Background Information for both Biology Faculty and Mathematics Faculty

# Biology Background

This module is designed to introduce students to the idea that cell size, shape, and structure are diverse. It is also designed to get students interested in learning more about the role of structure in the function of the cells. The human body is an obvious place to build intrigue for biology students, which is why we are focusing this activity on the various types of cells of the human body. Detailed information on each specific cell type is not needed for this activity, but an overview of the cells is provided.

* Erythrocytes, or red blood cells, are the cells in the body that carry oxygen from the lungs throughout the tissues. They are typically smaller cells due to the fact that they have no nucleus and lack many organelles. Their small size allows them to move through the various vessels and capillaries in the circulatory system.
* Adipocytes (add-ip-o-cytes) are cells that store and maintain fat in the body. They are typically large cells and can swell up to four times their size to store additional fat in the short term. White fat cells are primarily filled with types of lipids (fat) and have very little cytoplasm; the essential organelles are flattened around the periphery. Brown fat cells have smaller lipid storage compartments and have more centralized organelles. These cells can essentially be empty (of fat) although they make up a significant percent of a person’s mass (which explains why someone can have a low percent body fat but have more than 20% of their mass due to fat cells).
* Epidermal cells are skin cells that form the outer layers of epidermis or skin. These cells have a critical function as a barrier to infection, a chemical barrier, as well as helping to maintain body heat and hydration. There are many different types of cells within the epidermis (keratinocytes, melanocytes, immune cells, merkel cells); for this activity we are lumping them all together.
* Muscle cells, or myocytes, are cells that compose muscle tissue. There are three types of these cells: cardiac, smooth, and skeletal. These cells contain long chains of myofilaments that produce the contraction effect that does work for the muscles. There are also specialized structures that help provide energy to these myofilaments.
* Neurons, or nerve cells, are the cells responsible for sending electrical impulses throughout the body. They contain three main parts: 1) a cell body (soma) that has the nucleus and organelles, 2) dendrites that are projections with multiple branches that receive signals from other neurons, and 3) axons that are long projections that transmit electrical impulses from the neuron (they can be up to 1 meter in length).
* Glial cells make up the remaining portion of the central nervous system and include multiple cell types such as oligodendrocytes, astrocytes, ependymal cells, Schwann cells, microglia, and satellite cells.
* Leukocytes or white blood cells are the immune system cells of the body. These cells include monocytes, granulocytes and lymphocytes. These cells vary in size and number depending on whether a person is experiencing an infection.
* Hepatocytes, or liver cells, are the cells that make up the liver. The typical hepatocyte is cubical with sides of 20-30 [μm](https://en.wikipedia.org/wiki/Micrometre). A huge job of these cells is protein synthesis, producing cholesterol and bile salts, as well as detoxifying the blood.
* Vascular endothelial cells are the cells that line the blood vessels throughout the circulatory system. They play an important role in filtering fluids through the blood vessel walls, as well as allowing immune system cells to infiltrate the blood and tissue.
* Platelets are an important part of the blood clotting system and are not actually cells but pieces of cytoplasm from cells in the bone marrow. These “sacks” hold important clotting factors and are abundant in the blood.
* Bacteria are prokaryotic microorganisms that inhabit the body. Bacterial cells typically range from 0.5 um to 5 um making them much smaller than human cells. Bacterial cells do not have a nucleus or other major organelles and therefore are much smaller.

# Mathematics Background

This module is designed to allow students to practice scientific notation and basic algebra skills. Scientific notation is abundant in Biology, so it is a natural application that may be used in the classroom. The human body is an obvious place to build intrigue for all students, which is why we are focusing this activity on the various types of cells of the human body. Suggestions for addressing some of the more difficult mathematical concepts surrounding scientific notation can be found below.

Students will need to be familiar with the following quantitative concepts:

* Rules for Exponents
* Scientific Notation: $a x 10^{b}$, where $1\leq a<10$ and b is an integer
* Basic Algebra (solving for an unknown)

Students will have the most difficulty with the following quantitative concepts:

* Rules for Exponents:
	+ Why negative exponents “move” terms (from numerator to denominator and vice versa).
* Scientific Notation:
	+ Why $1\leq a<10$?
	+ The connection between positive/negative exponents and moving the decimal right/left.
	+ Applying the rules for exponents to perform operations on numbers in scientific notation.

Suggestions for addressing each of these quantitative concepts:

