1-031-T-Mma-CoolIt-TeacherVerstion
Modeling Cooling Water in Environment

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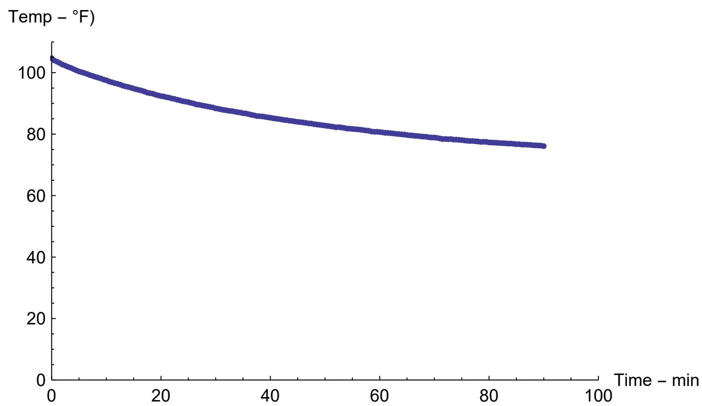
Collection of data with constant environmental temperature.

We collected this data with known and constant environmental temperature TE = 72 degrees Fahrenheit. The first coordinate is time in minutes and the second coordinate is the temperature of water in a 200 mL beaker.

```
data = {{0, 104.6926466}, {0.5, 103.9627984}, {1, 103.5536237},
   {1.5, 103.1002852}, {2, 102.6031473}, {2.5, 102.2874528}, {3, 101.8825621},
   {3.5, 101.568251}, {4, 101.165107}, {4.5, 100.762918}, {5, 100.4065269},
   {5.5, 100.1393454}, {6, 99.87256667}, {6.5, 99.56225007}, {7, 99.20788902},
   {7.5, 98.94247535}, {8, 98.63372458}, {8.5, 98.3691219}, {9, 98.10488516},
   {9.5, 97.75360889}, {10, 97.53402101}, {10.5, 97.13965855}, {11, 96.87741591},
   {11.5, 96.61515056}, {12, 96.39714649}, {12.5, 96.1354828}, {13, 95.78721512},
   {13.5, 95.52664815}, {14, 95.35275293}, {14.5, 95.09269732},
   {15, 94.83258687}, {15.5, 94.57277202}, {16, 94.35677678},
   {16.5, 94.14062631}, {17, 93.83883895}, {17.5, 93.49404562}, {18, 93.32165333},
   {18.5, 93.14973192}, {19, 92.84879779}, {19.5, 92.59143492}, {20, 92.37680763},
   {20.5, 92.24832504}, {21, 91.99119301}, {21.5, 91.86289318}, {22, 91.60612181},
   {22.5, 91.43530479}, {23, 91.17892155}, {23.5, 90.96573334}, {24, 90.70976182},
   {24.5, 90.58203259}, {25, 90.4118105}, {25.5, 90.15630517}, {26, 89.94383327},
   {26.5, 89.64626153}, {27, 89.56139036}, {27.5, 89.34896005}, {28, 89.17939052},
   {28.5, 89.00955806}, {29, 88.84015593}, {29.5, 88.67083537}, {30, 88.37437},
   {30.5, 88.2475355}, {31, 88.03589414}, {31.5, 87.86694293}, {32, 87.73993202},
   {32.5, 87.57110927}, {33, 87.48672478}, {33.5, 87.31766308}, {34, 87.14901666},
   {34.5, 87.02257663}, {35, 86.81158164}, {35.5, 86.7273506}, {36, 86.51649852},
   {36.5, 86.30609028}, {37, 86.13783299}, {37.5, 85.88521491}, {38, 85.8431822},
   {38.5, 85.75912761}, {39, 85.6327281}, {39.5, 85.4647038}, {40, 85.33872165},
   {40.5, 85.17044847}, {41, 85.04453629}, {41.5, 84.91865313}, {42, 84.79245461},
   {42.5, 84.62469179}, {43, 84.54082857}, {43.5, 84.37279415}, {44, 84.28896623},
   {44.5, 84.1213445}, {45, 83.99531403}, {45.5, 83.91153627}, {46, 83.78588962},
   {46.5, 83.66026646}, {47, 83.49246138}, {47.5, 83.45060213}, {48, 83.24134217},
   {48.5, 83.11547154}, {49, 83.03179791}, {49.5, 82.90630424}, {50, 82.78083019},
   {50.5, 82.65503248}, {51, 82.5295961}, {51.5, 82.4041777}, {52, 82.2366376},
   {52.5, 82.27843413}, {53, 82.15305009}, {53.5, 82.02768238}, {54, 81.81842854},
   {54.5, 81.77665131}, {55, 81.69310172}, {55.5, 81.60955849}, {56, 81.52602146},
   {56.5, 81.44214814}, {57, 81.31686264}, {57.5, 81.1915898}, {58, 81.10808134},
   {58.5, 80.85724443}, {59, 80.77375588}, {59.5, 80.77375588}, {60, 80.69027189},
   {60.5, 80.56471201}, {61, 80.43950362}, {61.5, 80.39776948}, {62, 80.31430403},
   {62.5, 80.23050021}, {63, 80.10531409}, {63.5, 80.02186079}, {64, 79.93841058},
   {64.5, 79.8549633}, {65, 79.72945554}, {65.5, 79.60429545}, {66, 79.56257659},
   {66.5, 79.4374232}, {67, 79.39536447}, {67.5, 79.27021682}, {68, 79.18678711},
   {68.5, 79.14507281}, {69, 78.97787695}, {69.5, 78.89445162}, {70, 78.89445162},
   {70.5, 78.76931524}, {71, 78.60212695}, {71.5, 78.39357054}, {72, 78.43528179},
   {72.5, 78.35185926}, {73, 78.39357054}, {73.5, 78.22672505}, {74, 78.22672505},
   {74.5, 78.14295983}, {75, 78.05953591}, {75.5, 77.93439854}, {76, 77.8509724},
   {76.5, 77.76720315}, {77, 77.76720315}, {77.5, 77.68377448}, {78, 77.60034429},
   {78.5, 77.47519581}, {79, 77.51691242}, {79.5, 77.47519581}, {80, 77.30798238},
   {80.5, 77.26626315}, {81, 77.22454333}, {81.5, 77.18282291}, {82, 77.09938017},
   {82.5, 77.0576578}, {83, 77.01593475}, {83.5, 76.93214449}, {84, 76.93214449},
   {84.5, 76.84869321}, {85, 76.7235103}, {85.5, 76.7235103}, {86, 76.59831968},
   {86.5, 76.59831968}, {87, 76.55658767}, {87.5, 76.51485471}, {88, 76.38930791},
   {88.5, 76.34757098}, {89, 76.30583301}, {89.5, 76.26409398}, {90, 76.0967846}};
```

```
dataPlot =
 ListPlot[data, PlotRange → {{0, 100}, {0, 110}}, AxesLabel → {"Time - min", "Temp - °F)"}]
```



