Mathematical Modeling Via Multiple Representations

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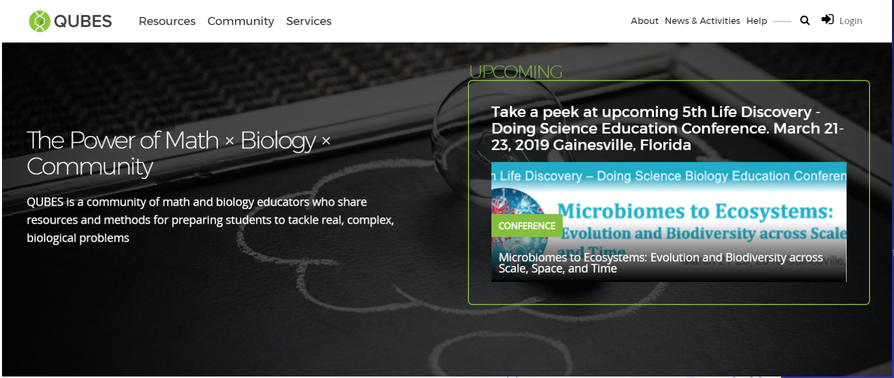
Annual Meeting for the Society for Mathematical Biology, 2019

Outline

QUBES
The Power of Biology × Math × Community

- Resource Hub - qubeshub.org is designed to support teaching at the interface of mathematics and biology no matter where you are
- Supported by NSF IUSE, NSF DBI, BIO SIGMAA, NCTM, SIAM, and COMAP and partner contracts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The Power of Math × Biology × Community

QUBES is a community of math and biology educators who share resources and methods for preparing students to tackle real, complex, biological problems

UPCOMING

Take a peek at upcoming 5th Life Discovery - Doing Science Education Conference. March 21-23, 2019 Gainesville, Florida

Life Discovery – Doing Science Biology Education Conferen
Microbiomes to Ecosystems:
Evolution and Biodiversity across Scale
CONFERENCE

Microbiomes to Ecosystems: Evolution and Biodiversity across Scale, Space, and Time

QUBES

The Power of Biology × Math × Community

- Run Peer Faculty Mentoring Networks, cloud based software, partner with professional societies and curriculum projects, organize conversations (like this one!).
- This takes an understanding of best practices in teaching at the interface of mathematics and biology.
- Proposed a working group to NIMBioS (National Institute for Mathematical Biology and Synthesis) to investigate and synthesize across domains how best to unpack the black box of models for biology students/teachers.

NIMBioS Working Group

Interdisciplinary Team:
1 math ed, 3 bio ed, 1 bio SoTL, 2 math SoTL, 1 bio faculty dev 2 biologists, 2 math bio, 3 mathematicians, 1 physics bio ed (blurry lines)

NIMBioS Working Group

- Discussed lots of issues:
    - What quantitative skills are needed?
    - What tools are available for assessment?
    - Math anxiety (instructor and student)
    - Language
- Additional research → language as key.
- **Model/modeling** - as the mathematicians were thinking about constructing an equation from a concept and the biologists were thinking about fitting an equation to data.
- Reflects other work done in physics for biology by Joe Redish

Models and Modeling

Definition

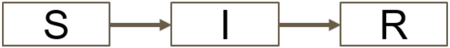
A **model** is a simplified, abstract or concrete representation of relationships and/or processes in the real world, constructed for some purpose.

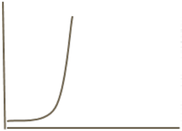
Model Representations:

- Verbal
- Visual
- Symbolic
- Numerical

May know this as the "Rule-of-4" in calculus reform.

"The rate of population growth is proportional to its current size."





$$\frac{dP}{dt} = rP$$

T (year)	P (per hectare)
0	56.1
1	63.2
...	...

