# Relative sizes on a logarithmic scale of organisms and the type of microscope needed to view each item. Electron microscope for atoms (0.1 nm) to viruses (100 nm). Light microscope for organelles (1 um) to frog cell (1 mm). Naked eye frog cell (1 mm) to human adult male (1 m).Biological Concepts:

# Learning Objectives

## Biology

1) Explain the relationship of surface area to volume

2) Describe the importance of a large surface area to volume ratio in the context of a living cell

## Mathematics

1) Calculate surface area of cubes and spheres

2) Calculate volume of cubes and spheres

3) Express two values as a ratio

## Data Literacy

1) Enter data into a table

2) Interpret Tables

3) Create a graph

4) Describe the axis labels on graphs

5) Interpret graphs

## Tools

GeoGebra-

<https://www.geogebra.org/m/xzzrgmqp>

<https://www.geogebra.org/m/q2pgh2ju>

Microsoft Excel (optional)

## Equations

Surface Area Cube

SA = 6a2

Surface Area Sphere

SA = 4πr2

Volume Cube

V = a3

Volume Sphere

Why are Cells Small?

At 0.1 to 5.0 μm in diameter, prokaryotic cells are significantly smaller than eukaryotic cells, which have diameters ranging from 10 to 100 μm. The prokaryotes' small size allows ions and organic molecules that enter them to quickly diffuse to other parts of the cell. Similarly, any wastes produced within a prokaryotic cell can quickly diffuse. This is not the case in eukaryotic cells, which have developed different structural adaptations to enhance intracellular transport. Small size, in general, is necessary for all cells, whether prokaryotic or eukaryotic. Let’s examine why that is so. First, we’ll consider the area and volume of a typical cell. Not all cells are spherical in shape, but most tend to approximate a sphere. The formula for the surface area of a sphere is 4πr2, while the formula for its volume is 4πr3/3. Thus, as the radius of a cell increases, its surface area increases as the square of its radius, but its volume increases as the cube of its radius (much more rapidly). Therefore, as a cell increases in size, its surface area-to-volume ratio decreases. This same principle would apply if the cell had a cube shape. If the cell grows too large, the plasma membrane will not have sufficient surface area to support the rate of diffusion required for the increased volume. In other words, as a cell grows, it becomes less efficient. One way to become more efficient is to divide. Another way is to develop organelles that perform specific tasks. These adaptations lead to developing more sophisticated cells, which we call eukaryotic cells. ([Copied and adapted from Biology 2e by OpenStax](https://openstax.org/books/biology-2e/pages/4-2-prokaryotic-cells))

# Mathematical Concepts:

## Thinking Questions

1. In eukaryotic cells, that make up tissues, there may be only one face of the cell that can absorb substances. What cell features can increase the surface area without dramatically increasing the volume?
2. What other shapes could a cell have that would maximize the Surface area to Volume Ratio?

## Volume and Surface Area

* [Volume](https://www.mathsisfun.com/definitions/volume.html): The amount of 3-dimensional space something takes up. Imagine how much water could be in it. Also called Capacity.
* [Surface Area](https://www.mathsisfun.com/definitions/surface-area.html): The total area of the surface of a three-dimensional object. Example: the surface area of a cube is the area of all 6 faces added together.

## Ratios

* Ratios are a way of comparing two numbers or quantities.  Using an easy example, like number of males in the class compared to number of females in the class. E.g. 14 males to 8 females, or 14:8.
* Ratios can be simplified to show how the quantities are related, without using the exact original amounts:   e.g. 14 males: 8 females is the same ratio as 7 males: 4 females. In other words, there are 7 males for every 4 females, if you were to group them.
* We simplify ratios the same way we do fractions, and that’s actually another way to write a ratio. 14:8 is the same as 14/8 which can be simplified to 7/4 or 7:4
* If you simplify the ratio or fraction down to the point where the second number is a 1, you get what’s called a Unit ratio.  E.g. 7:4 = 7/4 = 1.75 = 1.75/1. In other words, for every 1 female, there are 1.75 males.