We presume nothing here, unknown environmental temperature, unknown initial temperature of the water, and unknown decay constant. Indeed, we just presume a shape based somewhat on our view of decay, but not to zero, just to some equilibrium value.

```
sol = FindFit[data, a Exp[- b * t] + c, {a, b, c}, t]
{a → 32.0742, b → 0.0215747, c → 71.7388}
```

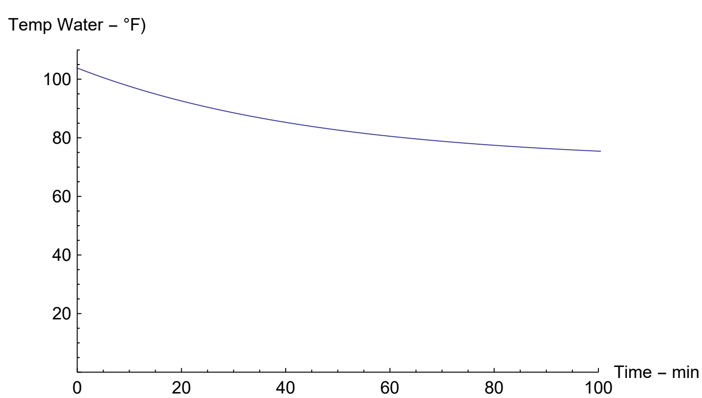
```
model[t_] = a Exp[- b * t] + c /. sol
```

$71.7388 + 32.0742\, e^{-0.0215747\, t}$

What is long term temperature value?

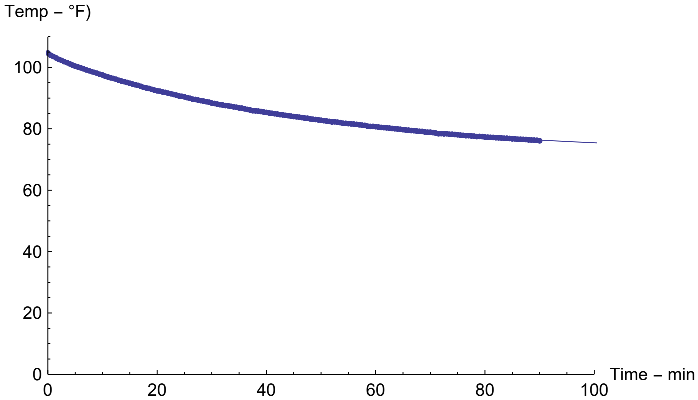
```
Limit[model[t], t → Infinity]
71.7388
```

```
modelPlot = Plot[model[t], {t, 0, 110},
  PlotRange → {{0, 100}, {0, 110}}, AxesLabel → {"Time - min", "Temp Water - °F)"}]
```



This is a reasonable model with reasonable values of TE (long term value of temperature of water) and T0 which are close to actual observed values of TE = 72.0 and T0 = 104.7.

```
Show[dataPlot, modelPlot]
```



We presume nothing here, i.e. we have unknown environmental temperature, unknown initial temperature of the water, and unknown decay constant. However, we now go with a differential equation with these unknown values: k - the rate constant, TE - the environmental temperature, presumed constant; and T0 - the initial temperature of the water in the beaker. We do this just to see if we can get these even without observations,e.g., one would assume TE and T0 cold be observed.

```
eqNC = T'[t] == -k (T[t] - TE)
```

$T'[t] == -k(-TE + T[t])$

```
Tsol[t_] = T[t] /. DSolve[{eqNC, T[0] == T0}, T[t], t][[1]]
```

$e^{-k t}\left(T0 - TE + e^{k t}\, TE\right)$

```
SSE[k_, TE_, T0_] = Sum[(Tsol[data[[i, 1]]] - data[[i, 2]])^2, {i, 1, Length[data]}];
```

```
solkTE = FindMinimum[SSE[k, TE, T0], {k, .1}, {TE, 70}, {T0, 100}]
```

