Project 1: Modeling a Pandemic

Math 238

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Allie Bellair, Carmen Ngo, Tal Pezzuco, Enrique Roman

1. Equations:

$$\frac{dS}{dt} =- aSI\,,\ \frac{dI}{dt} = aSI - rI$$

   a. According to the differential equation for dI/dt, the number of infected individuals I(t) will be increasing/decreasing in accordance to the transmission rate *a*, the recovery rate *r* and the fraction of susceptible individuals *S(t)*. I(t) will increase if *a*S(t)* is greater than *r*. In other words the fraction of infected individuals increases if the number of infected individuals and the rate at which they transmit the disease is much greater than the recovery rate. I(t) will decrease if the recovery rate is much greater than the transmitting rate, meaning that it will decrease if the number of infected individuals decreases. The spread of the disease can decrease if the rate of contact and probability of infection is decreased or limited. Limiting the rate of contact will decrease the probability of infection which ultimately reduces the fraction of infected individuals. This is essentially what quarantine is doing, it is decreasing the transmission rate by limiting the rate of contact.

   b. Use the chain rule to show $\frac{dI}{dS}$

$$\frac{dI}{dS} = \frac{dI}{dt}\left(\frac{dS}{dt}\right)^{-1} = \frac{aSI-rI}{-aSI} =- \left(1 - \frac{r}{aS}\right) = -1 + \frac{1}{R_0 S}$$

   where the reproduction number, $R_0 = \frac{a}{r}$

   Compute $\frac{d^2I}{dS^2}$. Determine when the number of infected will begin to decrease. Compare this to the solution from a.

$$\frac{d^2I}{dS^2} = \frac{R_0S*0-R_0}{R_0{}^2S^2} = \frac{-R_0}{R_0{}^2S^2} = -\frac{1}{R_0S^2}$$

This means that the number of infected will begin to decrease when the fraction of susceptible individuals increases. In part a. The number of infected increased when the fraction of the susceptible individuals decreased.

c. Show the form of I

$$I = \int -1 + \frac{1}{R_0S}\ dS = \int -1\ dS + \int \frac{1}{R_0}S^{-1}\ dS = -S + \frac{1}{R_0}ln(S)\ + C$$

where C is a constant.

d. From our answer in 1 c.

$$I = -S + \frac{1}{R_0}ln(S)\ + C \rightarrow \text{I + S - C} = \frac{1}{R_0}ln(S) \rightarrow \frac{I+S-C}{ln(S)} = \frac{1}{R_0} \rightarrow R_0 = \frac{lnS}{I+S-C}$$

From the given conditions, I(0) is approximately 0 and S(0) is approximately 1, at t=0.

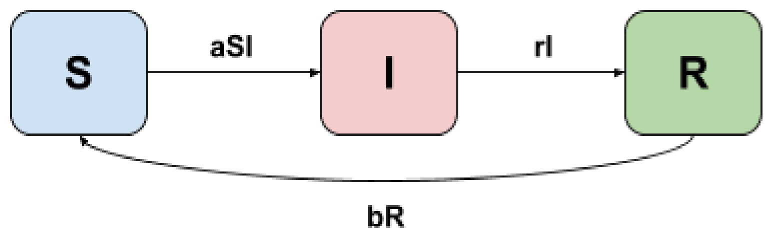
$$I(t) = -S(t) + \frac{1}{R_0}ln(S(t))\ + C \rightarrow I(0) = \ -S(0) + \frac{1}{R_0}ln(S(0))\ + C$$

$\rightarrow$ 0= -1+$\frac{1}{R_0}ln(1) + C \rightarrow$ C=1

And since I(∞) is approximately 0, $R_0 = \frac{lnS_\infty}{S_\infty - 1}$.

e. Herd immunity is when a large portion of a population becomes immune to an infectious disease which can protect the portion of people that are not immune because it will reduce the number of susceptible hosts to a level less than what the threshold is. Vaccination can lead to herd immunity because if enough people get vaccinated they will become immune to the disease, which will lower the chances of infection of the unvaccinated population a.k.a the definition of herd immunity.