# Learn by Doing:

In this module we are going to be examining the changes in surface area to volume ratio as we increase size of an object.

1. Build a cube to examine concept of surface area and volume.
2. Predict what the effect of changing the shape or size has on the surface area to volume ratio.
3. Go to the GeoGebra site for the cube and the sphere.
   1. <https://www.geogebra.org/m/xzzrgmqp>
   2. <https://www.geogebra.org/m/q2pgh2ju>
4. Record the data for the cube and sphere in the tables.
5. Using the data from the tables, plot the surface area and volume onto the graph.
6. Determine whether the data supported your predictions.
7. Answer the assessment questions.
8. Optional extension: Learning to use excel.

## Cube Exercise:

1. Cut out a 5 cm x 5 cm x 5 cm cube from your centimeter grid paper (found on next page).
2. Put the box together using glue or tape.
3. Answer the following questions
   1. What is the surface area of your cube? This is calculated by the equation SA = 6a2. The letter a represents the side length. Counting the number of 1 cm x 1 cm squares on the outside of the cube will give you the same answer.
   2. What is the volume of your box (V=a3)?
   3. What is the ratio of surface area to volume (SA:V)?
   4. Simplify the ratio for SA:V to the unit ratio (see the example in Mathematical Concepts).

## Predictions:

1. How do you think this ratio would change if your cube was larger, or smaller? (Hint: read the background information)?
2. Cells in the body are not often in the shapes of cubes. How do you think surface area to volume ratio might change if the shape of your object was a sphere and not a cube?

## Testing your predictions:

Go to the GeoGebra site for the cube and the sphere.

* 1. <https://www.geogebra.org/m/xzzrgmqp>
  2. <https://www.geogebra.org/m/q2pgh2ju>

Record the data for the cube and sphere in the tables (make sure you are entering data into the correct columns).

Using the data from the tables, plot the surface area and volume of the square and sphere onto the graph.

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## Data Sheets:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cube side (cm) | SA (cm2) | V (cm3) | Ratio (SA:V) | SA/V |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 | 150 | 125 | 150:125 | 1.2 |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Radius (cm) | SA (cm2) | V (cm3) | Ratio (SA:V) | SA/V |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 | 314 | 523.3 | 3.14:523.3 | 0.6 |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |

## Graphing Exercises:

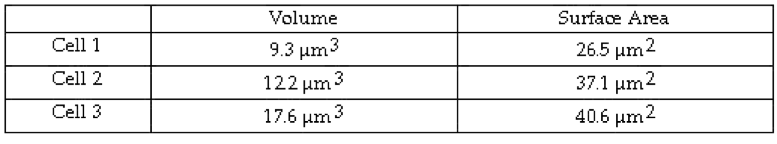
**\*only plot up to a radius of 6 cm**

## Graphing and Data Table Questions:

1. Describe the differences based on the data recorded from the square and the sphere.

1. Does the data you collected match your original predictions? Why or Why Not?
2. What surprised you about the results you collected?
3. Why did you need use two different y-axis labels in this exercise?

## Concept Review Questions:

1. Why is there an upper limit to the surface area-to-volume ratio in a cell? ​
   1. ​The volume of the cell maximizes the use of phospholipids.
   2. ​The surface area must be sufficient to provide adequate nutrient and waste exchange to sustain the chemical activities of the cell.
   3. ​The cells require a balance between endocytosis of precursor molecules to transporters in the membrane.
   4. ​The size of a cell is limited by the number of mitochondria.
   5. ​The organelles in cells require communication with the extracellular environment to support internal functions.
2. As the total surface area of a cell increases, the surface area/volume ratio decreases.
   1. True
   2. False
3. You have a cube of modeling clay in your hands. Which of the following changes to the shape of this cube of clay will decrease its surface area relative to its volume?
   1. Pinch the edges of the cube into small folds.
   2. Flatten the cube into a pancake shape.
   3. Round the clay up into a sphere.
   4. Stretch the cube into a long, shoebox shape.
4. Both the volume and the surface area for three different cells were measured. These values are listed in the following table:  
   

Using data from the table above, select the best explanation for why that cell will be able to eliminate waste most efficiently?