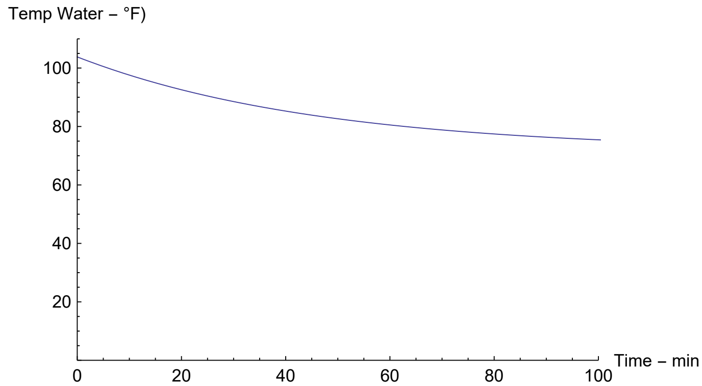
{3.68354, {k → 0.0215747, TE → 71.7388, T0 → 103.813}}

We see we obtain values for k, TE, and T0. These are quite reasonable and the TE and T0 are reasonably close to actual values of TE = 72 and T0 = 104.7.

```
TSSE[t_] = Tsol[t] /. solkTE[[2]] // Expand
```

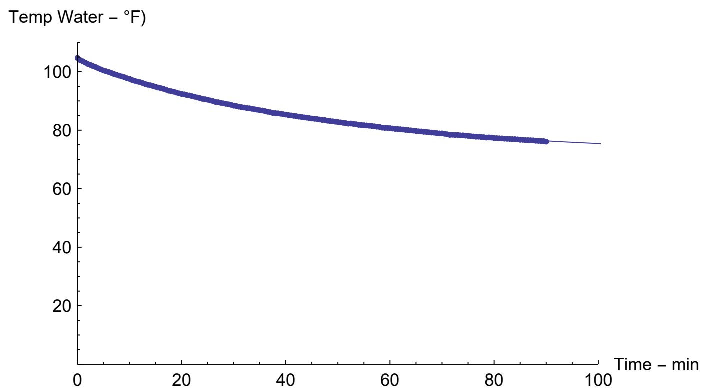
$71.7388 + 32.0742\, e^{-0.0215747\, t}$

```
SSEModelPlot = Plot[TSSE[t], {t, 0, 110},
  PlotRange → {{0, 100}, {0, 110}}, AxesLabel → {"Time - min", "Temp Water - °F)"}]
```



This model looks quite good.

```
Show[dataPlot, SSEModelPlot, AxesLabel → {"Time - min", "Temp Water - °F)"}]
```



We now use our observed values of TE = 72.0 and T0 = 104.7 and go with a differential equation with only one unknown, k - the rate constant.

```
T0 = 104.7; TE = 72.0;

eqNC = T'[t] == -k (T[t] - TE)
```

T′[t] == −k (−72. + T[t])

```
Tsol[t_] = T[t] /. DSolve[{eqNC, T[0] == T0}, T[t], t][[1]]
```

$72.\ e^{-1.\ k\ t} \left(0.454167 + 1.\ e^{1.\ k\ t}\right)$

Here we form the sum of square errors between our temperature at the time of observation as predicted by the model, Tsol[data[[i,1]]], and the observed temperature at that same time, data[[i,2]].

```
SSE[k_] = Sum[(Tsol[data[[i, 1]]] - data[[i, 2]])^2, {i, 1, Length[data]}];
```

We determine the k value which minimizes this the sum of square errors.

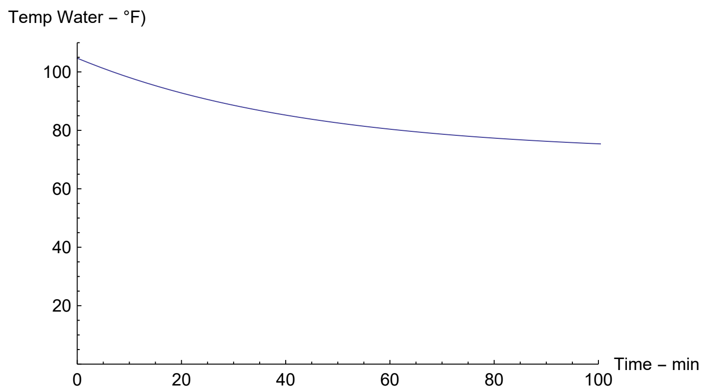
```
solk = FindMinimum[SSE[k], {k, .1}]
```

{17.3788, {k → 0.0226449}}

```
TSSEk[t_] = Tsol[t] /. solk[[2]] // Expand
```

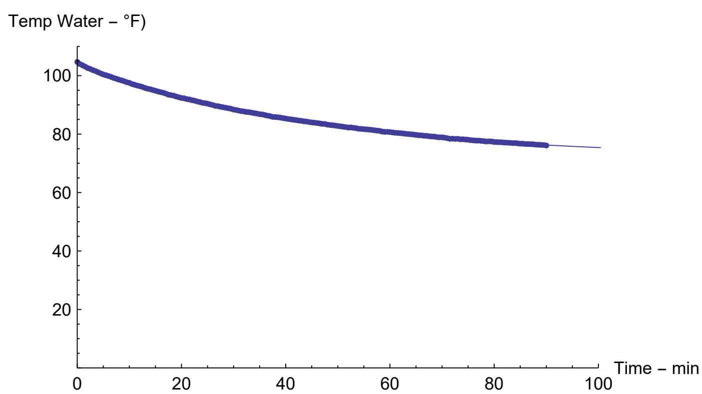
$72. + 32.7\, e^{-0.0226449\, t}$

```
SSEkModelPlot = Plot[TSSEk[t], {t, 0, 110},
  PlotRange → {{0, 100}, {0, 110}}, AxesLabel → {"Time - min", "Temp Water - °F)"}]
```



Again, this model is quite good as well.

```
Show[dataPlot, SSEkModelPlot, AxesLabel → {"Time - min", "Temp Water - °F)"}]
```



Collection of data with changing environmental temperature.