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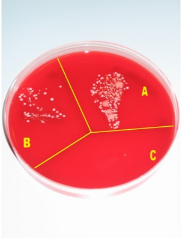

"Rule-of-Five" Model Representations:

- Verbal
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- Experiential

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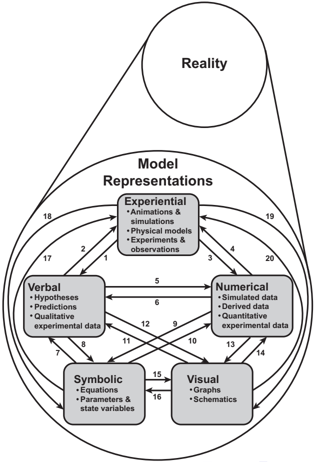

Reality

Model Representations

Experiential
- Animations & simulations
- Physical models
- Experiments & observations

Verbal
- Hypotheses
- Predictions
- Qualitative experimental data

Numerical
- Simulated data
- Derived data
- Quantitative experimental data

Symbolic
- Equations
- Parameters & state variables

Visual
- Graphs
- Schematics

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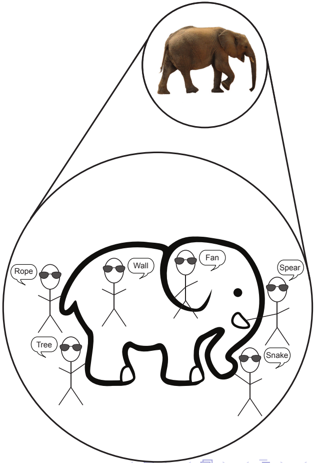


Models and Modeling

Definition

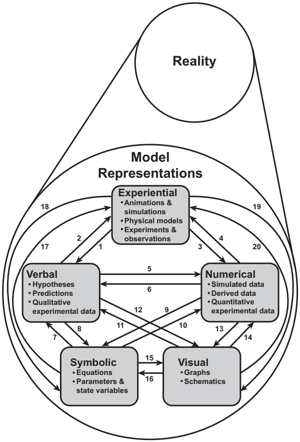
Modeling is the act of moving between representations/models (arrow), checking model with reality and/or revising model.

Modeling activities

- Moving between representations/models (arrow)
- Checking model with reality
- Creating and revising model

Modeling process

- A set of modeling activities from reality to "good enough."
- Reality & experiential are key!
- Defined to include approaches like data science



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The Challenge

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"**In theory, there is no difference between theory and practice. In practice, there is.**"

- So how do we implement this theory in the classroom?
  - With classroom projects that 'model' modeling with a directed sequence of modeling activities.

Modeling Activity Objectives

1. Illustrate the true nature of science
    - Theory without observation (natural and/or experimental) is mere speculation.
    - Observation without theory is just a collection of data.
    - **Scientific progress is due to the combination of theory and observation.**

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4. Teach the principles of density-dependent growth

Approach

- The real world is complicated.
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- The real world is complicated.
    - Hard to collect data.
    - Many confounding complications.
    - Demographic stochasticity.
- The nature of science is more easily discovered using real data from an artificial world. (e.g., C.S. Holling, 1959)
    - Easy to collect data.
    - Based on simple mechanisms.
    - Must have demographic stochasticity!

Experiential – Materials and Setup

			○	○			
		○	X	X	○		
		○	X	X	○		
			○	○			

X — square is occupied

○ — square is available

Time	Pop.	Prev.	Incr.
0	4	NA	NA
1		4	
2			
3			
4			
5			
⋮	⋮	⋮	⋮
20			

Experiential – Simulation Rules

		○	⊘	⊙			
	○	X	X	X	○		
		○	X	X	X	○	
			○	X	○		
				○			

X — square is occupied
○ — square is available

1. For each available square:
   a. Roll one die for each adjacent occupied square.
   b. If any die is 5 or 6, mark the square with a slash (/).
2. Change the slashes into X's. Record population.
3. Mark new available squares with a circle (○).

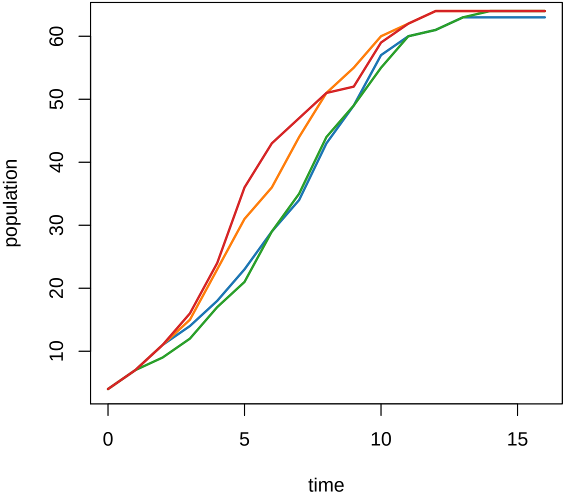
- Stop when nearly all squares are occupied.