* 1. Cell 1 since it has the smallest volume and will not produce as much waste as the other cells.
  2. Cell 2 since it has the highest surface area-to-volume ratio which facilitates the exchange of materials between a cell and its environment.
  3. Cell 3 since it has the largest surface area which will enable it to eliminate all of its wastes quickly.
  4. Cell 3 because it is big enough to allow wastes to easily diffuse through the plasma membrane.

1. Surface-area-to-volume ratios indicate
   1. cells must exceed a certain minimum size.
   2. as cells get larger, their surface area actually decreases.
   3. largest cells have less proportionate need for food intake and waste removal.
   4. a chicken egg is one cell thus demonstrating the upper limit for metabolizing cell size.
   5. as cells get larger, their surface area gets larger but at a slower rate than volume increases.
2. The surface area of an object is a measure of
   1. All the exterior surfaces of that object
   2. The interior and exterior surfaces
   3. The interior of the object only
   4. A cross-section of the object
3. The volume of an object is a measure of
   1. All the exterior surfaces of that object
   2. The interior and exterior surfaces
   3. The interior of the object only
   4. A cross-section of the object
4. Rank the shapes below from least surface area to most surface area.

(all are approximately 1cm x 1cm overall in size)

Radius = 0.5 cm L & W = 1 cm

1. For small objects, the volume is less than the surface area. As surface area increases, what happens to the volume.
2. Volume increases faster than surface area
   1. Both increase at an equal rate
   2. Surface area increases faster than volume
   3. At first surface area increases faster, then it switches to volume increasing faster.
3. Units for the measurement of surface area are \_\_\_\_\_\_\_\_ while units for the measurement of volume are \_\_\_\_\_\_
   1. squared; cubed
   2. cubed; squared
   3. squared; squared
   4. cubed; cubed
4. Cells are better to obtain nutrients and oxygen, and get rid of wastes and carbon dioxide, if
   1. They have a high surface area to volume ratio
   2. They have a low surface area to volume ratio
   3. Their surface area and volume are at or near equal
5. The ratio of two quantities or numbers tells you
   1. How many of each item there actually are
   2. How the quantities compare to each other
   3. How quickly the numbers change
   4. All of the above
6. If there are 12 cats and 32 dogs available for adoption at an animal shelter, the simplified ratio of cats to dogs is:
   1. 8:3
   2. 12:32
   3. 3:8
   4. 0.375
7. Matching:

\_\_\_\_\_Surface area of a cube

\_\_\_\_\_Surface area of a sphere

\_\_\_\_\_Volume of a cube

\_\_\_\_\_Volume of a sphere

a. 4πr2

b. 4/3 πr3

c. (L2)x6

d. L3

## Self-Reflection:

1. What was the most challenging part of this assignment?
2. What did you like best about this assignment?

## Excel Exercise:

1. Set up your spread sheet so that column A contains measures of cube sides of 0.5-10.   
   [Units don’t really matter, could be cm or micrometers. cm is more intuitive, but micrometers is more in keeping with cells actual size.]
2. In the next column, B, enter the formula that will calculate the surface area of a cube.
   1. in Excel it will look like this: [ = 6\*(B2)^2] where B2 is the cell where the first side measurement has been entered.   
      -When typing in any equation that uses a value you have already entered into the spread sheet, click on the cell (B2 for example) instead of typing in the cell name.
3. Column C will contain the formula for calculating the volume of a cube.
   1. In Excel it will look like this: [ = (B2)^3] where B2 is the cell where the first side measurement has been entered.
4. Once the equations are entered in the first row, each can be dragged down to populate the rows for all the side measurements.   
   [How to on a PC](https://support.office.com/en-us/article/fill-a-formula-down-into-adjacent-cells-041edfe2-05bc-40e6-b933-ef48c3f308c6)   
   [How to on a Mac](https://support.office.com/en-us/article/copy-a-formula-by-dragging-the-fill-handle-in-excel-for-mac-dd928259-622b-473f-9a33-83aa1a63e218)
5. Column D should show the actual ratio, 6:1, for example. Format this column as text to avoid having Excel change the format of the entry.  
   [How to on a PC](https://support.office.com/en-us/article/format-numbers-as-text-583160db-936b-4e52-bdff-6f1863518ba4)  
   [How to on a Mac](https://support.office.com/en-us/article/format-numbers-as-text-in-excel-for-mac-dfbe20dc-e7b1-44b2-80fe-2072d074e2a3)
6. In column E, display the quotient of the surface area divided by the volume.   
   The equation will be [ =B2/C2 ]