Here we collected data in our home office and we note that the environment' s temperature DOES NOT stay constant. Indeed, the office is warming up, no doubt due to the author' s body mass, the computer generated heat, and the warming outside the office due to rising sun.

So now we will model first a changing (increasing) environmental temperature TE(t) as a function of time and incorporating this into our cooling model.

data contains sets of observations first coordinate is time in minutes, second coordinate is temperature of environment in degrees Fahrenheit, and third coordinate is temperature of water in beaker in degrees Fahrenheit. NB: We did not record the temperature of the environment at time 0 and add in that data point later.

```
data = {{3.0, 56.7, 100.6}, {5.0, 58.3, 99.0}, {8.0, 59.0, 97.0}, {9.5, 59.7, 96.0},
   {12.0, 60.4, 94.5}, {15.0, 61.0, 92.8}, {17.5, 61.5, 91.3}, {20.5, 62.1, 89.9},
   {22.0, 62.2, 89.1}, {27.0, 62.8, 86.9}, {29.0, 63.0, 86.2}, {35.0, 63.7, 83.9},
   {38.5, 63.9, 82.7}, {44.0, 64.2, 81.0}, {47.5, 64.4, 80.1}, {54.0, 64.8, 78.4},
   {55.0, 64.9, 78.1}, {58.5, 64.9, 77.5}, {65.5, 65.3, 76.1}, {70.0, 65.5, 75.3},
   {73.5, 65.5, 74.7}, {75.0, 65.7, 74.6}, {78.5, 65.7, 74.0}, {80.0, 65.8, 73.8},
   {86.5, 66.0, 72.9}, {88.5, 66.0, 72.7}, {93.5, 66.2, 72.2}, {102.0, 66.2, 71.4},
   {102.5, 66.2, 71.3}, {106.5, 66.2, 70.9}, {109.0, 66.4, 70.8}, {112.5, 66.4, 70.5},
   {114.5, 66.4, 70.4}, {117.0, 66.4, 70.2}, {123.5, 66.6, 69.8}, {126.0, 66.6, 69.7},
   {127.0, 66.6, 69.4}, {130.5, 66.7, 69.4}, {132.0, 66.7, 69.3}, {139.5, 66.7, 69.0}};
```

Here is the set of environment temperatures with time of observations.

```
EnvTemp = Table[{data[[i, 1]], data[[i, 2]]}, {i, 1, Length[data]}]
```

```
{{3., 56.7}, {5., 58.3}, {8., 59.}, {9.5, 59.7}, {12., 60.4}, {15., 61.},
 {17.5, 61.5}, {20.5, 62.1}, {22., 62.2}, {27., 62.8}, {29., 63.}, {35., 63.7},
 {38.5, 63.9}, {44., 64.2}, {47.5, 64.4}, {54., 64.8}, {55., 64.9}, {58.5, 64.9},
 {65.5, 65.3}, {70., 65.5}, {73.5, 65.5}, {75., 65.7}, {78.5, 65.7}, {80., 65.8},
 {86.5, 66.}, {88.5, 66.}, {93.5, 66.2}, {102., 66.2}, {102.5, 66.2}, {106.5, 66.2},
 {109., 66.4}, {112.5, 66.4}, {114.5, 66.4}, {117., 66.4}, {123.5, 66.6},
 {126., 66.6}, {127., 66.6}, {130.5, 66.7}, {132., 66.7}, {139.5, 66.7}}
```

Here is the set of water temperatures with time of observations.

```
WaterTemp = Table[{data[[i, 1]], data[[i, 3]]}, {i, 1, Length[data]}]
```

```
{{3., 100.6}, {5., 99.}, {8., 97.}, {9.5, 96.}, {12., 94.5}, {15., 92.8},
 {17.5, 91.3}, {20.5, 89.9}, {22., 89.1}, {27., 86.9}, {29., 86.2}, {35., 83.9},
 {38.5, 82.7}, {44., 81.}, {47.5, 80.1}, {54., 78.4}, {55., 78.1}, {58.5, 77.5},
 {65.5, 76.1}, {70., 75.3}, {73.5, 74.7}, {75., 74.6}, {78.5, 74.}, {80., 73.8},
 {86.5, 72.9}, {88.5, 72.7}, {93.5, 72.2}, {102., 71.4}, {102.5, 71.3}, {106.5, 70.9},
 {109., 70.8}, {112.5, 70.5}, {114.5, 70.4}, {117., 70.2}, {123.5, 69.8},
 {126., 69.7}, {127., 69.4}, {130.5, 69.4}, {132., 69.3}, {139.5, 69.}}
```

