Math Instructor Teaching Notes

The intent of these activities is to provide supplemental resources that a math instructor could use to support learning in the unit on Exponential and Logarithmic Functions in college algebra. The concept behind the activities is to relate math content with science content. The various activities look at answering questions related to the concept of bacterial growth from various perspectives including numeric, verbal, algebraic, and graphical approaches. The concept of graphing on a semi-log graph is also part of one of the activities. Although graphing on a semi-log graph is generally not part of the college algebra curriculum, it may be used as an extension activity. Each activity in this module has an objective and instructors may use activities independent of each other or a combination of activities.

As a result of this project, we became aware of several concepts that need to be addressed to help students make connections between math concepts learned in a math classroom and how math concepts may be used in a science classroom. Through discussion between math/science faculty, it was noted that although the concepts of graphing exponential functions and logarithms are in the curriculum for college algebra, the skills needed in a microbiology course are very different from how the content is presented in college algebra. It was also noted that some of the formulas and terminology related to exponential growth models looked different in each discipline. We felt that it is important to discuss these differences and to provide activities that could be used by both biology and math instructors to help students see connections between what is taught in math and how it is used in science. It is suggested that math and science faculty have regular discussions throughout the semester regarding how math concepts are used in science and taught in math.

**Example of Summary of Differences**

|  |  |
| --- | --- |
| Math Context | Science Context |
| Students routinely graph on the coordinate plane using all 4 quadrants | Students graph data involving the first quadrant |
| The scale on both axes is linear and data points come from a given data set or an equation | The scale on either axes may vary. Students collect data or are given data values |
| Graphs generally do not have a chart title or axis labels other than x and y | Graphs need to have a title and specific labels on both axes to quantify a variable |
| Students are shown basic graphs of functions on a graph where both axes are linear | Semi-log graphs are needed to clearly display data points for exponential growth problems which makes the graph appear to be the model of a linear function if you don’t pay attention to the fact that the y-axis is on a log scale |
| Students use a function formula to graph a few data points using a table of values. Data points may contain positive and negative values for x  F(x) = 4x | Problems are connected to the real world and therefore data values are usually confined to quadrant 1 (usually contain only positive values for x) |
| Students solve problems involving logarithms  Log 100 = X | Student use the concept of a log to graph on a logarithmic scale |
| Students reference basic graphs of various functions and learn to associate a math function with the picture of a graph when the x- and y-axis are both linear | Using a semi-log graph to display data from an exponential function makes it easier to see data points that are growing rapidly but a math student may assume the data is linear based on the graph. Be sure to address this misconception |

The following pages contain copies of the student worksheets with instructor notes added in red. Student handouts are located in the resource files along with copies of graph paper and tables used for these lessons.

Exponential Growth Models

Objective 1: Use a function that models uninhibited growth to answer questions about bacterial growth

Uninhibited Growth of Cells

A model that gives the number N of cells in a culture after a time t has passed (in the early stages of growth) is N(t) = N0ekt k >0, where N0 is the initial number of cells and k is a positive constant that represents the growth rate. *Sullivan Algebra & Trigonometry*

Example: A colony of bacteria that grows according to the law of uninhibited growth is modeled by the function N(t) = 50e0.045t, where N is measured in grams and t is measured in days. Use the model to answer the following questions. Round to the nearest tenth of a gram.

Complete the table using the function N(t).

|  |  |
| --- | --- |
| Time (in days) | Population (in grams rounded to the nearest tenth) |
| 0 | 50 |
| 1 | 52.3 |
| 2 | 54.7 |
| 3 | 57.2 |
| 4 | 59.9 |
| 5 | 62.6 |

1) Determine the initial amount of the bacteria.

Algebraic Solution: N(0) = 50e0.045(0) = 50 grams----Notice the N0 in the formula represents the initial amount

Tabular Interpretation: 0 days represents the initial amount of the bacteria in grams

2) What is the population after 10 days? N(10) = 50e0.045\*10 = 78.4 g

3) According to the model, how long will it take the population to double? NOTE: If you start with 50 grams then you will have 100 grams when the bacteria have doubled. This means that you know N(t) and you need to solve for t. You are NOT finding N(100).

100 = 50e0.045t -------Divide both sides by 50

2= e0.045t -------- Take ln of both sides and use log property to rewrite

ln 2= 0.045t ln e----- ln e = 1 so divide both sides by .045 to get t

ln(2)/0.045 is approximately 15.4 days

Objective 2: Given a table of data answer questions about populations that obey the law of uninhibited growth and create a mathematical model for the data. (In this objective a student uses concrete data from a table to answer questions and create the model. Instead of the cells being measured in grams, the data in this table represents the viable cell count. The data can be changed for other bacterial growth examples)

A colony of *Staphylococcus* bacteria increases according to the law of uninhibited growth. Use the information in table A to answer each question.

1. What is the starting number of cells N0? 1 viable cell

2) What is N(108)? 16

Explain in words the meaning of N(108). At 108 minutes the viable cell count is 16

3) If N(t) = 1024 what is t? 270 minutes

Create an exponential function that follows the Law of Uninhibited growth to model the data from the table where N is the number of cells and t is time in minutes. Hint: You will need to solve for k.

