

Supplemental Maple worksheet for SIMIODE Project "Modeling Bubbles of Beer"
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Implement DE model of beer bubble rising.

> restart;
 with(plots) :

First equation from equation set (3) in paper. Here $n(t)$ is moles of gas in bubble.

$$\begin{aligned} > \text{eqn} := \text{diff}(n(t), t) = 4 \cdot \text{Pi} \cdot K \cdot \left(\frac{3 \cdot n(t) \cdot R \cdot T}{4 \cdot \text{Pi} \cdot (P - g \cdot \rho[\text{beer}] \cdot x(t))} \right)^{\frac{2}{3}}; \\ \text{eqn} := \frac{d}{dt} n(t) = \pi K 3^{2/3} 4^{1/3} \left(\frac{n(t) R T}{\pi (P - g \rho_{\text{beer}} x(t))} \right)^{2/3} \end{aligned} \quad (1)$$

Second equation (really a split on a second order DE for $x(t)$, the position (depth) of the bubble, $x = 0$ is surface.

$$\begin{aligned} > \text{eqx} := \text{diff}(x(t), t) = y(t); \\ \text{eqx} := \frac{d}{dt} x(t) = y(t) \end{aligned} \quad (2)$$

Third equation, coming from $F = ma$. Two versions, one with resistance proportional to radius, one for cross-sectional area.

$$\begin{aligned} > \text{eqy} := \text{diff}(y(t), t) = g \cdot \left(\frac{\rho[\text{beer}] \cdot R \cdot T}{m\text{co}2 \cdot (P - g \cdot \rho[\text{beer}] \cdot x(t))} - 1 \right) - \frac{d \cdot y(t)}{m\text{co}2 \cdot n(t)} \\ \cdot \left(\frac{3 \cdot n(t) \cdot R \cdot T}{4 \cdot \text{Pi} \cdot (P - g \cdot \rho[\text{beer}] \cdot x(t))} \right)^{\frac{1}{3}}; \# \text{Draw proportional to radius} \\ \# \text{eqy} := \text{diff}(y(t), t) = g \cdot \left(\frac{\rho[\text{beer}] \cdot R \cdot T}{m\text{co}2 \cdot (P - g \cdot \rho[\text{beer}] \cdot x(t))} - 1 \right) - \frac{d^2 \cdot y(t)}{m\text{co}2 \cdot n(t)} \\ \cdot \left(\frac{3 \cdot n(t) \cdot R \cdot T}{4 \cdot \text{Pi} \cdot (P - g \cdot \rho[\text{beer}] \cdot x(t))} \right)^{\frac{2}{3}}; \# \text{Drag proportional to cross section area} \\ \text{eqy} := \frac{d}{dt} y(t) = g \left(\frac{\rho_{\text{beer}} R T}{m\text{co}2 (P - g \rho_{\text{beer}} x(t))} - 1 \right) \\ - \frac{1}{4} \frac{d y(t) 3^{1/3} 4^{2/3} \left(\frac{n(t) R T}{\pi (P - g \rho_{\text{beer}} x(t))} \right)^{1/3}}{m\text{co}2 n(t)} \end{aligned} \quad (3)$$

Vital parameters from paper.

> P := 1.013e5;
 R := 8.3145;
 mco2 := 4.401e-2;
 rho[beer] := 1.01e3;
 eta := 1.3e-3;
 T := 281.0;
 g := 9.807;

```

#d := 4·Pi·eta; #Fudge factor in paper
d := 0.068; #For drag proportional to radius
d2 := 260.0 #For drag proportional to area

P := 1.013 105
R := 8.3145
mco2 := 0.04401
ρbeer := 1010.
η := 0.0013
T := 281.0
g := 9.807
d := 0.068
d2 := 260.0

```

(4)

Initial conditions for bubble.

```

> x0 := -0.156;
y0 := 0;
r0 := 1.7e-4;

x0 := -0.156
y0 := 0
r0 := 0.00017

```

(5)