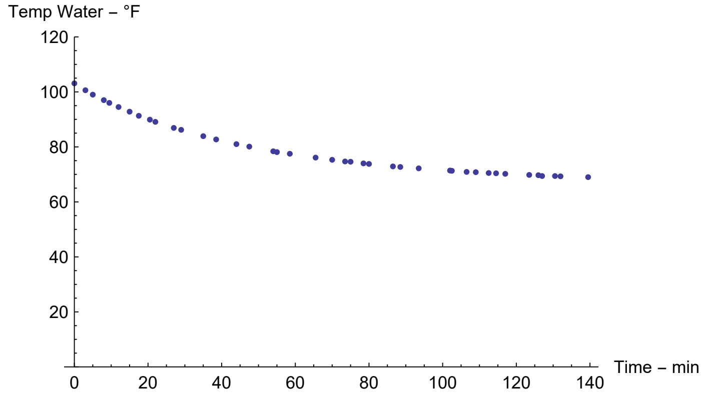
We add in one data point on the water temperature at time t = 0. We failed to capture the environmental temperature at time t = 0.

```
WaterTemp = AppendTo[WaterTemp, {0, 103.1}]
```

```
{{3., 100.6}, {5., 99.}, {8., 97.}, {9.5, 96.}, {12., 94.5}, {15., 92.8},
 {17.5, 91.3}, {20.5, 89.9}, {22., 89.1}, {27., 86.9}, {29., 86.2}, {35., 83.9},
 {38.5, 82.7}, {44., 81.}, {47.5, 80.1}, {54., 78.4}, {55., 78.1}, {58.5, 77.5},
 {65.5, 76.1}, {70., 75.3}, {73.5, 74.7}, {75., 74.6}, {78.5, 74.}, {80., 73.8},
 {86.5, 72.9}, {88.5, 72.7}, {93.5, 72.2}, {102., 71.4}, {102.5, 71.3}, {106.5, 70.9},
 {109., 70.8}, {112.5, 70.5}, {114.5, 70.4}, {117., 70.2}, {123.5, 69.8},
 {126., 69.7}, {127., 69.4}, {130.5, 69.4}, {132., 69.3}, {139.5, 69.}, {0, 103.1}}
```

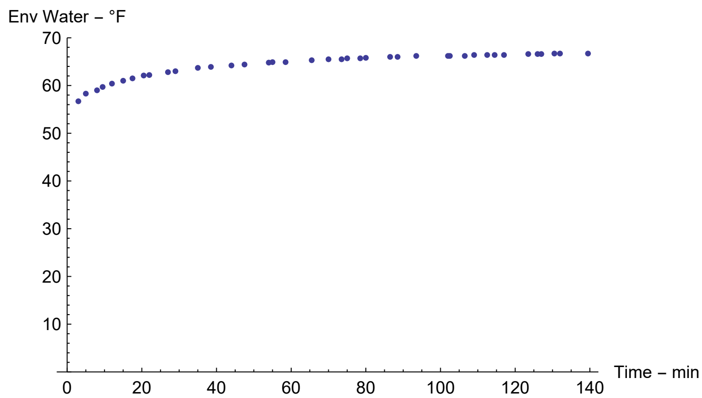
We plot the temperature of water.

```
WaterPlot =
 ListPlot[WaterTemp, PlotRange → {0, 120}, AxesLabel → {"Time - min", "Temp Water - °F"}]
```



We plot the temperature of the environment.

```
EnvTempPlot =
 ListPlot[EnvTemp, PlotRange → {0, 70}, AxesLabel → {"Time - min", "Env Water - °F"}]
```



It would appear that the environmental room temperature is leveling off to some value and we can use the model TModel(t) = $A(1 - e^{-m t}) + B$ to model this.

We estimate our parameters A, B, and m by fitting the function TE(t) = $A(1 - e^{-m t}) + B$ to the data.

```
solEnv = FindFit[EnvTemp, A (1 - Exp[-m t]) + B, {{A, 40}, {B, 30}, {m, .0001}}, t]
{A → 9.87664, B → 56.6497, m → 0.0350803}
```

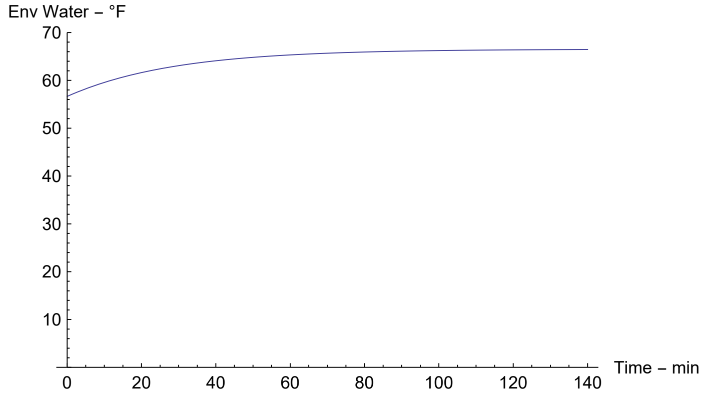
We use the best fitting parameters for a model of the environment' s temperature.

```
TModel[t_] = A (1 - Exp[-m t]) + B /. solEnv
```

$56.6497 + 9.87664 \times \left(1 - e^{-0.0350803\, t}\right)$

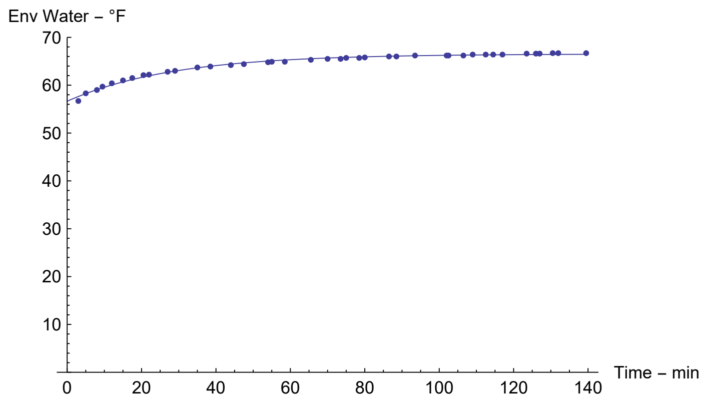
We plot the temperature of the environment offered by our fitted model.

```
TModelPlot = Plot[TModel[t], {t, 0, 140},
  PlotRange → {0, 70}, AxesLabel → {"Time - min", "Env Water - °F"}]
```



