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STUDENT VERSION
EBOLA OUTBREAK IN WEST AFRICA

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Abstract: Students will use data published by the World Health Organization to model the 2014 outbreak of the Ebola virus in West Africa. We begin with a simple exponential growth model and move through the modeling process to the logistic growth model. Both are separable differential equations. Students will investigate properties of the logistic growth model and will compare predicted values with observed values. Both quantitative and qualitative questions are asked throughout. This project can be used in a differential equations or modeling course as well as in the calculus sequence. The logistic growth model fits the data well and thus provides a simple introduction to modeling a real world situation.

Note that in the supplemental materials, an Excel file containing a complete data set with more dates and records is available which can be used for additional modeling projects. This Excel file includes the total number of cases of Ebola, the total number of deaths due to Ebola, the number of confirmed, probable, and suspected cases, and the number of cases per country for Guinea, Liberia, and Sierra Leone.

SCENARIO DESCRIPTION

Ebola virus disease or EVD is a potentially fatal disease affecting humans. The virus is thought to be carried by animals such as fruit bats and was discovered in 1976 when the first human outbreaks occurred [1]. The World Health Organization (WHO) recognizes March 2014 to be the start of the worst Ebola outbreak in history. The outbreak was primarily contained to West Africa, beginning in Guinea and spreading to Liberia and Sierra Leone [1].

The outbreak and swift spread of Ebola in West Africa sparked fears worldwide for its potential to reach other regions. In September 2014, the Center for Disease Control (CDC) cited model predictions for the number of Ebola cases that reached as high as 550,000 cases by January 20, 2015 in Liberia and Sierra Leone alone [2]. When the model was corrected for under reporting, the number of cases jumped to 1.4 million. At the time of the prediction, Liberia and Sierra Leone combined for 2,407 cases while the model fairly accurately estimated 2,618 cases [2].

In a January 2016 news release, the World Health Organization officially declared the Ebola outbreak over for all three countries [3] (although few flare-ups resulted in 14 additional cases since that declaration). Because we have the data for the actual epidemic, we now know that the predictions by the CDC vastly overestimated the impact of the outbreak. We will investigate our own models of the epidemic based on the data that we now have.

We will use data published by the WHO ([4], [5]) to determine parameters for simple models that describe the outbreak. The data for the total number of Ebola cases includes cases confirmed by test results as well as suspected cases (those showing symptoms resistant to treatment) and probable cases (suspected cases that have been seen by a clinician). Also note that the data provided for the total number of cases is a running total of the number of cases which includes those patients who may have since recovered or died from the disease. The data for the individual countries and the number of deaths due to Ebola is also included to provide the opportunity for additional modeling and comparisons.

Part I: Exponential Growth Model

Although some cases of Ebola were confirmed in countries such as Nigeria, Senegal, and Mali, the outbreak primarily occurred in Guinea, Liberia, and Sierra Leone. Thus, as we study the Ebola outbreak in West Africa, we will consider only the data from these three countries.

1. Construct a model.
   - Suppose the number of cases of persons in West Africa infected with the Ebola virus grows at a rate proportional to the total number of cases. Write a differential equation to represent the situation described above. Use N for number of cases of Ebola and t for the time in months.
   - In March 2014, there were 49 cases and after three months, the number of cases had grown to 528. Solve the differential equation from above and use the given data to find an expression for the total number of Ebola cases, $N(t)$.
2. Interpret and verify the model.
   - Use your exponential growth model to estimate the time it will take for the outbreak to reach a total number of 5,000 cases.
   - Comment on how well your answer to the previous part matches the data in Table .
   - Use your model to estimate the number of cases after 5 months, 7 months, and 10 months.
   - Compare the predicted values from above to the observed values in Table .
   - Remark on whether you expect this model be accurate for the duration of the epidemic.
3. Further examine the scenario.

- Recall the predictions noted by the CDC for Liberia and Sierra Leone for January 2015. How do your prediction and the actual data for all three countries compare to the prediction of the CDC?
- Consider factors that may affect the behavior of the epidemic and its growth. Describe at least four such factors and identify whether the influences are environmental, economic, social, cultural, political, or other.
- Identify the inconsistency in the data for Liberia in Table . Speculate reason(s) for the early discrepancy.
- Describe the behavior you may expect for the epidemic as time progresses. Sketch a rough graph to illustrate your expectations. While the horizontal axis of your graph is time, what does the vertical axis represent?

4. Reflect on initial results.

   - Give a brief summary of your findings from Part I including strengths and weaknesses of using exponential growth to model the Ebola virus outbreak in West African countries. Consider the initial assumption that the virus spreads at a rate proportional to the number of infected individuals and address why this assumption is reasonable and include its faults, if any.

Part II: Limited Growth Model

From the explorations in the previous section, you likely realized that an exponential model is not appropriate for predicting the behavior of the epidemic beyond the first few months after the initial outbreak. A limited growth model can be used to more accurately describe the total number cases of Ebola in West Africa.

1. Limiting the spread of the disease.

   - Suppose the countries of Guinea, Liberia, and Sierra Leone are quarantined so that no person is allowed to enter or leave the countries. What would be an upper bound on the number of total cases of Ebola? Are you making additional assumptions about the disease when determining this upper bound?
   - List a few other factors that could limit the total number of individuals who may become infected with the disease during the outbreak.

One model that incorporates limited growth is the logistic or Verhulst model:

$$\frac{dN}{dt} = kN\left(1 - \frac{N}{M}\right). \tag{1}$$

In the logistic growth model (1), k is the *intrinsic growth* rate while M is referred to as the *carrying capacity.* This model is often associated with population growth, however it is also sometimes used to reflect the spread of contagious diseases.