Derived on p. 417 of paper.

```

> n0 := evalf( ( 4·Pi·r03·(P - g·ρ[beer]·x0) ) / ( 3·R·T ) )
n0 := 9.058930018 10-10

```

(6)

We guess at K.

```

> #K := 7.5e-3;
K := 2e-3;

K := 0.002

```

(7)

Numerical solution of the ODE system.

```

> sol := dsolve( {eqn, eqx, eqy, n(0) = n0, x(0) = x0, y(0) = y0}, {n(t), x(t), y(t)}, numeric,
maxfun = 108);
sol := proc(x_rkf45) ... end proc

```

(8)

Max time for solver

```

> Tf := 3.78;

Tf := 3.78

```

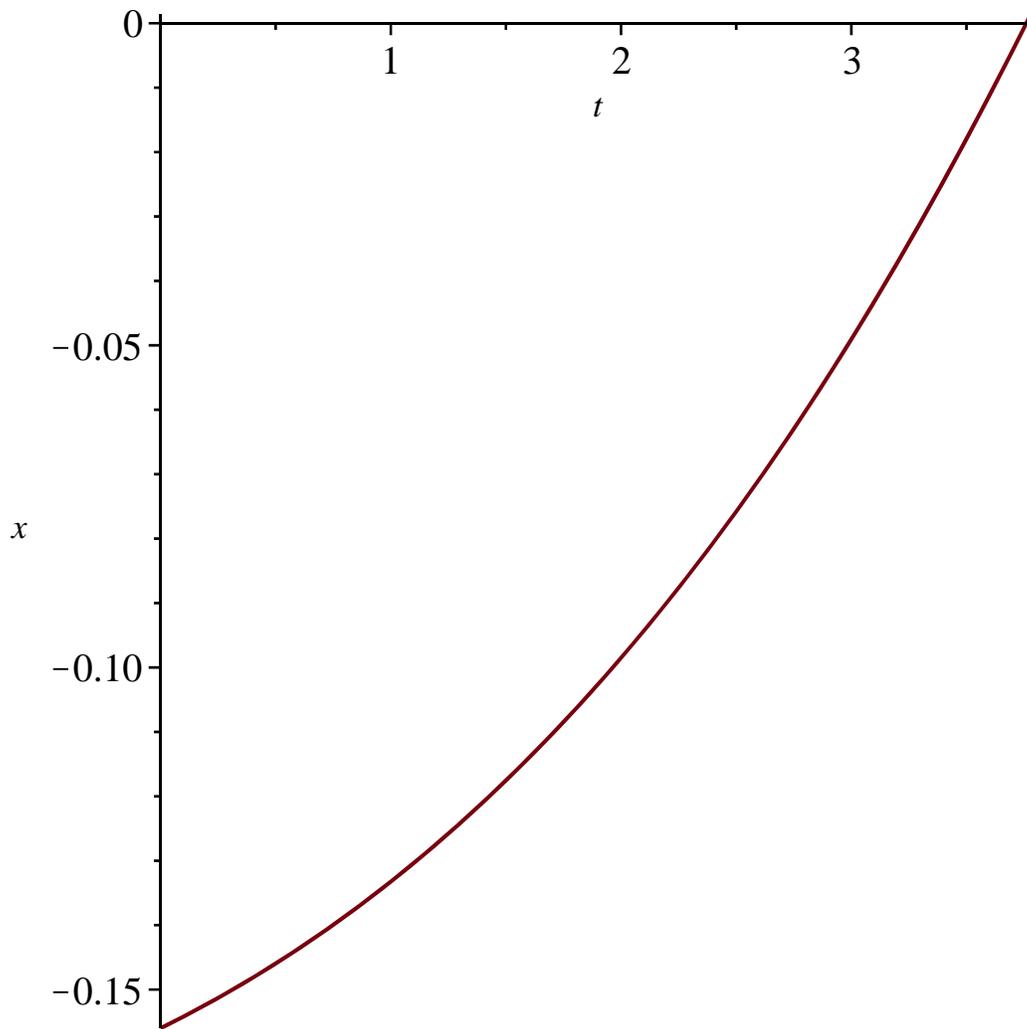
(9)