We see that our environmental temperature mode, TModel(t), is quite good at modeling the data.

```
Show[EnvTempPlot, TModelPlot, PlotRange → {0, 70},
 AxesOrigin → {0, 0}, AxesLabel → {"Time - min", "Env Water - °F"}]
```



We use our changing environmental temperature model TModel(t) in our Newton's Law of Cooling model,

T'(t) == - k (T(t) - TModel(t)),   with T(0) = 103.1.

```
Tsol[t_] =
 T[t] /. DSolve[{T'[t] == - k (T[t] - TModel[t]), T[0] == 103.1}, T[t], t][[1]] // Simplify
```

$$\frac{1}{0.0350803 - 1.\ k}\left(2.33377 + 1.28301\ e^{0.-1.\ k\ t} - 66.5264\ k + 9.87664\ e^{-0.0350803\ t}\ k - 46.4503\ e^{0.-1.\ k\ t}\ k\right)$$

We seek to determine best fitting value of k using sum of square errors between our model at each of the times of observations Tsol[WaterTemp[[i,1]]] and our observed water temperature at each of the times of observations, WaterTemp[[i,2]].

```
SSE[k_] = Sum[(Tsol[WaterTemp[[i, 1]]] - WaterTemp[[i, 2]])^2, {i, 1, Length[WaterTemp]}];
```

Here we find the value of the parameter k, whose units are 1/time, which minimizes our sum of square errors.

```
solk = FindMinimum[SSE[k], {k, .6}]
```

{0.493792, {k → 0.0174862}}

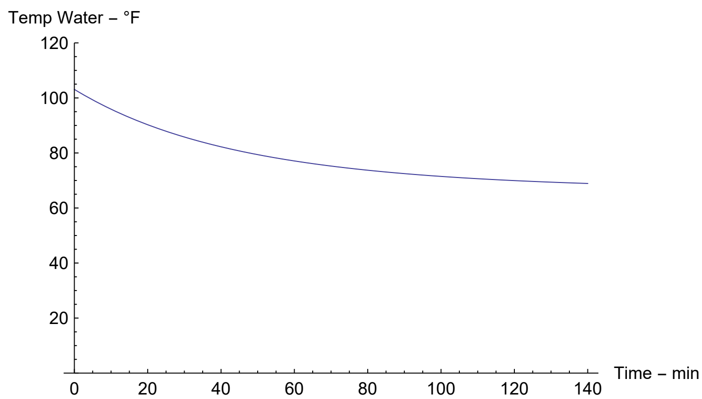
We put this value of
k into our new model.

```
TMod[t_] = Tsol[t] /. solk[[2]] // Expand
```

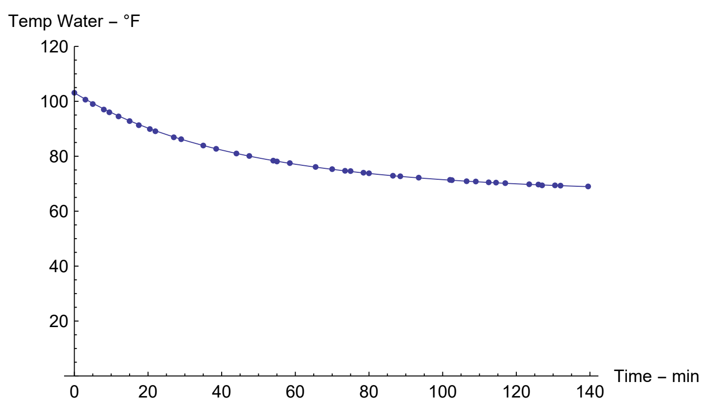
$66.5264 + 26.7576\, e^{0. - 0.0174862\, t} + 9.81603\, e^{-0.0350803\, t}$

We compare our new model and the data for the temperature of the water in this changing environment.

```
ModPlot = Plot[TMod[t], {t, 0, 140},
  PlotRange → {0, 120}, AxesLabel → {"Time - min", "Temp Water - °F"}]
```




```
Show[WaterPlot, ModPlot]
```

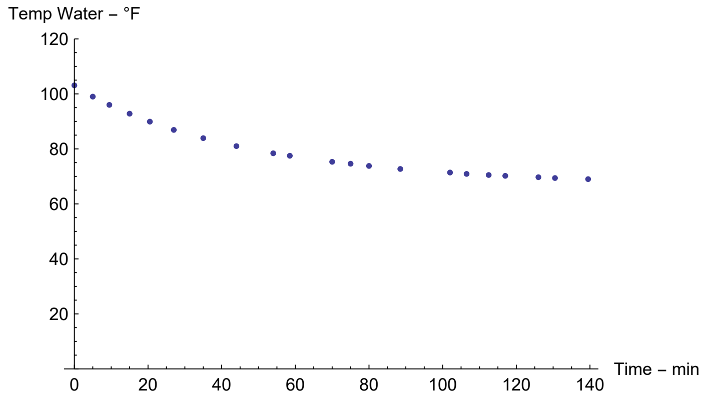


This looks quite good.