N(t) = N0ekt

2 = 1 ek(27) Solve for k.

ln 2 = lne27k Take the ln of both sides

ln 2 = 27k lne Use a log property to rewrite and divide by 27 to solve for k….remember that ln e = 1

ln2/27 = k round to 3 decimal places k≈ .026

Model: N(t) =N0e026t

Use your model to predict the number of cells at the end of 330 minutes. Round ***down*** to the nearest whole number.

N(330) = 1e.026(330) ≈ 5324 cells

Table A

|  |  |  |  |
| --- | --- | --- | --- |
|  | Maximal Growth Rates *Staphylococcus aureus* | |  |
|  | Example of exponential growth (doubling time) |  |  |
|  | Time (Min) | Viable Cell Count |  |
|  | 0 | 1 |  |
|  | 27 | 2 |  |
|  | 54 | 4 |  |
|  | 81 | 8 |  |
|  | 108 | 16 |  |
|  | 135 | 32 |  |
|  | 162 | 64 |  |
|  | 189 | 128 |  |
|  | 216 | 256 |  |
|  | 243 | 512 |  |
|  | 270 | 1024 |  |
|  | 297 | 2048 |  |
|  | 324 | 4096 |  |
|  | 351 | 8192 |  |
|  | 378 | 16384 |  |
|  | Adapted from M. M. Mason/ Graham-Smith |  |  |

Objective 3: Find the equation of a population that obeys the law of uninhibited growth using a verbal description. (This is the same concept as the previous objective, but instead of starting with a table of data the information is presented using a verbal description. The students will then use the model created to answer questions related to viable cell count when the initial cell count is not 1)

A colony of *Staphylococcus* bacteria increases according to the law of uninhibited growth. If the number of bacteria doubles in 27 minutes, find a function that gives the number of cells in the culture if N is the number of cells and t is time in minutes.

N(t) = N0ekt

Regardless of the starting amount of the bacteria if N0 is the initial amount when it doubles you will have 2N0 and we know that it takes 27 minutes to double so t = 27. We can use this information to solve for k.

2N0 = N0e27k divide both sides by N0

2 = e27k take ln of both sides and use log property

ln(2) = 27k(lne) remember lne = 1 so divide by 27 to get k

ln(2)/27 = k leave exact

The function would be N(t) = N0e(ln(2)/27)t where N(t) is the number of cells after t minutes.

If you start with 5 cells, use your function to complete the table for various time periods to estimate the number of viable cells.

|  |  |
| --- | --- |
| Time in minutes | Number of Viable Cells |
| 0 | 5 |
| 13.5 | 7.5 |
| 27 | 10 |
| 54 | 20 |
| 81 | 40 |
| 100 | 65 |
| 135 | 160 |
| 162 | 320 |

The time in minutes and the starting number of cells may be changed once the model is created. Data generated from the model may also be used to generate discussion questions related to data in the table and what is happening with the cells at different time periods. For example, why did the cell count not double at 13.5 minutes? For what time intervals do we see the data double and how does that relate to the numbers used for time? Answers will vary but since the doubling time is 27 minutes at 13.5 minutes we would expect the number of cells to increase by only .5 since 13.5 represents half of the doubling time. Whereas there is a difference of 27 minutes between 0 and 27, 27 min and 54, 54 and 81 min as well as 135 minutes and 162 minutes so we saw the number of viable cells double between those time increments.

Objective: Relating math skills to science. Graphing exponential functions.

In science, the formula, Nn = N02n is used to model bacterial growth where n represents “number of generations”, N0 represents the initial number of cells, and Nn represents the total number of cells after n generations. (Emphasize the difference between N0, Nn, and lowercase n)

1. Compare and contrast the above science formula to the math function f(x) = abx? Answers will vary. Both formulas model exponential growth. Different variables are used in the equation used in science. The b value in science is 2 since we are dealing with the concept of binary fission which involves doubling whereas in math examples b may take on different values depending on the data. Science uses subscripts in the variables represented by the same letter that indicate different values. The math formula uses function notation. There is a difference between an uppercase variable and a lowercase value.

2. Using the information from table B, determine the amount of time that it takes for cells to double. This is referred to in science as the generation time. 27 minutes

3. Complete the “number of generations” column in the table.

4. a) How long does it take for the cell to complete 5 generations? 135 minutes (27)(5)

b) How long does it take for the cell to complete 10 generations? 270 minutes (27)(10)

c) How long does it take for the cell to complete 15 generations? 405 minutes (27)(15)

d) How long does it take for the cell to complete 30 generations? 810 minutes (27)(30)

Note that the information for 4a and 4b can be found by looking at the table. 4c can be found by using the pattern to find the next row of the table, but if students can relate the relationship between (generation time)(number of generations) it is much easier to use the model rather than completing the remainder of the table to get to 30 generations.

5. Create an exponential function to model the relationship between the *Number of Generations* and *Viable Cell Count* using the function f(x) = abx .

