Running the Same Pace During a Marathon

June 30, 2018

Long distance running isn't just about the training. There is a lot of strategy that goes into racing a long distance event. One general strategy most long distance runners follows is attempting to run the same speed for the entire race. In this activity we will be looking at data from long distance running, focusing on the idea of running the same speed for the entire race. Given below is a table of splits for a runner competing in a long distance race.

Distance(km)	Time(mins)
5	21
10	42
15	63
20	84
25	105
30	126
35	147
40	168
Finish(42.25)	177.45

1. What race distance in miles is the above data from?

   Note: 1600 meters = 1 mile

2. Is this a realistic data set? Why or Why not?

Given below is a table of splits for a non-professional runner competing in the Boston Marathon. The splits are found when the runner crosses a mat (located at every 5 kilometers that corresponds with a chip in the bib they are wearing.

Distance(km)	Time(seconds)
5	1251
10	2491
15	3765
20	5057
25	6368
30	7731
35	9161
40	10740
Finish(42.25)	11466

1. Graph a scatter plot of the points in the table where distance is your independent variable and determine what type of relationship is shown(linear, quadratic, exponential, logarithmic)?

2. What type of relationship would approximate this data (linear, quadratic, exponential, logarithmic)?

3. Find the mean $\bar{t}$ (average) of the time data and the mean $\bar{y}$ of the distance data.

4. On your scatter plot on the previous page, draw a line of best fit.
   Remember that the point $(\bar{t}, \bar{y})$ is on the line of best fit.

5. What are the slope and y-intercept of your line of best fit?

6. Write the equation of your line of best fit.

7. What does the slope represent? What would it mean if the slope was higher?

8. Use your calculator to find the line of best fit.

9. Determine the average (minutes per mile) pace of the runner. How does this relate to your slope above?

10. Suppose this marathon runner decided to start running ultra-marathons (races longer than a marathon). What would we expect this runners time to be in a 50 kilometer race assuming they were able to run the same pace.

11. (a) Many runners are interested in their half-marathon time. Using the equation of the line of best fit, determine the half marathon split for the runner (21.13 kilometers).

    (b) Is there a different way you could have determined this?

12. Given below is a table of splits for a professional runner competing in the Boston Marathon. Determine if the professional or non-professional did a better overall job at running the same speed for the entire race.

Distance(km)	Time(seconds)
5	1158
10	2052
15	3250
20	4572
25	5745
30	6961
35	8788
40	10200
Finish(42.25)	13300

Prepared by Cameron Cook, Suzanne Lenhart, and Greg Wiggins for NIMBioS

Running the Same Pace During a Marathon Key

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Uses Algebra concepts for simple models

Long distance running isn't just about the training. There is a lot of strategy that goes into racing a long distance event. One general strategy most long distance runners follows is attempting to run the same speed for the entire race. In this activity we will be looking at data from long distance running, focusing on the idea of running the same speed for the entire race. Given below is a table of splits for a runner competing in a long distance race.

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40	168
Finish(42.25)	177.45

1. What race distance in miles is the above data from?

   Note: 1600 meters = 1 mile

   The marathon-26.2 miles

2. Is this a realistic data set? Why or Why not?

   No, it is unlikely a runner would runner every 5km in exactly 21 minutes.

Given below is a table of splits for a non-professional runner competing in the Boston Marathon. The splits are found when the runner crosses a mat (located at every 5 kilometers that corresponds with a chip in the bib they are wearing.

Distance(km)	Time(seconds)
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1. Graph a scatter plot of the points in the table where distance is your independent variable and determine what type of relationship is shown(linear, quadratic, exponential, logarithmic)? -linear

2. What type of relationship would approximate this data (linear, quadratic, exponential, logarithmic)?

   linear

3. Find the mean $\bar{t}$ (average) of the time data and the mean $\bar{y}$ of the distance data.

   $\bar{y} = 5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 42.25 = 24.69$

   $\bar{t} = 6446.67$

4. On your scatter plot on the previous page, draw a line of best fit.
   Remember that the point $(\bar{t}, \bar{y})$ is on the line of best fit.

5. What are the slope and y-intercept of your line of best fit?

   y-intercept = 0 (distance traveled at 0 secs is 0)

   slope = m = $\frac{24.69-0}{6446.67-0} = .00383$

6. Write the equation of your line of best fit.

   $y = .00383t$

7. What does the slope represent? What would it mean if the slope was higher?

   The slope of this distance vs time graph is the average speed of the runner. We also can say on average this runner travels 0.00383 kilometers every second. If the slope was higher that would mean that the runner was running at a higher speed (faster).

8. Use your calculator to find the line of best fit.

   $y = 0.003654t + 1.13214$

9. Determine the average (minutes per mile) pace of the runner. How does this relate to your slope above?

   We compute this by using our average speed we calculated in the previous and converting our units from Km/sec to mins/mile. After a chance in units, we see that the average pace = 1/slope.

   $\frac{0.00383Km}{1Sec} * \frac{1Mile}{1.6Km} * \frac{60Sec}{1Min} = .1370 Miles/Min$

   Note, we want Min/Mile so we need the inverse of the above: $\frac{1}{.1370 Miles/Min} = 7.30 Minutes/Mile$ (7:18 Minutes per Mile)

10. Suppose this marathon runner decided to start running ultra-marathons (races longer than a marathon). What would we expect this runners time to be in a 50 kilometer race assuming they were able to run the same pace.

$y = 0.003654t + 1.13214$

$50 = .003654t + 1.13214$

$t = 13364$

11. (a) Many runners are interested in their half-marathon time. Using the equation of the line of best fit, determine the half marathon split for the runner (21.13 kilometers).

    $y = 0.003654t + 1.13214$

    $21.13 = 0.003654t + 1.13214$

    $t = 5473$

    (b) Is there a different way you could have determined this?

    1/2 marathon time = total time/2

    $\frac{11466}{2} = 5733$

    Note the difference, as one method used the line of best fit. Discuss with students why the line of best fit method gave a higher value here.

12. Given below is a table of splits for a professional runner competing in the Boston Marathon. Determine if the professional or non-professional did a better overall job at running the same speed for the entire race.

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correlation coefficient: non-professional = .99915 , professional = .97710

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