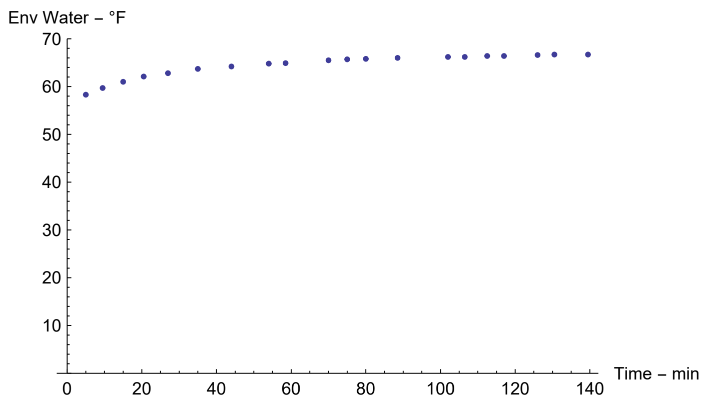
We offer a smaller sample for students to use in their work. We do this analysis without comments as it is similar to the analysis above, only with less data.

```
data = {{0, , 103.1}, {3.0, 56.7, 100.6}, {5.0, 58.3, 99.0},
   {8.0, 59.0, 97.0}, {9.5, 59.7, 96.0}, {12.0, 60.4, 94.5}, {15.0, 61.0, 92.8},
   {17.5, 61.5, 91.3}, {20.5, 62.1, 89.9}, {22.0, 62.2, 89.1}, {27.0, 62.8, 86.9},
   {29.0, 63.0, 86.2}, {35.0, 63.7, 83.9}, {38.5, 63.9, 82.7}, {44.0, 64.2, 81.0},
   {47.5, 64.4, 80.1}, {54.0, 64.8, 78.4}, {55.0, 64.9, 78.1}, {58.5, 64.9, 77.5},
   {65.5, 65.3, 76.1}, {70.0, 65.5, 75.3}, {73.5, 65.5, 74.7}, {75.0, 65.7, 74.6},
   {78.5, 65.7, 74.0}, {80.0, 65.8, 73.8}, {86.5, 66.0, 72.9}, {88.5, 66.0, 72.7},
   {93.5, 66.2, 72.2}, {102.0, 66.2, 71.4}, {102.5, 66.2, 71.3}, {106.5, 66.2, 70.9},
   {109.0, 66.4, 70.8}, {112.5, 66.4, 70.5}, {114.5, 66.4, 70.4},
   {117.0, 66.4, 70.2}, {123.5, 66.6, 69.8}, {126.0, 66.6, 69.7}, {127.0, 66.6, 69.4},
   {130.5, 66.7, 69.4}, {132.0, 66.7, 69.3}, {139.5, 66.7, 69.0}};

dataSample = Table[{data⟦i, 1⟧, data⟦i, 2⟧, data⟦i, 3⟧}, {i, 1, Length[data], 2}]

{{0, Null, 103.1}, {5., 58.3, 99.}, {9.5, 59.7, 96.}, {15., 61., 92.8}, {20.5, 62.1, 89.9},
 {27., 62.8, 86.9}, {35., 63.7, 83.9}, {44., 64.2, 81.}, {54., 64.8, 78.4},
 {58.5, 64.9, 77.5}, {70., 65.5, 75.3}, {75., 65.7, 74.6}, {80., 65.8, 73.8},
 {88.5, 66., 72.7}, {102., 66.2, 71.4}, {106.5, 66.2, 70.9}, {112.5, 66.4, 70.5},
 {117., 66.4, 70.2}, {126., 66.6, 69.7}, {130.5, 66.7, 69.4}, {139.5, 66.7, 69.}}

data = dataSample

{{0, Null, 103.1}, {5., 58.3, 99.}, {9.5, 59.7, 96.}, {15., 61., 92.8}, {20.5, 62.1, 89.9},
 {27., 62.8, 86.9}, {35., 63.7, 83.9}, {44., 64.2, 81.}, {54., 64.8, 78.4},
 {58.5, 64.9, 77.5}, {70., 65.5, 75.3}, {75., 65.7, 74.6}, {80., 65.8, 73.8},
 {88.5, 66., 72.7}, {102., 66.2, 71.4}, {106.5, 66.2, 70.9}, {112.5, 66.4, 70.5},
 {117., 66.4, 70.2}, {126., 66.6, 69.7}, {130.5, 66.7, 69.4}, {139.5, 66.7, 69.}}

EnvTemp = Table[{data⟦i, 1⟧, data⟦i, 2⟧}, {i, 2, Length[data]}]

{{5., 58.3}, {9.5, 59.7}, {15., 61.}, {20.5, 62.1}, {27., 62.8}, {35., 63.7}, {44., 64.2},
 {54., 64.8}, {58.5, 64.9}, {70., 65.5}, {75., 65.7}, {80., 65.8}, {88.5, 66.}, {102., 66.2},
 {106.5, 66.2}, {112.5, 66.4}, {117., 66.4}, {126., 66.6}, {130.5, 66.7}, {139.5, 66.7}}

WaterTemp = Table[{data⟦i, 1⟧, data⟦i, 3⟧}, {i, 1, Length[data]}]

{{0, 103.1}, {5., 99.}, {9.5, 96.}, {15., 92.8}, {20.5, 89.9},
 {27., 86.9}, {35., 83.9}, {44., 81.}, {54., 78.4}, {58.5, 77.5}, {70., 75.3},
 {75., 74.6}, {80., 73.8}, {88.5, 72.7}, {102., 71.4}, {106.5, 70.9},
 {112.5, 70.5}, {117., 70.2}, {126., 69.7}, {130.5, 69.4}, {139.5, 69.}}
```

```
WaterPlot =
 ListPlot[WaterTemp, PlotRange → {0, 120}, AxesLabel → {"Time - min", "Temp Water - °F"}]
```




```
EnvTempPlot =
 ListPlot[EnvTemp, PlotRange → {0, 70}, AxesLabel → {"Time - min", "Env Water - °F"}]
```



It would appear that the environmental room temperature is leveling off to some value and we can use the model TModel(t) = $A(1 - e^{-m t}) + B$ to model this.