Plot the position

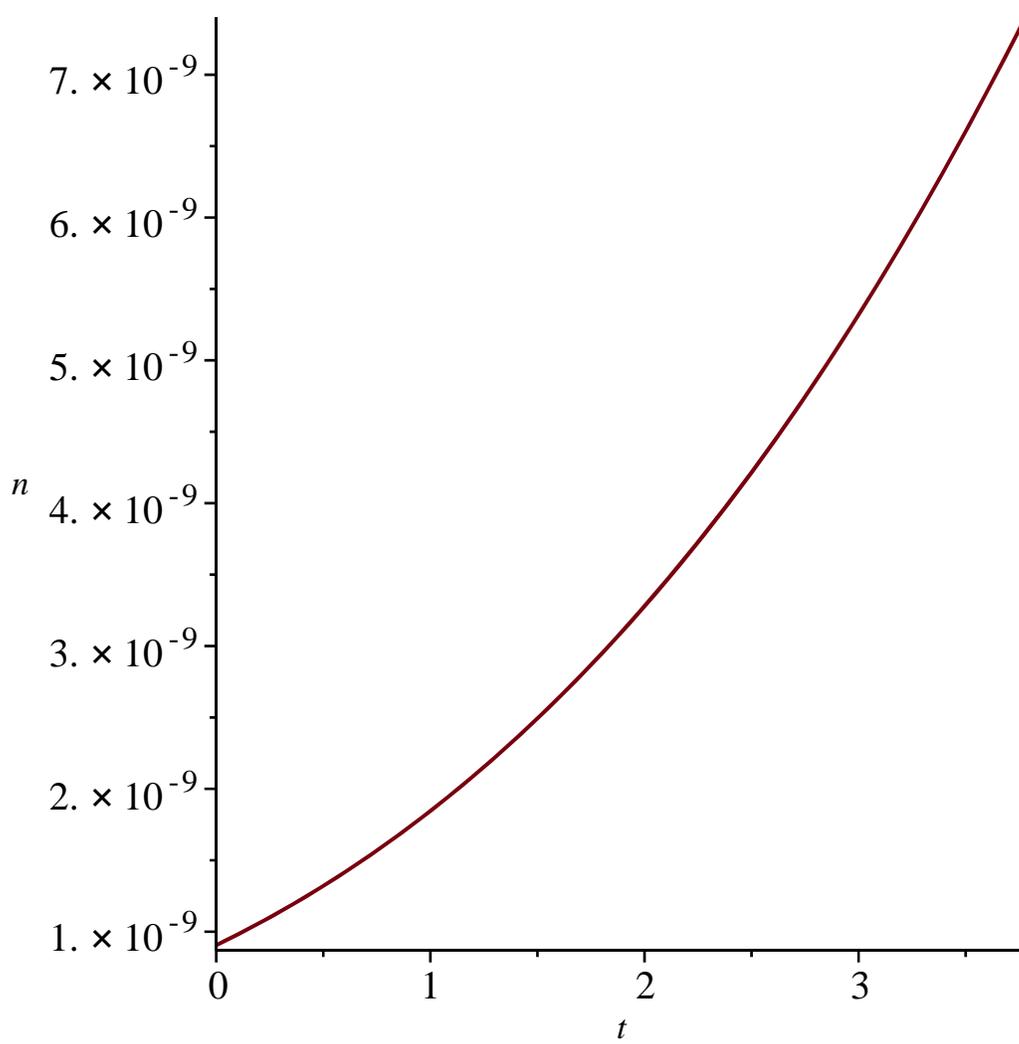
```

> p1 := odeplot(sol, [t, x(t)], t=0..Tf)

```



And the bubble molar volume
> `odeplot(sol, [t, n(t)], t=0..Tf)`



```

Compute bubble radius change
> n1 := subs(sol(Tf), n(t)) :
      (n1/n0)^(1/3)

```

2.01428032936266