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STUDENT VERSION
TUNED MASS DAMPERS - PART I

Keith Landry
Georgia Southern University
Statesboro GA USA
klandry@georgiasouthern.edu

Brian Winkel, Director SIMIODE
Chardon OH 44024 USA
BrianWinkel@SIMIODE.org

Abstract: We offer an opportunity to build mathematical models to mitigate dangerous displacements in structures using structural improvements called Tuned Mass Dampers. We model the motion of the original structure as a spring-mass-dashpot system (one degree of freedom) with stiffness replacing the spring constant in the model. We add another, smaller spring-mass-dashpot system whose frequency is tuned to reduce the overall oscillation of the original structure and consider the effects of this two degree of freedom model. Here we do not include resistance or damping forces which are often portrayed as being proportional to velocity.

SCENARIO DESCRIPTION

History - Need for Reducing Oscillation in Buildings and Related Infrastructure

In the 1800s and early 1900s, most large civil engineering infrastructure (buildings, dams, bridges, etc.) was designed and built using rather conservative design processes which resulted in fairly stiff, rigid structures. Vibrations in structural components such as the floor beams caused by dynamic loads were rarely a concern. In the late 1900s, significant improvements in engineering design, engineering science, and construction methods resulted in lighter, more slender structures which proved far more susceptible to large deflections resulting from dynamic wind or seismic loads caused when the dominant frequency in the loading or driving function neared the natural frequency of the structure. This undesirable condition (for typical infrastructure facilities) is known as *resonance* or near-resonance.

Engineers knew that damping served to remove energy from physical systems, but noted that providing too much damping acted to increase the stiffness of a system. In an attempt to mitigate the potentially catastrophic effects of near-resonance in buildings, structural engineers took note of the "passive" tuned mass dampers (TMD) being used by mechanical engineers to reduce vibration amplitudes in machinery, ships, automobiles, and electrical transmission lines among other things. By tuning the natural frequency of the damper to match that of the structure to which it is attached, engineers found that the TMD acted to significantly reduce the large displacements associated with resonance. Simply put, the energy that would have normally led to large displacements in the building itself is now being used to drive a large mass in the damping device, but in a direction opposing the building motion. Building goes to the left; TMD goes to the right. Pretty cool!

Structural Engineering in the News

Most of the advancements in the application of TMDs in structures are found in the field of earthquake engineering. A list of significant structures which utilize TMDs is available from the National Information Service for Earthquake Engineering (NISEE) at UC Berkeley [13]

Passive TMDs (some of which use a liquid [14] instead of solid mass) started appearing in structures more frequently in the early 1970s [11]. Since Passive TMDs are tuned to a single frequency, engineers looked to Active TMDs in an attempt to provide more protection to structures across a range of frequencies starting in the early 1990s.

We describe the types of TMDs and give some examples of their ubiquitous use in engineering.

Types – Passive (PTMD), Semi-active (SATMD), Active (ATMD)

Here is the definition [3] of a Tuned Mass Damper given by DEICON: Dynamics & Controls, a firm founded in 1998. DEICON is an engineering firm specializing in Sound and Vibration Control (active and passive), as well as Advanced Control Design and Prototyping.

> A tuned mass damper (TMD) is a vibrating mass that moves out of phase with the motion of the structure it is suspended to. With its out of phase motion, the inertial force of the TMD mass abates the resonant vibration of the structure by dissipating its energy. The ideal extent of phase difference between the motion of the TMD mass and that of the structure, i.e. 90 degrees, is attained by tuning the TMD to the natural frequency of the structural mode targeted for damping.
>
> Tuned mass dampers have been used for adding tuned damping to various structures, successfully. Considering that the first vibrational mode of a structure plays a dominant role in its dynamic response, a TMD is normally (but not always) tuned to the first natural frequency or mode of the structure. The energy dissipation effectiveness of a TMD depends on a) the accuracy of its tuning, b) the size of its mass compared to the modal mass of its target mode, i.e. its mass ratio, and c) the extent of internal damping built into the tuned mass damper.[3]

Engineers also have to be cognizant of the economics as well as the structural issues, for one could mount a massive structure atop a tall office building which is itself the size of that office building just to mitigate vibrations, but such a structure would have no occupancy or income generating capabilities.