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| measure one side of cube (cm) | SA | V | ratio | SA/V |
| 0.5 | 1.5 | 0.125 | 1.5:.125 | 12.0 |
| 1 | 6 | 1 | 6:1 | 6.0 |
| 2 | 24 | 8 | 24:8 | 3.0 |
| 3 | 54 | 27 | 54:27 | 2.0 |
| 4 | 96 | 64 | 96:64 | 1.5 |
| 5 | 150 | 125 | 150:125 | 1.2 |
| 6 | 216 | 216 | 216:216 | 1.0 |
| 7 | 294 | 343 | 294:343 | 0.9 |
| 8 | 384 | 512 | 384:512 | 0.8 |
| 9 | 486 | 729 | 486:729 | 0.7 |
| 10 | 600 | 1000 | 600:1000 | 0.6 |

1. Use the [scatter plot](https://www.excel-easy.com/examples/scatter-chart.html) with a line to make a graph of volume (y axis) to surface area (x axis)
2. Choose the measure of a cube side, surface area, & volume and make a skater plot with two lines.
3. Alter the graph to have two Y axis labels, on the left, one for the surface area (cm2) and on the right, another axis for the volume (cm3). [Instructions to get 2nd Y axis.](https://www.nhaustralia.com.au/blog/Create-a-chart-with-a-secondary-axis-in-Excel-2016/)
4. Why is it important to have two different Y axis labels?
5. Note the point at which the volume and surface area are equal in these graphs.
6. Describe the situation before and after the surface are and volume are equal (1:1 Ratio) in terms of the relative size of the surface area and volume.

|  |  |  |
| --- | --- | --- |
|  | Surface Area | Volume |
| Before Ratio is 1:1 |  |  |
| After Ratio is 1:1 |  |  |

1. When the quotient of surface area divided by volume [column E] is plotted against the measure of one side of a cube [column A], or radius of a sphere, this illustrates the important concept that as cells increase in size (surface area), their surface area to volume ratios decrease.
2. Repeat for a sphere, using radius instead of the measured length of one side of the cube.
3. The equation, in Excel, for surface are of a sphere is [ =4\*3.14\*(A2)^2 ]
4. The equation, in Excel, for volume of a sphere is [ =(4/3)\*(3.14)\*(A2)^3 ]

## References:

Clark, M., Douglas, M., Choi, J. (2020). Biology 2e by OpenStax. <https://openstax.org/details/books/biology-2e>

Gross, L., Beals, M., Harrell, S. (2019). [Sizes of Organisms: The Surface Area to Volume Ratio](http://dx.doi.org/10.25334/Q44J03). Quantitative Biology at Community Colleges, /groups/qbcc, QUBES Educational Resources. [doi:10.25334/Q44J03](http://dx.doi.org/10.25334/Q44J03)

## Module Authors:

John Starnes, Southcentral Kentucky Community and Technical College

Jennifer Buntz, Central New Mexico Community College

Christianne Nieuwsma, South Mountain Community College

Sondra LoRe, National Institute for STEM Evaluation and Research

Vedham Karpakakunjaram. Montgomery College