Using the function f(x) abx, a represents the initial cell count, b= common ratio represented by viable cell count for consecutive generations, and x represents the number of generations. Model f(x) = 2x

6. Graph the number of generations vs the cell count using a linear scale on the graph paper provided. What is your independent variable? Number of generations

What is the dependent variable? Viable cell count

Label your axes accordingly.

7. What variable represents the domain of this graph? Number of generations

What variable represents the range of this graph? Viable cell count

8. Discuss possible drawbacks to graphing this data on the coordinate plane using a linear scale. Answers will vary. The viable cell count grew more quickly than the number of generations so it was difficult to graph the y values on the graph.

Table B

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Maximal Growth Rates *Staphylococcus aureus* | | |  |
|  | Example of exponential growth (doubling time) |  |  |  |
|  | Time (Min) | Number of generations | Viable Cell Count |  |
|  | 0 | 0 | 1 |  |
|  | 27 | 1 | 2 |  |
|  | 54 | 2 | 4 |  |
|  | 81 | 3 | 8 |  |
|  | 108 | 4 | 16 |  |
|  | 135 | 5 | 32 |  |
|  | 162 | 6 | 64 |  |
|  | 189 | 7 | 128 |  |
|  | 216 | 8 | 256 |  |
|  | 243 | 9 | 512 |  |
|  | 270 | 10 | 1024 |  |
|  | 297 | 11 | 2048 |  |
|  | 324 | 12 | 4096 |  |
|  | 351 | 13 | 8192 |  |
|  | 378 | 14 | 16384 |  |
|  | Adapted from M. M. Mason/ Graham-Smith |  |  |  |

Objective: Graphing exponential functions using a semi-log graph

Instructors may want to use these graphs to introduce the concept of a semi-log graph. A semi-log graph is a graph that has one axis that uses a linear scale and one axis that uses a log scale. Depending on student knowledge a discussion of the semi-log graph and how to graph on the semi-log scale may be needed. The instructor may want to use one of the YouTube videos either as a pre-assignment before this activity or as an introduction in class to graphing on a semi-log graph.

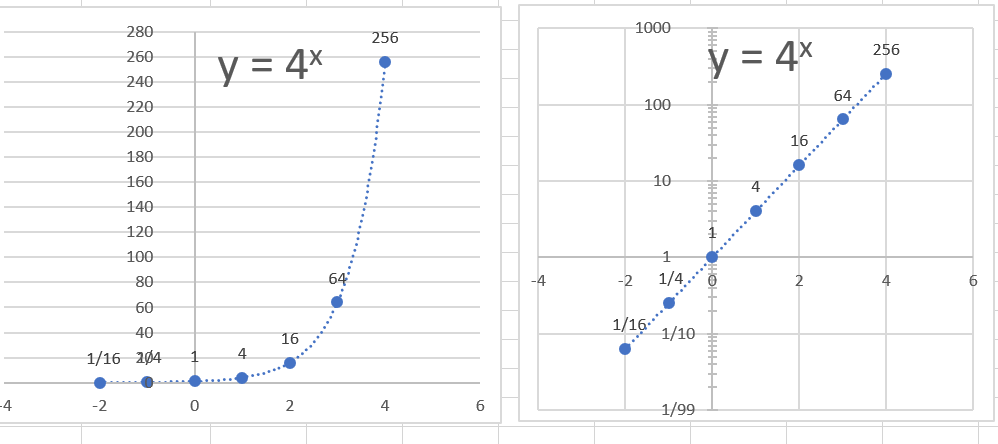
<https://www.youtube.com/watch?v=FNAnPvncwWY&t=4s>

<https://www.youtube.com/watch?v=sBhEi4L91Sg&t=30s>

<https://www.youtube.com/watch?v=eJF9hiv3c-A&t=605s>

Compare and contrast the graph of y = 4x using both a linear(arithmetic) scale and a semi-log scale.

|  |  |
| --- | --- |
| x | f(x)= 4x |
| -2 | 1/16 |
| -1 | 1/4 |
| 0 | 1 |
| 1 | 4 |
| 2 | 16 |
| 3 | 64 |
| 4 | 256 |



Using the indicated information from table B and your handout of semi-log graph paper, graph the generation time vs the viable cell count and answer the following questions:

1. What information should be displayed on the x-axis and what information would go on the y-axis? The number of generations would go on the x-axis and would use a linear scale and the number of viable cells would go on the y-axis and use the log scale.

2. What do you notice about the shape of the graph once it is graphed on semi-log paper? The graph with the semi-log scale makes the graph look linear.

3. Think about decisions scientists often must make concerning data. What do you perceive the benefits to be of graphing on a semi-log scale vs. a linear (arithmetic) scale? Answers will vary. When the data values are increasing exponentially the semi-log scale makes it easier to read data points on the graph.

Extension questions if activities related to the Law of Inhibited Growth model and the exponential model were completed.

1. Compare and contrast the two models that were used to represent the data from tables A and B from previous activities.

Table A: N(t) =N0e026t Table B: f(x) = 2x

2. Compare the domain and range of the two models:

In the first model, t represents time and N(t) represents the number of cells.

In the second model, x represents the number of generations and f(x) represents the number of cells.