Checking the Beer Paper with Mathematica

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Run this command once per experiment to clear out model solution variables stored in memory.

In[1]:=
```
Clear[solution, bubbledata]
```

Enter parameters in model.

In[2]:=
```
ρbeer = 1.010 * 10^3;
g = 9.807;
R = 8.3145;
P = 1.013 * 10^5;
mCO2 = 4.401 * 10^-2;
η = 1.3 * 10^-3;
T = 281;
```

Solve the system of ordinary differential equations (ODE) with initial data numerically.

In[9]:=
```
solution[k_] := NDSolve[
  {n'[t] == 4 * π * k * (3 * n[t] * R * T / (4 * π * (P - g * ρbeer * x[t])))^(2/3),
   x'[t] == y[t],
   y'[t] == g * (ρbeer * R * T / (mCO2 * (P - g * ρbeer * x[t])) - 1) -
     (4 * π * η * y[t] / (mCO2 * n[t])) * (3 * n[t] * R * T / (4 * π * (P - g * ρbeer * x[t])))^(1/3),
   x[0] == -0.156,
   y[0] == 0,
   n[0] == 10^-10},
  {x, y, n}, {t, 5}];
```

Use the solution to the system of ODE to find bubble position and radius at each time increment and display bubble data in a table.

In[10]:=
$$r[t_] := \left(\frac{3 * n[t] * R * T}{4 * \pi * (P - g * \rho_{beer} * x[t])}\right)^{1/3};$$

In[11]:=
```
bubbledata[k_] :=
  Table[Evaluate[{t, 100 * x[t], r[t] / r[0]} /. solution[k][[1]]], {t, 0, 3.78, 0.54}];
```

In[12]:=
```
TableForm[bubbledata[0.0075],
 TableHeadings → {None, {"time (sec)", "depth (cm)", "radius/initial radius"}}]
```

Out[12]//TableForm=

time (sec)	depth (cm)	radius / initial radius
0.	−15.6	1.
0.54	−13.2741	2.13045
1.08	−6.5433	3.27041
1.62	7.04505	4.43269
2.16	30.1908	5.63737
2.7	66.0876	6.91556
3.24	118.869	8.31778
3.78	194.509	9.93474