Numerical – Lots of Data

Time	Pop.	Prev.	Incr.
0	4	NA	NA
1	7	4	3
2	11	7	4
3	15	11	4
4	23	15	8
5	31	23	8
6	36	31	5
7	44	36	8
8	51	44	7
9	55	51	4
10	60	55	5
11	62	60	2

Time	Pop.	Prev.	Incr.
0	4	NA	NA
1	7	4	3
2	11	7	4
3	14	11	3
4	18	14	4
5	23	18	5
6	29	23	6
7	34	29	5
8	43	34	9
9	49	43	6
10	57	49	8
11	60	57	3
12	61	60	1

Visual – Population Graphs



Visual – What Else?

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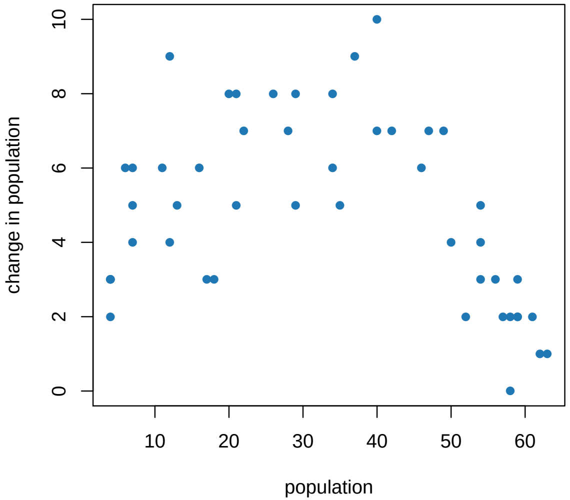
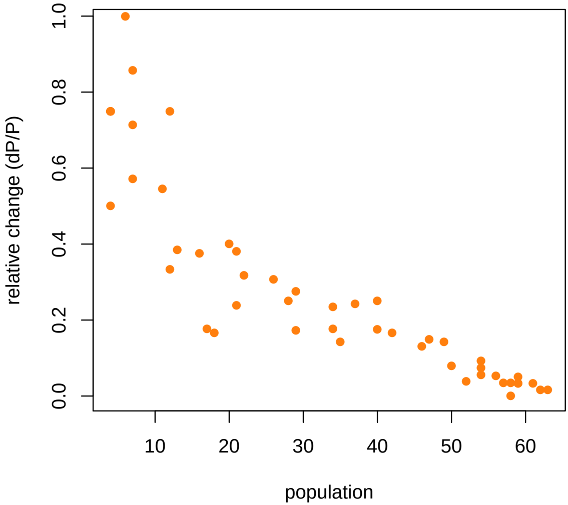
  - Notice that slopes of the orange and green lines are the same for the same populations?
- How about plotting population change vs population?
- Other ideas?

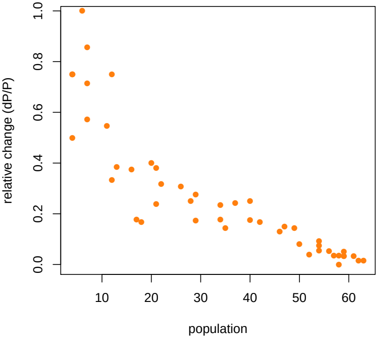
Visual – Change vs Population



Visual – Relative Change vs Population



Verbal – An Empirical Hypothesis



- Ignore the demographic stochasticity (scatter).
    - Is there a signal hiding under the noise?

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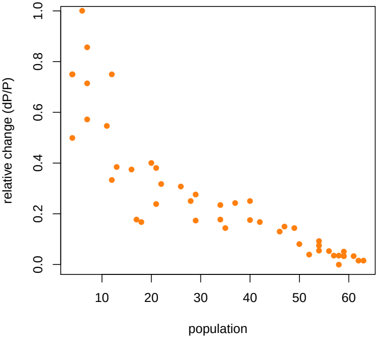

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- **Maybe the relative change is a linear function of the population, reaching 0 when the space is full.**

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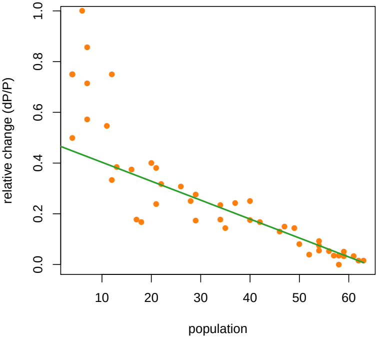

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Symbolic – Dynamic Equation

- Discrete

$$\frac{\Delta P}{P} = r\left(1 - \frac{P}{K}\right)$$

or

$$\Delta P = rP\left(1 - \frac{P}{K}\right)$$

- Continuous

$$\frac{dP/dt}{P} = r\left(1 - \frac{P}{K}\right)$$

or

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

We need numerical implementation of a statistical method to fit r to the data (given K).

