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STUDENT VERSION

Fishing - Make the most of it!

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Abstract: We offer students a harvesting model for operating a fishery over a 25 year horizon and ask them to write a report on optimal harvesting policy with their analyses for fishing industry experts (not necessarily mathematicians).

SCENARIO DESCRIPTION

You have been hired to manage a fishery for the next twenty-five years. This means you have to choose the rate of harvesting of the fish. After carefully studying the previous behavior of the fishery, you write down a logistic growth model (1) with for $y(t)$, the number of fish in the population at time t:

$$y'(t) = 0.2y(t)\left(1 - \frac{y(t)}{10000}\right) - h(t)\,, \quad y(0) = 2500\,. \tag{1}$$

Here $h(t)$ is the harvesting rate, with units of fish per year. The harvesting rate is chosen by you. It does not have to be a constant. The fishery has been depleted before you arrive, so the initial population is below the maximum value.

Your goal is to balance the desire to harvest lots of fish with the imperative to maintain a viable fish population for the future. One way to combine and mathematize these goals is to try to maximize the function J given by (2):

$$J = 2000\alpha + \int_0^{25} 14e^{-0.1t}h(t)\,dt\,. \tag{2}$$

The term before the integral sign is a sustainability bonus which you can earn by leaving at least 1500 fish alive at the end of your term. More precisely, take $\alpha = 1$ if $y(25) \geq 1500$ and $\alpha = 0$ otherwise. The integral term will be large if lots of fish are harvested during your term; we are assuming a price of \$14 per fish, and the decaying exponential term refects the fact that most people would rather have a fish right now than next year!

If you immediately harvest all of the fish, you will have $J = \$35{,}000$, so try to do better than that. I have not done much better than \$50,000 just by making guesses. To find the best possible answer, rather than just a good one, you should explore *control theory*.

To answer this question, please write a report with the following ingredients:

- Write an introduction explaining the meaning and units of the numbers 0.2, 10000, and 2500 in the initial value problem.
- Calculate this fishery's *maximum sustainable yield* (MSY), and explain your calculation. To do this, suppose that the population level is constant, so the right hand side of the differential equation (1) is zero, and the harvest rate is constant, $h(t) = H$. The MSY is the largest possible value of H under these circumstances.
- Propose a harvesting plan by defining a function $h(t)$. This will mostly likely be an explicit formula like $h(t) = t + \sin(t)$, but in principle I would accept any construction of a function which is defined for $t \in [0, 25]$.
- Report the value of J that you achieve with your harvesting policy $h(t)$. Bigger is better! A small part of your grade depends on making J large.
- Include two graphs in your report, showing $h(t)$ and $y(t)$ from $t = 0$ to $t = 25$. These graphs should have captions of two or three sentences narrating what you see happening in the graphs.
- Write a discussion of how you arrived at your function $h(t)$ and why you think it is effective. Give an interpretation of your proposal targeted at fishing industry experts (not mathematicians). This is a much larger part of the problem score than the value of J.

The report should include the two figures generated in your software package. If you are not sure how to begin, you can start with the guess $h(t) = 100 + 10t$ and then compute y and J, and then change h to try to improve your score. You might also use other function types, like polynomials or Ae^{bt} or $Ae^{b(t-25)}$.

An iconic text in the area of fishery management[1] appears in the references.

REFERENCES

[1] Clark, Colin W. 1976. *Mathematical Bioecomonimcs: The Optimal Management of Renewable Resources*. New York: John Wiley & Sons.

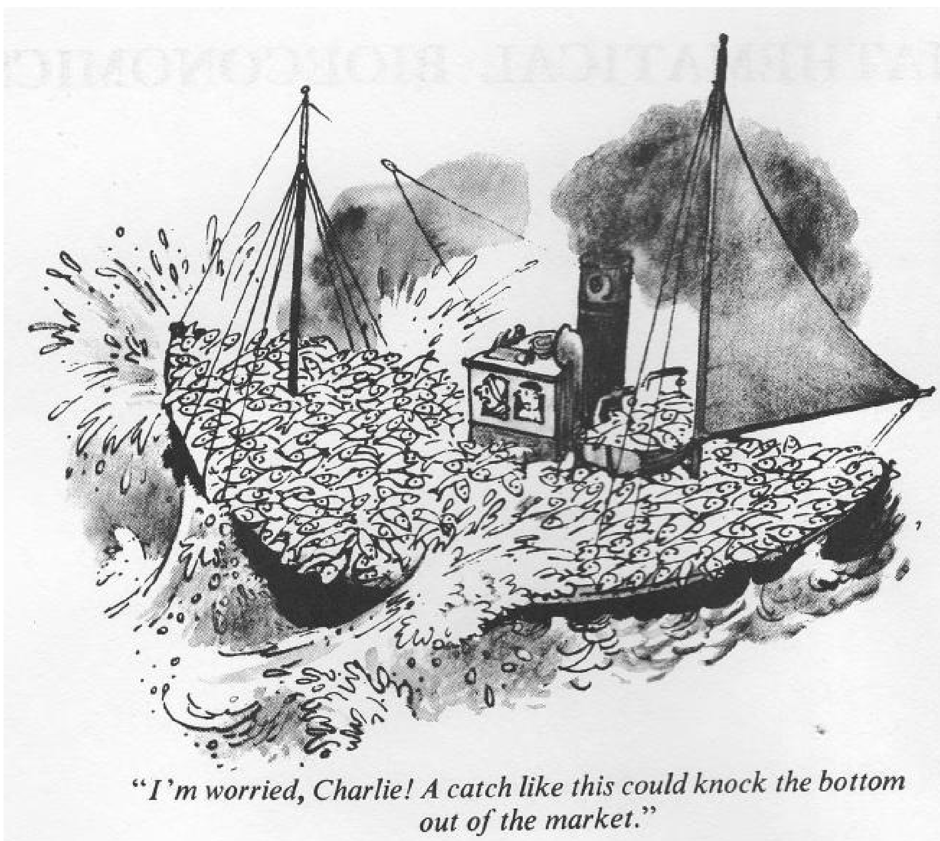

Figure 1. From the frontispiece of [1] as produced from *Punch*.