’s Devices: the Physical World of Animals and Plants*. Princeton University Press, Princeton, NJ.

Wainwright, S. A., W. D. Biggs, J. D. Currey, and J. M. Gosline. 1976. *Mechanical Design in Organisms*. Princeton University Press, Princeton, NJ.

# About this Resource

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This material is now being revised as part of the “Resources for Improving Quantitative Skills in Community College Biology[[2]](#endnote-2)” project. As part of that project is also aligned with the OpenStax Biology Textbook[[3]](#endnote-3).

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1. http://www.tiem.utk.edu/~gross/bioed/ [↑](#endnote-ref-1)
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