(See Ledder, Coll Math J, 47 (109), 2017.)

Improvements

- So far, we're working with very limited data (like real ecologists) and a very simple setting. With a computer simulation, we can add detail and collect much more data quickly.

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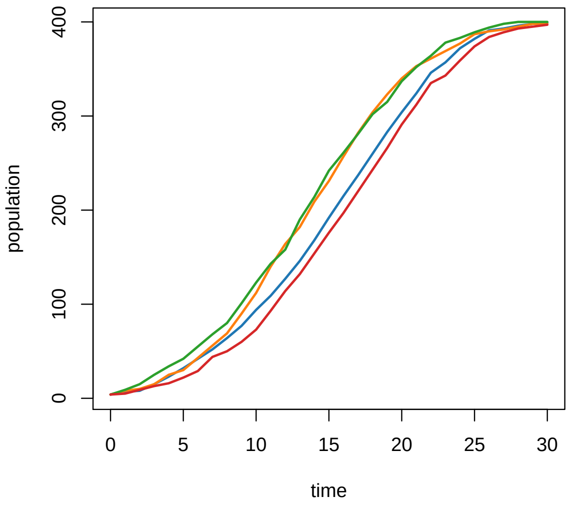
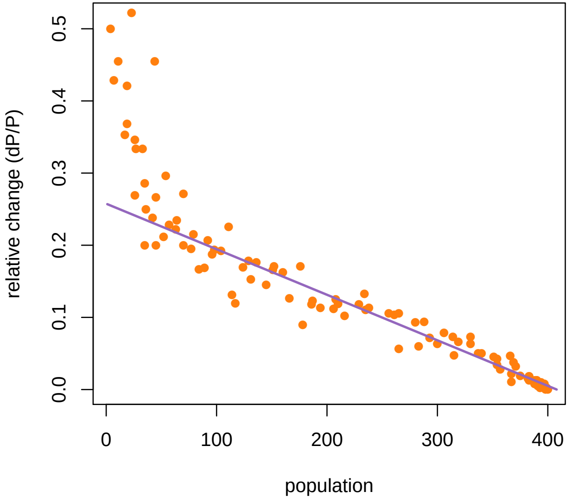
- PopGrowth.R (GL) and LogGrowth.nlogo (M.D. LaMar)
    - arena size, $8 \leq s \leq 50$, best at about 20
    - birth probability, $0 < b < 1$, best at 0.1 to 0.8
    - death probability, must be $0 \leq d < b$, best at 0 to b/4
    - number of trials, 1 to 4
    - starting setup: center or edge
    - curve fit options: none, r only, r and K



population
0
100
200
300
400
0
5
10
15
20
25
30
time


relative change (dP/P)
0.0
0.1
0.2
0.3
0.4
0.5
0
100
200
300
400
population

Wrapping it Up

Remember our objectives:

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- In this study, we developed a model for a synthetic system.
    - We actually know the true biological processes, which are completely different from the model.
- But the model does a great job of predicting the results.

Acknowledgement

- **Special thanks to Carrie for her kind offer to share her talk slot with someone whose first priority had to be giving a research talk.**

Acknowledgements

- Thank you to SMB for the day of education events.
- For more information, see our work here:

A Framework for Modeling to Encourage Interdisciplinary Conversations

Dr. Carrie Diaz Eaton
Dr. Hannah Callender
Dr. Kam Dahlquist
Dr. Drew LaMar
Dr. Glenn Ledder
Dr. Richard Schugart

Special issue on **Interdisciplinary Conversations** in *PRIMUS*

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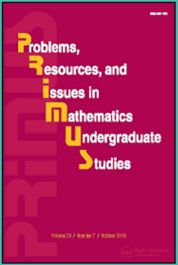
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Problems, Resources, and Issues in Mathematics Undergraduate Studies

Dr. Victor Piercey, Dr. Carrie Diaz Eaton, Dr. Stella Hoffreneig, & Dr. Susan Ganter

NSF #1446258, 1446269, and 1446284.

Bates

THE UNIVERSITY of TENNESSEE UT KNOXVILLE

NIMBioS National Institute for Mathematical and Biological Synthesis