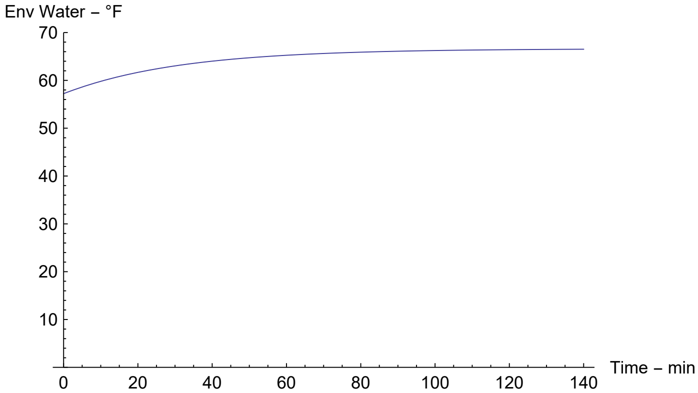
We estimate our parameters A, B, and m by fitting the function TE(t) = $A(1 - e^{-m t}) + B$ to the data.

```
solEnv = FindFit[EnvTemp, A (1 - Exp[-m t]) + B, {{A, 40}, {B, 30}, {m, .0001}}, t]
{A → 9.39247, B → 57.2237, m → 0.0321087}
```

```
TModel[t_] = A (1 - Exp[-m t]) + B /. solEnv
```

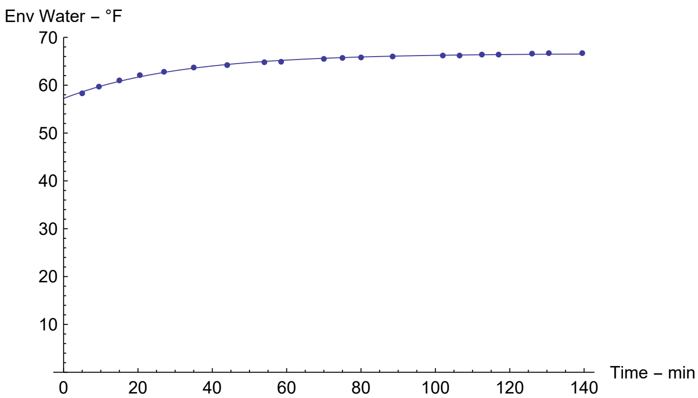
$57.2237 + 9.39247 \times \left(1 - e^{-0.0321087\, t}\right)$

```
TModelPlot = Plot[TModel[t], {t, 0, 140},
  PlotRange → {0, 70}, AxesLabel → {"Time - min", "Env Water - °F"}]
```



We see that our environmental temperature mode, TModel(t), is quite good.

```
Show[EnvTempPlot, TModelPlot, PlotRange → {0, 70},
 AxesOrigin → {0, 0}, AxesLabel → {"Time - min", "Env Water - °F"}]
```



We use our changing environmental temperature model TModel(t) in our Newton's Law of Cooling model, T'(t) == - k (T(t) - TModel(t)), with T(0) = 103.1.

```
Tsol[t_] = T[t] /. DSolve[{T'[t] == - k (T[t] - TModel[t]), T[0] == 103.1}, T[t], t][[1]]
```

$$\frac{1}{-0.0321087 + 1.\ k} 45.8763\ e^{-1.\times(0.0321087-1.\ k)\ t-1.\ k\ t} \left(-0.0466244\ e^{0.0321087\ t} - 0.0255349\ e^{1.\times(0.0321087-1.\ k)\ t} - 0.204734\ k + 1.45208\ e^{0.0321087\ t}\ k + 1.\ e^{1.\times(0.0321087-1.\ k)\ t}\ k\right)$$

We seek to determine best fitting value of k using sum of square errors.

```
SSE[k_] = Sum[(Tsol[WaterTemp[[i, 1]]] - WaterTemp[[i, 2]])^2, {i, 1, Length[WaterTemp]}];

solk = FindMinimum[SSE[k], {k, .6}]

{0.372556, {k → 0.0174899}}
```

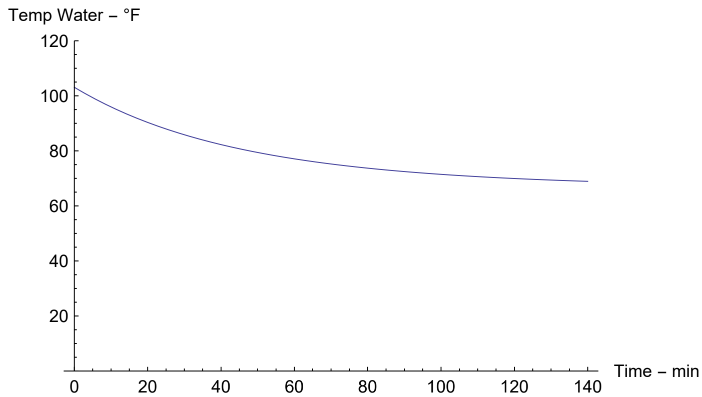
We put this value of k into our new model.

```
TMod[t_] = Tsol[t] /. solk⟦2⟧ // Expand
```

$66.6161 + 11.2372\, e^{-0.0321087\, t} + 25.2466\, e^{-0.0174899\, t}$

We compare our new model and the data for the temperature of the water in this changing environment.

```
ModPlot = Plot[TMod[t], {t, 0, 140},
  PlotRange → {0, 120}, AxesLabel → {"Time - min", "Temp Water - °F"}]
```




```
Show[WaterPlot, ModPlot, AxesLabel → {"Time - min", "Temp Water - °F"}]
```