2. The new schematic is shown below when people are reinfected.

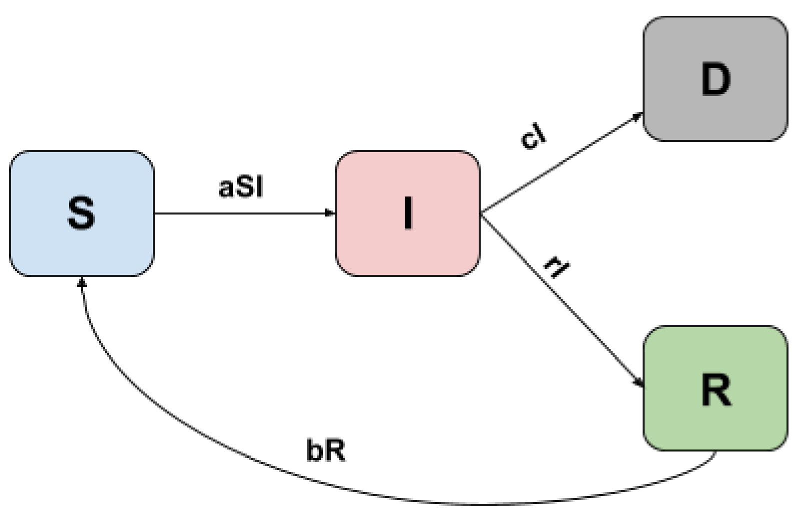


$$S'(t) = -aSI + bR$$

$$I'(t) = aSI - rI$$

$$R'(t) = rI - bR$$

3. The schematic when deaths are taken into consideration is shown below.



a. The following equations can be used when death is taken into consideration.

$$S'(t) = -aSI + bR$$

$$I'(t) = aSI - rI - cI$$

$$R'(t) = rI - bR$$

$$D'(t) = cI$$

b. The following is the data obtained from a study of Ebola completed in 2014 for Sierra Leone. The table below presents the parameters obtained using the gathered information.

parameter	calculation	value
R_0	-	1.63
r	1/average time to recover = 1/10.4	0.09615
a	R_0*r	0.15673
b	-	0
c	1/days from infected to death = 1/6.2	0.16129
S(0)	susceptible/total population	0.99932178994
I(0)	infected/total population	0.00067821006
R(0)=D(0)	0	0
Fatality rate	-	43%

c. https://www.geogebra.org/classic/jrm7t9zs

At 419 days, the fraction of infected individuals reaches 0, the fraction of susceptible individuals stabilizes to 0.6137, the fraction of recovered individuals stabilizes to 0.1705, and the fraction of individuals who have passed away stabilizes to 0.2157.

According to the CDC, “The dramatic variation in number of cases reported from the end of October to the beginning of November 2014 is due to a change in the data sources used. Prior to October 2014, the cumulative total numbers were derived from a combination of patient databases and country situation reports. Later, the revised approach used numbers compiled by the Ministries of Health and WHO country offices.” For this reason, we decided to take t = 0 to be day 160/610 (November 5, 2014 - the first day of November when the new numbers were derived) with 4759 cases instead of day 0 with 1 case on May 27, 2014. The total population of Sierra Leone at this time was 7.017 million. So, the “initial” fraction of infected individuals I0 was calculated to be 0.0007 and as a result the fraction of susceptible individuals S(0) was 0.9993.

To justify our parameter settings, the basic reproduction number R_0 , death rate c, and average time to recover r, was found from a study conducted in the Pujehun District, in the Southern Province of Sierra Leone. R_0 was found to be 1.63, c was calculated to be 1/6.2 = 0.1612903226 by taking the inverse of the average number of days of symptom onset to death in hospital: 6.6 days and symptom onset to death in community: 5.8 days, and r was calculated to be 1/10.4 = 0.09615384615 using the fact that the days of symptom onset to hospital discharge: 10.4 days. The transmission rate a was calculated using the formula $R_0 = \frac{a}{r}$, where a = R_0 *r = 0.1567307692.

The first observation that comes from the behavior of the solutions is that the fatality rate is not 43% as researched, but rather ~22%. This might be explained by the parameter b representing re-infection at a rate proportional to the population of recovered individuals being set to 0 due to the fact that according to the CDC, "In most cases, people who have completely recovered from EVD do not become reinfected."

To continue comparing our results to the real data, it took 610 days from May 24, 2014 to January 27, 2016 for there to be 0 new cases. In our results, it took 419+160 (initial starting day) = 579 days for the fraction of infected individuals to reach 0, which is a 31 day discrepancy. The number of days it took for there to be 0 new cases declared the end of the Ebola outbreak in Sierra Leone and so the fraction of people susceptible at the end is (total population-total cases)/total population = (7017000-14124)/7017000 = 0.997987174. In our results, we found the fraction of susceptible individuals on the same day the fraction of infected individuals reached 0 to be 0.6137, which is a notable difference. It took 603 days from May 24, 2014 to January 20, 2016, which is 7 days before the date of 0 new cases, for the number of deaths to stabilize. In our results, we matched the fraction of deaths, 0.2157, to 419-7 (January 27th-7 days) = 413 days and found the first instance of that fraction at 399+160 = 559 days, which is a 44 day discrepancy. The fraction of recovered individuals is (total cases-total deaths)/total population = (14124-3956)/7017000 = 0.0014490523, which is a lot less than the predicted 0.1705. This could also be due to the reinfection rate being set to 0. These discrepancies could also be due to interventions taken in the real situation that are not considered in the model such as increased testing or government decisions.

References

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