Basically, one can think of a TMD as a mechanical counterweight for a structure consisting of a moving mass (roughly 1–2% of the structure's mass) which is usually placed in the upper portion of the structure. The purpose of the TMD is to reduce the effects of motion caused by wind or seismic loads. This, and other TMDs, can be passive, with no energy input to regulate the damper motion or Semi-Active or Active with more and more energy inputs to control the motion of the damper and hence the structure.

The first uses of TMD's in the United States for large structures were in the John Hancock Building in Boston in 1977 [1, 4] and City Corp Center [11] in New York in 1978. Since that time many different styles, including active TMD's and pendulum TMD's, have been employed, while diverse applications have been found through retro-fitting on large-span bridges and highways. Indeed, the current TMD exemplar is the 800-ton wind-compensating damper built into the center of the 508 meter tall Taipei 101 [19] in Taiwan. The TMD consists of a huge spherical mass hung as a pendulum, which is visible from the restaurant on the 88^{th} and 89^{th} floors. A very recent use of a TMD is in the construction of the Grand Canyon Skywalk [6, 12]. As an example of the diversity of uses, TMDs are also used in the design of surgery tables to mitigate the vibrations of surroundings during surgery [9] - think eye surgery and the New York City subway rumbling down below the building.

An excellent narrative, "What is a Tuned Mass Damper," [16] with both technical, laboratory, and cultural elements is offered by the Practical Engineering Project at its YouTube site.

Illustrations of the wide variety of instances of TMDs, both big and small

We illustrate some TMD's which are found in practice.

Stockbridge dampers in power transmission lines illustrate a very common use of TMDs. An example is shown in Figure 1.

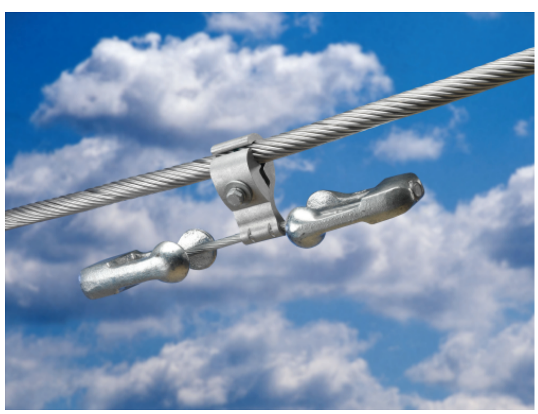

Figure 1. An example of a Stockbridge Damper from a commercial catalog [17].

They are featured in the popular cultural media [23], are a significant commercial product [15], and have been studied in the professional literature [24] for some time. These TMDs are typically placed near insulators to minimize displacements which could damage those devices. The TMD is essentially a flexible rod with weights on each end, like a dumbbell, clamped to the transmission line. The vertical motion of the transmission line is conveyed through the rigid clamp to the flexible rod. Energy is dissipated through the motion of the cantilevered weights, thus reducing vibrations in the lines.

Consider wind loads on buildings and bridges. Perhaps the most famous structural failure due to such vibration was the Tacoma Narrows Bridge collapse in 1940. Known as "Gallopin' Gertie," the ultimate demise of the bridge has been captured on film as it happened [20].

When finished in the summer of 1940, this bridge was the third longest suspension bridge in the world. However, it was built with a far too slender and flexible roadway. For bridges of this size, traffic loads are typically inconsequential. The bridge design is typically governed by its own self-weight (gravity load) and anticipated lateral loads (e.g., earthquake and wind). In this case, the bridge design was sufficient to carry the gravity loads, but not the lateral wind loads. As the high winds in the region hit the vertical profile of the bridge deck, the deck would flex vertically and horizontally which allowed the wind to alternately flow across the top and bottom of the roadway and supporting plate girders. The frequency of the alternating flow turned out to be close to a natural frequency of the bridge so the vertical displacements of the deck grew until the bridge tore itself apart. These large displacements were noticed during the bridge construction and wind-tunnel tests were commissioned in an attempt to mitigate the significant vibrations. The study recommended improving the aerodynamic profile of the bridge deck, but the bridge collapse occurred before any modifications could be made.