Date	t (months)	Total Cases	Total Deaths	Cases in Guinea	Cases in Liberia	Cases in Sierra Leone
3/22/2014	0	49	29	49	-	-
4/14/2014	1	194	121	197	27	-
5/12/2014	2	260	182	248	12	-
6/16/2014	3	528	337	398	33	97
7/14/2014	4	982	613	411	174	397
8/13/2014	5	2,115	1,144	519	786	810
9/14/2014	6	5,335	2,662	942	2,710	1,673
10/14/2014	7	9,191	4,546	1,472	4,249	3,252
11/11/2014	8	14,383	5,165	1,919	6,878	5,586
12/14/2014	9	18,569	6,900	2,416	7,797	8,356
1/11/2015	10	21,261	8,414	2,806	8,331	10,124

Table 1. Total number of Ebola cases in West Africa by date during the first ten months of the outbreak. Also included are the number of deaths as well as cases reported by individual countries. All case totals are cumulative and do not reflect patient recovery or death. Data retrieved from the World Health Organization Disease Outbreak News [4] (March 2014-August 2014) and Situation Reports [5] (September 2014-January 2015).

2. Basic properties of the logistic growth model.
   - Based on the differential equation (1) above, for what intervals of N will the function $N(t)$ be increasing? Decreasing?

     For the given situation and interpretation of N, is it realistic for N to decrease?
   - When the number of cases N is close to the carrying capacity M, what can you say about the rate (dN/dt) at which the disease is spreading? Write your answer in words and mathematically as a limit.
   - How does your upper bound for the total number of cases found in Part I relate to M in the differential equation (1)? Which value will be greater?
   - When the number of cases is small, what can you say about the rate (dN/dt) at which the disease is spreading? Compare to the exponential growth model from Part I.

From the last observation and our work from the previous section, we can conclude that in the early months after the outbreak, the spread of the Ebola virus is exponential. Thus, we may use the constant of proportionality determined in Part I for the parameter k here.

However, we know that the outbreak of Ebola in West Africa did not reach the entire populations of the three countries discussed here. Thus, to develop a model with better accuracy, we must determine a more realistic value for M. To do so, we continue investigating the properties of the logistic growth model.

In order to demonstrate limited growth, or a maximum number of total cases, the growth rate must transition from a rate that increases to a decreasing rate. That is, the number of new cases per month must begin to decline. This transition occurs at an inflection point on the graph of $N(t)$.

3. Inflection point and carrying capacity in the logistic growth model.
   - Use a plot of the data from Table or compute the average growth rate from month to month to determine the coordinates (t, N) for the inflection point in the data.
   - Use implicit differentiation and substitution to determine $\dfrac{d^2N}{dt^2}$. Determine the value(s) of N at which the inflection point(s) occur.
   - Combine the previous two parts to give an estimate for the carrying capacity, M.
4. Expression for $N(t)$.
   - Solve the separable differential equation (1).
   - Use your growth rate from the exponential model for k and the carrying capacity from above for M to write $N(t)$ representing the total number of cases of Ebola in West Africa at time t months. Graph $N(t)$ for $t = 0, 1, 2, \dots, 24$.
   - Compare the graph of $N(t)$ to your sketch (in the previous section on exponential growth) depicting the behavior of the epidemic that you expect.
5. Model predictions and verification.
   - Use your equation for $N(t)$ to estimate the total number of cases of Ebola for times $t = 5, 7,$ and 10 months.
   - Compare the predicted values from the previous part to the data in Table and to the results from the exponential model in Part I.
   - Remark on the performance of the logistic model when used to depict the Ebola outbreak during its first ten months. State your expectations for the performance and accuracy of the model for the duration of the epidemic.
   - Compare the results of the logistic growth model to the data reported by the World Health Organization for $1 \leq t \leq 24$ in Tables and 5. Show comparisons using both a table and a graph.
6. Conclusion.
   - Give a brief summary of your findings from Part II including strengths and weaknesses of using logistic growth to model the Ebola virus outbreak in West African countries. Consider the impact of the carrying capacity on the accuracy of the logistic model and address whether it is reasonable to develop and use this model during an outbreak, in particular early on before the inflection point appears in the data.

Date	t (months)	Total Cases	Total Deaths	Cases in Guinea	Cases in Liberia	Cases in Sierra Leone
2/15/2015	11	23,218	9,365	3,108	9,007	11,103
3/15/2015	12	24,666	10,179	3,389	9,526	11,751
4/12/2015	13	25,791	10,689	3,548	10,042	12,201
5/10/2015	14	26,724	11,065	3,597	10,604	12,523
6/14/2015	15	27,305	11,169	3,674	10,666	12,965
7/12/2015	16	27,642	11,261	3,760	10,673	13,209
8/16/2015	17	27,952	11,284	3,786	10,672	13,494
9/13/2015	18	28,220	11,291	3,792	10,672	13,756
10/11/2015	19	28,454	11,297	3,800	10,672	13,982
11/15/2015	20	28,598	11,299	3,804	10,672	14,122
12/13/2015	21	28,604	11,300	3,807	10,675	14,122
1/17/2016	22	28,602	11,301	3,804	10,675	14,123
2/14/2016	23	28,603	11,301	3,804	10,675	14,124
3/13/2016	24	28,603	11,301	3,804	10,675	14,124

Table 2. Total number of Ebola cases in West Africa by date during the second year of the epidemic. Also included are the number of deaths as well as cases reported by individual countries. All case totals are cumulative and do not reflect patient recovery or death. Data retrieved from the World Health Organization Situation Reports [5] (February 2015-March 2016).

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