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SIMIODE
A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS
& OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
CHORD PATH TIME

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Abstract: Using Newton's Second Law of motion students can determine the time it takes for a mass to slide along a chord of a vertical circle from high point to any point along the circle. They come to a nonintuitive conclusion through their analysis, but upon reflection the conclusion does make sense.

SCENARIO DESCRIPTION

A mass particle starts from rest at the highest point, $O = (0, 1)$, of a vertical circle and slides along a chord only effected by the acceleration due to gravity to another terminal point A on the circle. See Figure 1. What can you say about the time(s) it takes for the mass to hit the terminal points in terms of a changing angle θ? Reflect on your answer and make comments on the reasonableness and interpretations we might offer for your solution. This modeling opportunity appears in [1, p. 59] and other textbooks. However, we believe the use of elementary vector mathematics to obtain a differential equation and then solve the equation and interpret the solution is a worthy modeling task.

REFERENCES

[1] O'Neil, P. V. 2003. *Advanced Engineering Mathematics, Fifth Edition*. Pacific Grove CA: Brooks/Cole-Thomson Learning. p. 59.

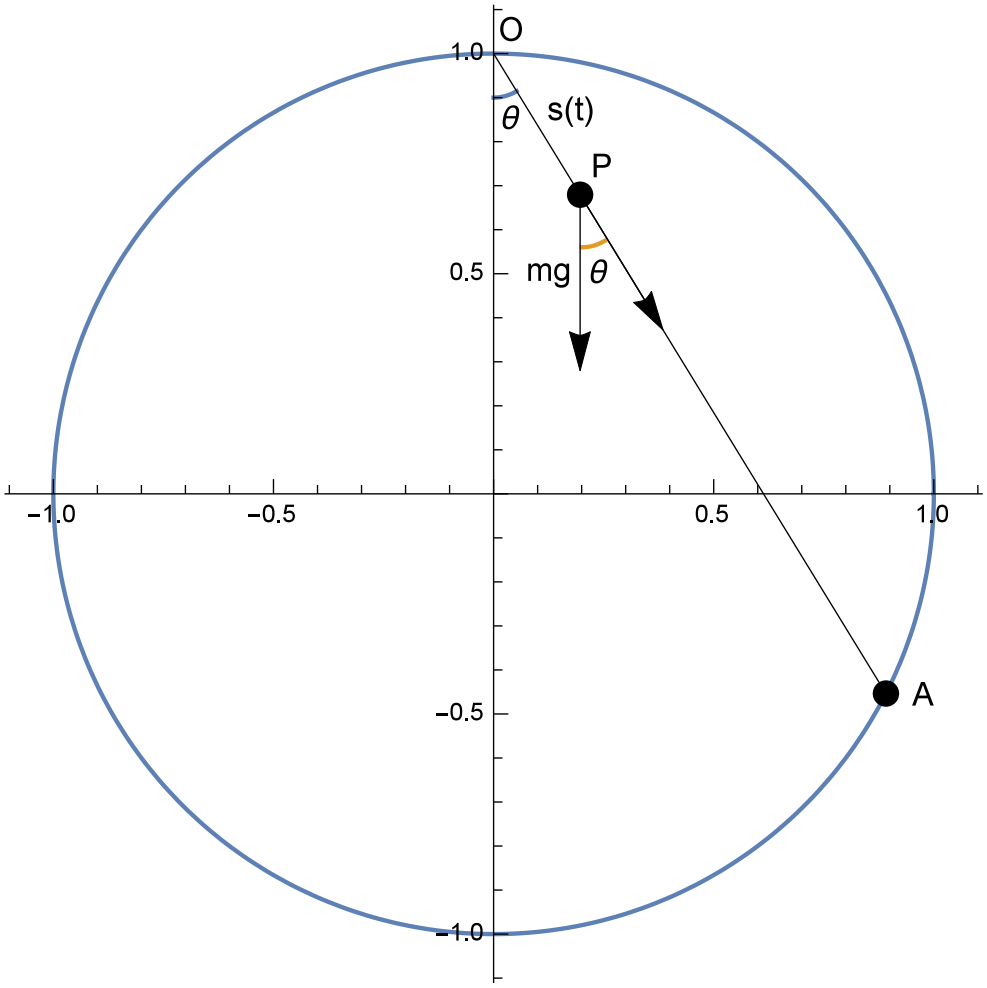

Figure 1. Useful figure in determining the time it takes for a mass particle to slide along the chord from point) at the top of a circle to an arbitrary point A along the circle.