TMDs were not considered, but what if engineers had treated the bridge as if it were a long, slender power transmission line? Could you place a Stockbridge damper on the bridge deck at critical locations, say near joints between plate girders, to minimize displacements? Would these TMDs need to be located so they vibrated in a vertical or horizontal plane to be effective? These are questions engineers address in design and construction.

One use of TMDs would be to reduce the effect of wind on a building, say, in Chicago. In this YouTube video shot from the window of an office in a Chicago building [21] one can clearly see the proximity of the tall building which is the subject of the video to the shore of Lake Michigan. The motion of the rod attached to the window blinds gives a hint as to the principal direction of the lateral motion or "drift" the building is experiencing at that floor level. Do you think the wind is pushing against the window in the video or flowing across it? Why? Does the magnitude of the displacement appear to be a structural building frame concern? What impact might it have on the building's veneers and windows?

TMDs would help in this case if they were attached to the building frame. Assuming the building has a fairly uniform, symmetric profile you could develop a mathematical model of the structure (basically a beam cantilevered out of the ground) and load the beam with a dynamic wind load that increases in magnitude with building height according to local design codes. Analysis of the model would give you an idea of what frequencies cause the largest displacements. In turn, you can then design a TMD to effectively reduce those vibrations. Typically TMDs are integrated into the building frame in such a way that the TMD's mass moves 180 degrees out of phase with the building. The energy needed to produce motion in the TMD is then unavailable to produce motion in the building itself.

We point to just a few diverse applications of TMDs: seismic loads on buildings and bridges, vibration dampers in surgery rooms, and reduction of oscillations on cannon barrels, while noting how many sources of materials there are on TMDs.[25, 5] Indeed, if you search YouTube on "Tuned Mass Dampers" [25] you will find 7,210 results with Google offering up some 103,000 results [5].

In this YouTube video [2] one can actually see the vibrations of tall buildings in Japan as well as learn something about the modeling approaches to building design while in [8] one can see the Ibáñez Suspension Bridge, located in Puerto Aysén, Chile, shaken by a 6.2 magnitude earthquake that took place in April 21, 2007. It did not suffer structural damages."

Vibration control design is featured in trade publications, e.g., [18], in which reduction of vibrations of floors under sensitive medical or research equipment is discussed. There is application of barrell tuners or harmonic dampers for various types of rifles as seen in [22]. Thus the uses of TMDs is pervasive and productive in many fields.

Modeling a Tuned Mass Damper

We represent the stiffness of the original mass of a system using a spring and the energy absorption using a damper. We will attach a spring and damper (TMD) to the original mass and explore how much it can reduce vibrations due to external forces. Displacement of the mass will be characterized about an initial equilibrium position by a single degree-of-freedom (e.g., Left-to-Right in Figure 3, Up-and-Down in Figure 4). In the case of the TMD though, we will add a second mass and spring to the primary mass. Displacement of the TMD mass will be characterized by a separate degree of freedom about its own initial equilibrium position. Thus, a TMD is a pair of coupled damped harmonic oscillators.

A large mass m_1 on a "spring" with spring constant k_1 is coupled to a smaller mass m_2 by a spring with spring constant k_2 and damper with damping coefficient c_2. The large mass might also be naturally damped with damping or restoring coefficient c_1. For our study in Part I here we take $c_1 = 0$ and $c_2 = 0$. That is, we will presume there is no resistance or damping of either the original mass or of the TMD. By tuning the values of m_2 and k_2 the maximum amplitude of m_1's oscillations can be lowered. We shall consider the case $c_1 \neq 0$ and $c_2 \neq 0$ in Part II.

