**Exponential Growth Models**

Uninhibited Growth of Cells

A model that gives the number N of cells in a culture after a time t has passed (in the early stages of growth) is N(t) = N0ekt k >0, where N0 is the initial number of cells and k is a positive constant that represents the growth rate. *Sullivan Algebra & Trigonometry*

Example: A colony of bacteria that grows according to the law of uninhibited growth is modeled by the function N(t) = 50e0.045t, where N is measured in grams and t is measured in days. Use the model to answer the following questions. Round to the nearest tenth of a gram.

Complete the table using the function N(t).

|  |  |
| --- | --- |
| Time (in days) | Population (in grams rounded to the nearest tenth) |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

**Part 1:**

1) Determine the initial amount of the bacteria.

2) What is the population after 10 days?

3) According to the model, how long will it take the population to double?

**Part 2:**

A colony of *Staphylococcus* bacteria increases according to the law of uninhibited growth. Use the information in table A (see next page) to answer each question.

1. What is the starting number of cells, N0?

a. What is N(108)?

b. Explain in words the meaning of N(108).

3) If N(t) = 1024, what is t?

Table A

|  |  |  |  |
| --- | --- | --- | --- |
|  | Maximal Growth Rates *Staphylococcus aureus* |  |  |
|  | Example of Binary Fission (doubling time) growth |  |  |
|  | Time (Min) | Viable Cell Count |  |
|  | 0 | 1 |  |
|  | 27 | 2 |  |
|  | 54 | 4 |  |
|  | 81 | 8 |  |
|  | 108 | 16 |  |
|  | 135 | 32 |  |
|  | 162 | 64 |  |
|  | 189 | 128 |  |
|  | 216 | 256 |  |
|  | 243 | 512 |  |
|  | 270 | 1024 |  |
|  | 297 | 2048 |  |
|  | 324 | 4096 |  |
|  | 351 | 8192 |  |
|  | 378 | 16384 |  |
|  | Adapted from M. M. Mason/ Graham-Smith |  |  |

4. Create an exponential function that follows the Law of Uninhibited growth to model the data from the table where N is the number of cells and t is time in minutes. Hint: (You will use your answer from #1, and you will also need to use data from the table to solve for k. Round to 4 places)

N(t) = N0ekt

$$N\left(t\right)=\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

5. Use your model to predict the number of cells at the end of 330 minutes ($N(330)$). Round ***down*** to the nearest whole number.

**Part 3:**

A colony of *Staphylococcus* bacteria increases according to the law of uninhibited growth. If the number of bacteria doubles in 27 minutes, find a function that gives the number of cells in the culture if N is the number of cells and t is time in minutes.

N(t) = N0ekt

If you start with 5 cells, use your function to complete the table for various time periods to estimate the number of viable cells.

|  |  |
| --- | --- |
| Time in minutes | Number of Viable Cells |
| 0 |  |
| 13.5 |  |
| 27 |  |
| 54 |  |
| 81 |  |
| 100 |  |
| 135 |  |
| 162 |  |

In science, the formula, Nn = N02n is used for bacterial growth problems where n represents “number of generations”, N0 represents the initial number of cells, and Nn represents the total number of cells after n generations.

1. Compare and contrast the above science formula to the math function f(x) = abx?

2. Using the information from table B (see page that follows), determine the amount of time that it takes for cells to double. This is referred to in science as the generation time.

3. Complete the “number of generations” column in the table on the next page.

4. a) How long does it take for the cell to complete 5 generations?

 b) How long does it take for the cell to complete 10 generations?

 c) How long does it take for the cell to complete 15 generations?

 d) How long does it take for the cell to complete 30 generations?

5. Create an exponential function to model the relationship between the *Number of Generations* and *Viable Cell Count* using the function f(x) = abx .

6. Graph the number of generations vs the cell count using a linear scale on the graph paper provided. What is your independent variable?

What is the dependent variable?

 Label your axes accordingly.

7. What variable represents the domain of this graph ?

 What variable represents the range of this graph?

8. Discuss possible drawbacks to graphing this data on the coordinate plane using a linear scale.

Table B

|  |  |  |
| --- | --- | --- |
|  | Maximal Growth Rates *Staphylococcus aureus* |  |
|  | Example of Binary Fission (doubling time) growth |  |  |  |
|  |  Time (Min) | Number of generations | Viable Cell Count |  |
|  | 0 |  | 1 |  |
|  | 27 |  | 2 |  |
|  | 54 |  | 4 |  |
|  | 81 |  | 8 |  |
|  | 108 |  | 16 |  |
|  | 135 |  | 32 |  |
|  | 162 |  | 64 |  |
|  | 189 |  | 128 |  |
|  | 216 |  | 256 |  |
|  | 243 |  | 512 |  |
|  | 270 |  | 1024 |  |
|  | 297 |  | 2048 |  |
|  | 324 |  | 4096 |  |
|  | 351 |  | 8192 |  |
|  | 378 |  | 16384 |  |
|  | Adapted from M. M. Mason/ Graham-Smith |  |  |  |

**Part 4:**

Compare and contrast the graph of y = 4x using both a linear(arithmetic) graph and a semi-log graph.

|  |  |
| --- | --- |
| x | f(x)= 4x |
| -2 |  1/16 |
| -1 |  1/4 |
| 0 | 1 |
| 1 | 4 |
| 2 | 16 |
| 3 | 64 |
| 4 | 256 |



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Using the indicated information from table B and your handout of semi-log graph paper, graph the generation time vs the viable cell count and answer the following questions:

1. What information should be displayed on the x-axis and what information would go on the y-axis?

2. What do you notice about the shape of the graph once it is graphed on semi-log paper?

3. Think about decisions scientists often must make concerning data. What do you perceive the benefits to be of graphing on a semi-log scale vs. a linear (arithmetic) scale?

**Part 5:**

1. Compare and contrast the two models that were used to represent the data from tables A and B from previous activities.

Table A: N(t) =N0e026t Table B: f(x) = 2x

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2. Compare the domain and range of the two models:

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