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STUDENT VERSION
SUBLIMATION OF CARBON DIOXIDE

Brian Winkel, Director SIMIODE
Chardon OH 44024 USA
BrianWinkel@simiode.org

Abstract: We offer data on the sublimation of dry ice (carbon dioxide) collected in a classroom setting so that students can model the rate of change in the mass of a small solid carbon dioxide block with a differential equation model, solve the differential equation, estimate the parameters in the model using the data sets, and validate the model.

SCENARIO DESCRIPTION

Matter exists in several states: plasma, gas, liquid, and solid. When a solid becomes liquid the object melts. When a liquid becomes a gas the object vaporizes. When a solid becomes gas directly the object *sublimates*. A readily available example of sublimation occurs when the solid, dry ice (solid carbon dioxide - CO_2), becomes gaseous.

Now if we had a solid block of dry ice we would notice it "disappear" as it sublimates into its gaseous state. Indeed, the mass of the block of dry ice would decrease. A reasonable question to ask is, "At what rate does the mass decrease as the dry ice sublimates?" A related question is, "What does the rate of change of the mass depend upon?"

Some years ago in a course we were teaching we too wondered whether we could model the sublimation of a piece of carbon dioxide. At the time (over 30 years ago) dry ice was used to pack ice cream shipments. One day, because we had received some things in a shipment containing dry ice, we thought of collecting data on the sublimation of dry ice. While we had colleagues who were true engineers and scientists on our teaching team, we decided, as a mathematician who was pretty much an inept lab fellow, to go this one alone.

We set up a modest device consisting of a tray, in which we placed our dry ice on a scale which could render mass in grams to the thousandth of a gram. We zeroed out the scale (i.e. we noted the mass of the tray and then noted the mass with the initial dry ice amount on it so that we could tell from total mass data exactly what our dry ice mass was as time progressed) and began to collect

data. We should say we really lived dangerously, for we did this all for the first time in class with two students in the first row recording the mass every 30 sec. We have since lost that data, but there was a very unexpected thing that happened. The mass of the dry ice, rather we should say the mass of the zeroed dry ice and tray, went up(!) in the first few minutes. We continued to collect data on that run and we all went home to ponder what was going on. I had the advantage of holding the equipment and some more dry ice. After class I tried the experiment again and found the problem. The dry ice was so cold, below sublimation temperature of dry ice, -78.5^o C, that it was causing water to condense on the tray, and upon inspection, I could see the ice film on the tray. That was why the mass went up on this configuration for a while.

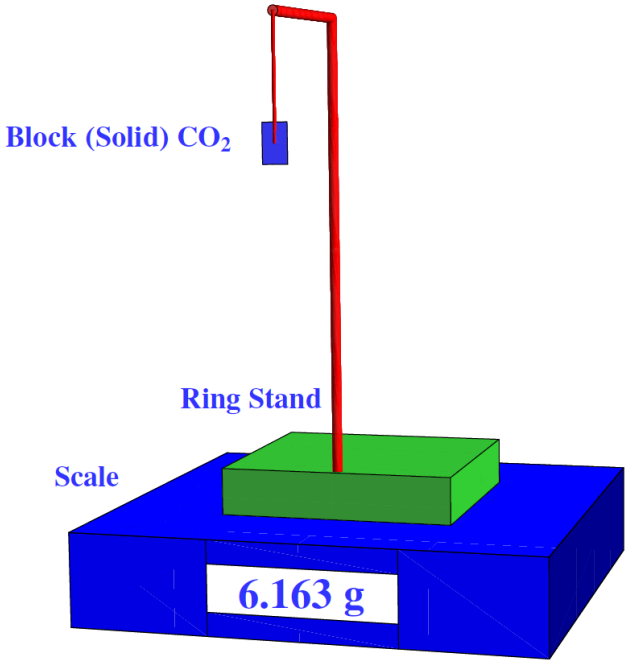

Figure 1. Sketch of apparatus for data collection consisting of a scale on which is mounted a ring stand to hold the solid block of CO_2 out from the base. This configuration will not allow condensed moisture realized by the falling cold sublimated CO_2 to collect on any surface involved in the scale apparatus.

Most of the students were able to explain exactly what we saw, only without seeing it - they were sharpies! We all realized that we needed to somehow do the experiment by, perhaps, suspending the dry ice so that when the sublimated (but still very cold) CO_2, which was heavier than air, fell it did not fall on any part of our scale device. The next day the teacher arrived with a proper set of equipment (see Figure 1) and we began to collect data again.

We collected the data in a room of temperature 19.3^o C using a small box shape piece of dry ice we had carved (wearing protective gloves) to the size of two sugar cubes, 1 cm $\times$ 1 cm $\times$ 2 cm.

Time (s)	Mass (g)		Time (sec)	Mass (g)
0	25.525		1800	13.553
120	24.512		1930	12.910
240	23.524		2040	12.331
360	22.639		2170	11.689
480	21.765		2280	11.188
600	20.890		2410	10.566
720	20.043		2530	10.043
840	19.221		2690	9.377
960	18.431		2780	9.011
1080	17.677		2880	8.616
1200	16.936		3060	7.945
1320	16.220		3220	7.404
1440	15.548		3380	6.877
1570	14.828		3480	6.593
1680	14.213		3600	6.244

Table 1. Data collected by students Masood Makkar and Paul Werner (3 December 1992) on successful run for mass of dry ice (g) as a function of time (s).

This time we did not experience any unusual phenomena. We present the collected data in Table 1.

Time (s)	Mass (g)		Time (sec)	Mass (g)
0	7.570		480	5.871
30	7.457		510	5.776
60	7.338		540	5.682
90	7.220		570	5.589
120	7.110		600	5.497
150	6.995		630	5.405
180	6.885		660	5.313
210	6.778		690	5.224
240	6.673		720	5.136
270	6.571		750	5.048
300	6.464		780	4.960
330	6.363		810	4.878
360	6.265		840	4.790
390	6.163		870	4.707
420	6.067		900	4.625
450	5.969			

Table 2. Data collected by Dr. Ed Mottel on successful run for mass of dry ice (g) as a function of time (s).

Incidentally, a few days later Dr. Ed Mottel, a chemist in our teaching group, collected additional

data in his lab using the same set up. We show this in Table 2. We do not have the room temperature data for this second collection, though.

So where are we? Well, we are going to ask you to model the amount of mass of dry ice as a function of time by producing a differential equation which models the *rate of change in mass*. That is, if $m(t)$ is the mass in grams (g) at time t in seconds (s) we seek to find, solve, and use the differential equation:

$$m'(t) = _____ \quad \text{where} \quad m(0) = m_0 \quad \text{is the initial condition,} \tag{1}$$

to determine $m(t)$ and then use one of the two data sets (Table 1 or Table 2) to estimate any parameters introduced into your model.

You should be asking yourself what the rate of sublimation depends upon.

Produce a model differential equation with initial condition using reasonable and defended assumptions; solve it; estimate the parameters in your model for one of the two data sets (Table 1 or Table 2); and validate that your model is reasonable. Offer up a discussion on your modeling process. Try your approach on the second data set to confirm your modeling skills.