In this case with no resistance ($c_1 = 0$ and $c_2 = 0$) when we apply an external load or driver force, $f(t)$, we have the following single degree of freedom differential equation model for the displacement, $y(t)$, of our mass from its static equilibrium, i.e. a spring-mass (no dashpot) system:

$$my''(t) + ky(t) = f(t)\,, \quad y(0) = y_0\,, \quad y'(0) = v_0. \tag{1}$$

The natural frequency of mass m_1 can be obtained from the solution of the homogeneous portion, (2), of our differential equation (1),

$$my''(t) + ky(t) = 0, \quad y(0) = y_0\,, \quad y'(0) = v_0. \tag{2}$$

From (3) we see that the natural frequency of the undriven or homogeneous solution of (2) is $\omega_0 = \sqrt{\frac{k}{m}}$ while the complete homogeneous solution to (1) is:

$$y(t) = \frac{v_0\sqrt{m}\sin\left(\sqrt{\frac{k}{m}}t\right) + y_0\sqrt{k}\cos\left(\sqrt{\frac{k}{m}}t\right)}{\sqrt{k}}\,. \tag{3}$$

Recall what happens if the driver function $f(t)$ has the same frequency as the natural frequency, ω_0, of the spring-mass system - RESONANCE and trouble. To see this we divide both sides of (1) by m and rearrange the resulting equation using $\omega_0^2 = \frac{k}{m}$ to obtain (4):

$$y''(t) + \omega_0^2 y(t) = \frac{1}{m}f(t)\,, \quad y(0) = y_0\,, \quad y'(0) = v_0\,. \tag{4}$$

Now if $f(t) = \sin(\omega t)$ and this driver has the same natural frequency as our mass system, i.e. $\omega = \omega_0$, then our solution of (4) is

$$y(t) = -\frac{t\cos(\omega_0 t)}{2m\omega_0} + \frac{\sin(\omega_0 t)}{2m\omega_0^2} - \frac{\sin(\omega_0 t)\cos^2(\omega_0 t)}{2m\omega_0^2} + \frac{\sin(2\omega_0 t)\cos(\omega_0 t)}{4m\omega_0^2} + \frac{v_0\sin(\omega_0 t)}{\omega_0} + y_0\cos(\omega_0 t)\,. \tag{5}$$

The key term in this solution is the very first term. All the other terms are of period $\frac{2\pi}{m}$ with finite amplitudes. The first term $-\frac{t\cos(\omega_0 t)}{2\omega_0}$ grows without bound and accordingly we refer to this solution phenomenon as *resonance*. For the following values, $\omega_0 = 1, y_0 = 2, v_0 = 0$, and $m = 1$ we plot our solution (5) over the time interval $[0, 100]$ in Figure 2 and see the solution grows indefinitely; this is resonance.

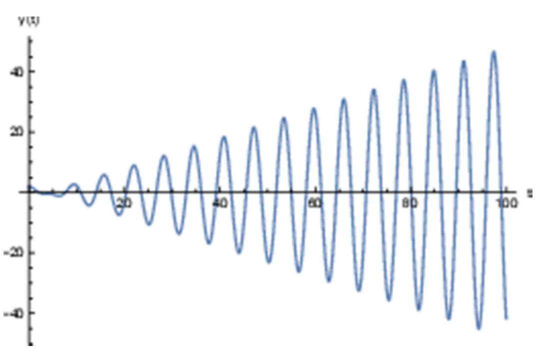

Figure 2. Plot of solution of (4) with values, $\omega_0 = 1$, for the natural frequency of the system and the driver as well, and $y_0 = 2, v_0 = 0$, and $m = 1$. In this case $f(t) = \sin(\omega t)$ is the driver function.