* Rules for Exponents:
	+ Students who are having trouble “memorizing” the rules will likely benefit from seeing the powers written out in long form (e.g. $3^{5}=3∙3∙3∙3∙3$). This helps them make connections between the operations and the “rules”.
	+ Product Rule: $(a^{n})\left(a^{m}\right)=a^{n+m}$
		- Ex.: $(3^{2})\left(3^{5}\right)=\left(3∙3\right)\left(3∙3∙3∙3∙3\right)=3^{2+5}=3^{7}$
	+ Quotient Rule: $\frac{a^{n}}{a^{m}}=a^{n-m}$
		- Ex.: $\frac{3^{9}}{3^{4}}=\frac{3∙3∙3∙3∙3∙3∙3∙3∙3}{3∙3∙3∙3}=3^{9-4}=3^{5}$
		- Ex.: $\frac{3^{4}}{3^{9}}=\frac{3∙3∙3∙3}{3∙3∙3∙3∙3∙3∙3∙3∙3}=\frac{1}{3∙3∙3∙3∙3}=\frac{1}{3^{5}}=3^{4-9}=3^{-5}$
	+ Power rule: $\left(a^{n}\right)^{m}=a^{nm}$
		- Ex.: $(3^{5})^{2}=\left(3^{5}\right)\left(3^{5}\right)=\left(3∙3∙3∙3∙3\right)\left(3∙3∙3∙3∙3\right)=3^{5∙2}=3^{10}$
	+ Negative Exponents: $a^{-n}=\frac{1}{a^{n}}$
		- Ex.: $3^{-5}=\frac{1}{3^{5}}$
		- Ex.: $\frac{1}{3^{-5}}=\frac{3^{5}}{1}=3^{5}$
		- Why do negative exponents “move and become positive”?
			* Note (multiple representations of a number): $a=a∙1=\frac{a}{1}$
				+ Ex.: $5=5∙1=\frac{5}{1}$
			* Note: $1=\frac{a}{a}=\frac{a^{1}}{a^{1}}=a^{1-1}=a^{1}∙a^{-1}=a^{0}$ (i.e. anything to the zero power is 1)
				+ Ex.:$ 1=\frac{5}{5}=\frac{5^{1}}{5^{1}}=5^{1-1}=5^{1+ -1}=5^{1}∙5^{-1}=5^{0}$
			* Note: $a^{n}∙a^{-n}=a^{n+ -n}=a^{n-n}=a^{0}=1$
				+ Ex.: $5^{3}∙5^{-3}=5^{3+ -3}=5^{3-3}=5^{0}=1$
			* Multiplicative Inverses (Reciprocals): values that multiply to 1
				+ Example: 5 and $\frac{1}{5}$ are reciprocals of one another because $5∙\frac{1}{5}=1$
			* Then we see that $1=\frac{a^{n}}{a^{n}}=a^{n}∙\frac{1}{a^{n}}=a^{n}∙a^{-n}=a^{0}$
				+ Ex.: $1=\frac{5^{n}}{5^{n}}=5^{n}∙\frac{1}{5^{n}}=5^{n}∙5^{-n}=5^{n-n}=5^{0}$
* Scientific Notation:
	+ Most commonly defined as $a x 10^{b}$ where $1\leq a<10$ and $b\in Ζ$ (i.e. b is an integer).
		- Common question: why is $1\leq a<10$?
			* Consider a “number line” of the powers of 10 (and their values):

$$10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}$$

$$\frac{1}{1000} \frac{1}{100} \frac{1}{100} 1 10 100 1000$$

* + - * Note that $1=10^{0}$ is in the “center” of our “number line”. Multiplying a number by 1 does not change the number (i.e. 1 is the multiplicative identity). Therefore, it makes sense for this to be our “starting point” since $a x 10^{1}=a$ and our decimal does not move.
	+ Most common question: How do I know whether to move the decimal left or right?
		- Most common answer: If the exponent is negative, move the decimal to the left. If the exponent is positive, move the decimal to the right.
			* But why?! Consider your rules for exponents. We see in the comment above (under Scientific Notation: Common question) that if the exponent is positive, we are multiplying by a power of 10 (thus, the number should get larger), so we move the decimal to the right. Likewise, if the exponent is negative, we are really dividing by a power of 10 (thus, the number should get smaller), so we move the decimal to the left.
				+ Ex.: $1.7 x 10^{3}=1.7 x 1000=1700$
				+ Ex.: $1.7 x 10^{-3}=1.7 x \frac{1}{1000}=\frac{1.7}{1000}=.0017$
	+ Operations on Scientific Notation:
		- Adding and Subtracting:
			* Most “straight-forward” approach:
				+ Write the numbers in decimal notation (clearly this option is a poor one if you have extremely large or small numbers).
				+ Add or subtract the numbers.
				+ Convert the answer to scientific notation.
			* Slightly more efficient:
				+ If necessary, rewrite one of the numbers (in scientific notation) so that both numbers have the SAME exponent.
				+ Add or subtract the “a” values (i.e. the decimal values); keep the common exponent.

Be sure to line up the decimals.

* + - * + Convert your answer to scientific notation (by moving the decimal and changing the exponent accordingly, if necessary).
		- Multiplying and Dividing
			* Multiply or divide the “a” values (i.e. decimal values).
			* Add (when multiplying) or subtract (when dividing) the exponents.
			* Convert your answer to scientific notation (by moving the decimal and changing the exponent accordingly, if necessary).
* Basic Algebra:
	+ Students will need to know the relationship between density, mass, and volume: $density= \frac{mass}{volume}$. Students will also need to know the relationship between total mass and the mass of one cell of a particular type: (# cells) x (mass of one cell) = total mass contributed by cell type. Students may apply basic algebra to manipulate these relationships to solve for any ONE of these unknowns. Basic algebra dictates that we may solve for one unknown by “undoing” the operations necessary to get an unknown alone. For example, given 3x+2y=6, students may recall solving for y in an algebra class by first subtracting the 3x (to get 2y=6-3x), then dividing by the 2 that is “attached to” y (to get y=6-(3/2)x). We can do the same thing with formulas. For example, to isolate mass, we can “undo” division by multiplying both sides by the volume (to get density x volume = mass). Likewise, we can isolate volume by then dividing by density (to get volume = mass/density).

## Explaining Density with an Elevator

Density can also be explained with a thought experiment. Imagine an elevator with one person on it. The elevator goes up a few floors and picks up two more people. As it rises to the top of the building, more and more people get on the elevator until it is very crowded with fifteen people. The space inside the elevator car, its volume, never gets bigger. As each new person gets on, the total mass in the elevator increases and the space between the people becomes smaller and smaller, like the atoms in a high density object. The density of the elevator becomes greater as it rises and collects people in our thought experiment.