Resonance Spells Trouble

In general, engineers do not like resonance, as it means things are getting out of control, going beyond tolerance levels, and possibly causing havoc. Suppose (4) were a model of a tall structure of mass m and k represented a restorative force or stiffness in the structure. If $y(t)$ is the horizontal displacement of the corner of the structure from its static equilibrium due to a force, $f(t)$, from tremors of an earthquake then resonance would mean that the structure would swing wildly and eventually break. Recall (4) does not have any resistance or damping force so this is somewhat unrealistic. Nevertheless, we shall study this situation and see if we cannot prohibit resonance by applying appropriate force from another system at the top of the structure. We shall address the situation in which both initial mass and damper mass experience resistance, i.e. $c_1 \neq 0$ and $c_2 \neq 0$, in Part II of this scenario study.

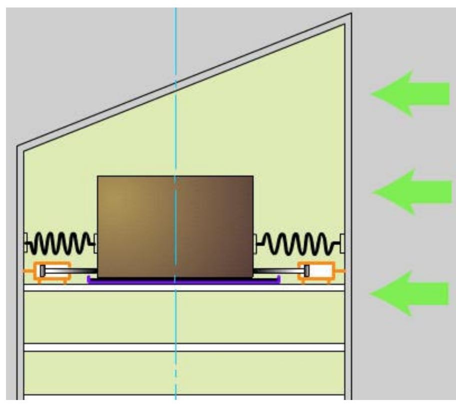

Figure 3. Schematic of the TMD atop the Citicorp building in New York City from [7].

In Figure 3 from [7] we show a schematic of a TMD used to reduce the sway in the Citicorp building in New York City. This was one of the first installations of a TMD in a large structure. In the diagram the green (horizontal) arrows indicate the external forces applied to the structure and the brown (large dark) box, with springs and dashpots attached, indicate the TMD which has a large mass sliding back and forth to mitigate the sway of the structure.

A good diagram from which to work in developing a two degree of freedom system of differential equations model for damping an oscillator gone wild is offered in Figure 4. Often, it is easier to see and build a model in the case the diagram is “righted” to vertical, bkut the forces are the same in either horizontal or vertical view.

Here the oscillator to be damped has mass m_1 with spring or restoring constant k_1 and resistance or damping constant c_1. This could be a large structure. k_1 would be its *stiffness* while c would reflect internal frictional resistance to motion proportional to the velocity of the structure. Now, the damping oscillator would be a large mass which sits atop the structure and is permitted to slide or roll as the structure sways in response to wind or seismic forces. This damper has mass m_2 with

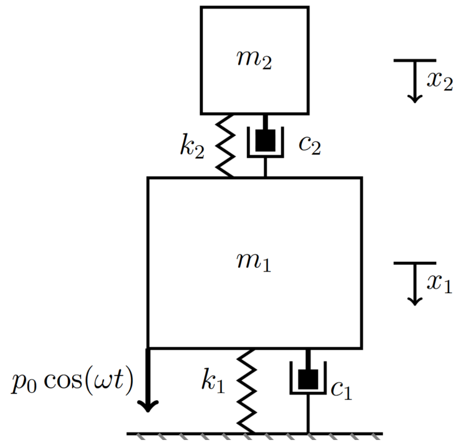

Figure 4. Diagram of the Tuned Mass Damper for developing full set of two degrees of freedom differential equation model [10].

spring or restoring constant k_2 and resistance or damping constant c_2. While the model in Figure 4 appears to be orthogonal to that offered in Figure 3, the identifying features of structure mass, m_1; tuned damper mass (m_2); stiffness or spring constants (k_1 and k_2), and dampers (c_1 and c_2), are the same and so we will work from Figure 4.

The distances, while represented vertically in Figure 4, are actually horizontal in the case of our building situation and the distances x_1 and x_2 are the distances from the static equilibrium of the mass of the structure and the mass of the TMD, respectively.

Activities

Activity 1

Using Newton's Second Law, which says that the mass times the acceleration of that mass is equal to the sum of all external forces acting on the mass, and Figure 4 as a Free Body Diagram source, show that the governing equations for the motions of the structure and the damper are given by (6). Hint: For consistency all downward forces, positions, velocities, and accelerations are positive.

$$\begin{aligned} m_1 x_1''(t) &= -k_1 x_1(t) - c_1 x_1'(t) - k_2(x_1(t) - x_2(t)) - c_2(x_1'(t) - x_2'(t)) + p_0 \cos(\omega t) \\ m_2 x_2''(t) &= -k_2(x_2(t) - x_1(t)) - c_2(x_2'(t) - x_1'(t)) \end{aligned} \tag{6}$$

In our study here we will first presume an "ideal" structure with no resistance or damping coefficient, i.e. $c_1 = 0$, and the same for the damper, i.e. $c_2 = 0$. As we can see there is a driving force applied to the structure of $f(t) = p_0 \cos(\omega t)$. Our interest will be in seeing if we can tune the added mass damper to reduce or eliminate the possibility of resonance when the initial structure is driven by a force with a frequency equal to that of the natural frequency of the initial mass system.

Activity 2

a) Determine the form of (6) when $c_1 = 0$ and $c_2 = 0$.

b) In the resulting system use $m_1 = 10$ and $k_1 = 90$ with $y(0) = 0$ and $y'(0) = 0$, while $f(t) = 10\cos(3t)$ and set $m_2 = 1$ with $k_2 = 0$, i.e. withdraw the second mass system. Explain the physical significance and impact results of the numbers 10, 90, and 3 in the above sentence in terms of the motion of the initial oscillator.

c) Solve the resulting single differential equation for the motion of mass m_1 over a time interval of 20 units and plot the displacement of that mass, $x_1(t)$, over that time interval. Explain what you see.

d) In (6) use the values of $m_1 = 10$ and $k_1 = 90$ with $y(0) = 0$ and $y'(0) = 0$ and $f(t) = 10\cos(3t)$ and set $m_2 = 1$ with varying values of k_2. Keep $c_1 = 0$ and $c_2 = 0$. What do you observe in the maximum amplitude of the initial mass m_1 as one changes k_2? Defend your observation with data or plot.

e) From (d) what is the "best" value of k_2? Be sure you define the word "best."

f) For the best value of k_2 determine the maximum amplitude displacement for mass m_1, $x_1(t)$ over a range of frequencies.

Activity 3

We consider another, comparable configuration as Activity 2 for practice. Notice that mass m_2 is only 1% of mass m_1 which is quite realistice in structural design when usingTMDs.

a) Determine the form of (6) when $c_1 = 0$ and $c_2 = 0$.

b) In the resulting system use $m_1 = 10$ and $k_1 = 90$ with $y(0) = 0$ and $y'(0) = 0$ while $f(t) = 10\cos(3t)$ and set $m_2 = 0.1$ with $k_2 = 0$, i.e. withdraw the second mass system. Explain the physical significance and impact results of the numbers 10 ,90, and 3 in the above sentence in terms of the motion of the initial oscillator.

c) Solve the resulting single differential equation for the motion of mass m_1 over a time interval of 20 units and plot the displacement of that mass, $x_1(t)$, over that time interval. Explain what you see.

d) In (6) use the values of $m_1 = 10$ and $k_1 = 90$ with $y(0) = 0$ and $y'(0) = 0$ and $f(t) = 5\cos(3t)$ and set $m_2 = 0.1$ with varying values of k_2. Keep $c_1 = 0$ and $c_2 = 0$. What do you observe in the maximum amplitude of the initial mass as one changes k_2? Defend your observation with data or plot.

e) From (d) what is the "best" value of k_2? Be sure you define the word "best."

f) For the best value of k_2 determine the maximum amplitude displacement for mass m_1, $x_1(t)$ over a range of frequencies.

Activity 4

From the introductory material for this scenario offered above and the analyses in Activities 2 and 3 offer a description of how to build a TMD to stop the resonance phenomena in the case of (7) which is the case of (6) where there is no damping, i.e. $c_1 = 0$ and $c_2 = 0$,

$$\begin{aligned} m_1 x_1''(t) &= -k_1 x_1(t) - k_2(x_1(t) - x_2(t)) + p_0 \cos(\omega t) \\ m_2 x_2''(t) &= -k_2(x_2(t) - x_1(t)) \end{aligned} \tag{7}$$

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